

Simulation of the behavior of the flow of electrons between the electrodes of a thermionic converter

Simulación del comportamiento del flujo de electrones entre los electrodos de un convertidor termoiónico

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Abstract

Thermionic converters have been shown to be an alternative to generate electrical power from thermal energy, however, efficiency is reduced by its internal impedance. This impedance has not been possible to suppress because thermionic converters require an ionized cesium atmosphere to transport the electrons. Different authors, have proposed alternatives to reduce this impedance, however, this continues being the main limitation to improve the efficiency of thermionic converters. Recently Fitzpatrick et. al have been working with cesium diodes of near spacing which have shown the possibility of decreasing this impedance when working with thermionic converters in Knudsen mode. In this paper, a theoretical analysis of high vacuum thermionic converters based on the fundamental laws of electronic emission is presented. The most important result was the explanation of the curve I vs V of a thermionic diode from its birth to its saturation. And the development of the algorithms that allow the use of the equations that predominantly explain the emission of electrons.

Resumen

Los convertidores termoiónicos han mostrado ser una alternativa para generar potencia eléctrica a partir de la energía térmica, sin embargo, la eficiencia se ve reducida por su impedancia interna. Esta impedancia no ha sido posible suprimirla debido a que los convertidores termoiónicos requieren de una atmósfera de cesio ionizado para transportar los electrones. Diferentes autores, han propuesto alternativas para reducir esta impedancia, sin embargo, continúa siendo ésta la principal limitante para mejorar la eficiencia de los convertidores termoiónicos. Recientemente Fitzpatrick et. al han estado trabajando con diodos de cesio de espaciamiento cercano los cuales han mostrado la posibilidad de disminuir esta impedancia cuando se trabaja con convertidores termoiónicos en el modo Knudsen. En este trabajo se presenta un análisis teórico sobre los convertidores termoiónicos de alto vacío a partir de las leyes fundamentales de emisión electrónica. Se obtuvo como resultado más importante la explicación de la curva I vs V de un diodo termoiónico desde su nacimiento hasta su saturación. Y el desarrollo de los algoritmos que permiten el empleo de las ecuaciones que de manera predominante explican la emisión de electrones.

Simulation, Thermionic, Convert

Simulación, Termoiónico, Convertidor

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Introduction

More than two hundred years ago Du Fay observed that the space surrounding an incandescent body is a conductor of electricity. In 1853 Edmon Becquerel devoted an article to this topic. One of the observations he published was that a potential of a few volts was sufficient to make current flow into a galvanometer that closed the circuit with the hot air between the red-heated platinum electrodes.

Between the years 1882 to 1889 Elster and Geitel worked on a sealed device containing two electrodes, one that could be heated and one that could be cooled; they noted by means of an electrometer connected to the cold electrode that charge flowed from the hot electrode to the cold electrode; they also found that at relatively low temperatures the current passed more easily if the hot filament was positively charged. At moderately high temperature; they noted that; charge could be transferred just as easily from the hot filament to the cold filament as from the cold filament to the hot filament. At higher temperature they noted that the negatively charged filament predominated.

In 1883 in a patent application, Thomas Alva Edison stated that he observed the thermionic effect in a vacuum. Seven years later, Preece and Fleming demonstrated that the thermionic effect was due to electric flux leaving the main filament, passing through the vacuum, and collecting at the relatively positive electrode.

The nature of the negative charge carriers was determined until 1899 by J.J. Thomson, who found that the ratio of charge to mass agreed with the value he found for the electron.

Basically the thermionic effect consists of the emission of electrons that are generated in a hot electrode with respect to a cold electrode that collects it. The former is called emitter and the latter collector. According to reports, this phenomenon occurs in materials at temperatures above 1000 K.

To explain this phenomenon, it is necessary to be clear about the concepts of Fermi level and work function.

The Fermi level of a material is defined as the maximum energy level that the free electrons can have when the material is at an absolute temperature of zero degrees. The free electrons are distributed in the energy states below the Fermi level from a minimum energy level.

On the other hand, the work function is defined as the energy required to release an electron that is confined within the material. This energy basically overcomes the Coulombian interaction generated by the effect of the image charge in the material due to the absence of the electron.

Taking these two concepts into consideration, when the temperature of a material is raised, it is capable of yielding thermal energy to the electrons; when this is greater than or equal to the work function, the electrons will be free, that is, they will be outside the material. By external factors, energy can be given up to the emitted electrons so that they can move through the vacuum to another material. When two plates of different materials separated by a certain distance are used in which the emitter has a higher work function than the collector, then the electrons that are received in the latter will have a higher energy than the former. This generates a surplus of energy which is translated as electromotive force capable of powering a charge in the system. The potential in the charge is generated by the energy difference in the Fermi levels of the emitter and collector.

Ideally, it is required that the electron flow does not lose energy during its path, however, in practice, energy losses occur in the interelectronic region, as well as when the flow impinges on the collector.

The emission of electrons in the absence of an external electric field is explained by the Richardson-Dushman equation. On the other hand, the behavior of the electron flow in the interelectronic region is obtained from the Langmuir - Child law. This equation determines the current density as a function of the applied electric field and the distance between the electrodes. Furthermore, considering the effect of the applied field on the confining surface potential barrier of the material, it is found that the Schottky equation correctly describes this phenomenon.

Electronic valves and thermionic converters present similar I - V curves that can be explained from their origin to their saturation by means of these equations.

Figure 1 shows a cesium thermionic converter whose emitter and collector are separated by an insulator; the separation distance between the two electrodes is in the order of tenths of a millimeter.

The converter housing is hermetically sealed, so the internal atmosphere can be controlled. In the conventional converter, the interelectrode space is filled with cesium vapor, and the cesium working pressure is of the order of 1 torr.

Cesium, in thermionic converters, performs two basic functions; the first is that it adsorbs on the electrode surfaces to improve their emission capacity, and the second is to generate positive ions on the surface of the emitter with which it is possible to neutralize the electron space charge generated by heating in the space adjacent to the surface of the emitter. The elimination of this space charge and the ease of electron transport make it possible to obtain practical current densities.

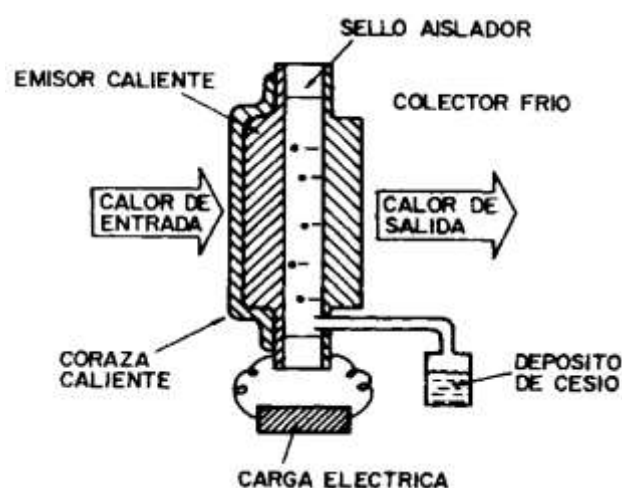


Figure 1 Thermionic converter

The thermionic converter, from the thermodynamic point of view, is a heat engine that uses the electron gas as working fluid, therefore, its efficiency cannot exceed that established by the Carnot cycle. The temperature difference between emitter and collector pulls the electrons through the system charge.

For a given set of electrodes, the output power of the thermionic converter is a function of the collector and emitter temperature, the interelectrode spacing and the pressure of the cesium atmosphere. Usually, thermionic converters operate at high temperature. Typically, the emitter temperature is in the range of 1600 to 2400K, and the collector temperature varies from 800 to 1100K, thus making it possible to obtain current densities ranging from 5 to 10 A/cm² at an output potential of the order of 0.5V. The efficiency of this type of converters to generate electricity from heat varies between 10 and 15%.

The objective of the present work is: to simulate the behavior of the electrons emitted between the electrodes of a thermionic converter, in order to find the emission capacity of a thermionic converter as a function of the parameters involved.

The simulation of the electron flow behavior will show the emission capacity of a thermionic converter, as a function of the main parameters involved.

1. Basic theory of electron emission

It is known that a metal contains a large number of free electrons in its interior, which move easily in the presence of applied electric fields, even if these are of low intensity. However, when the electrons approach the surface, they cannot escape due to the surface confinement potential which can be modified by external factors. The following is a brief review of the theories that have been developed to explain the phenomenon of electron emission in a thermionic converter.

1.1 I - V characteristics of a vacuum thermionic diode

The vacuum diode is an electronic device with two terminals, which are connected to the emitter and collector. The intermediate space between emitter and collector can be vacuum (high vacuum diodes) or cesium plasma (cesium diodes). The emitter is characterized because it emits electrons when it is at a high temperature, when an external voltage is applied between it and the collector, the current flowing through the device behaves according to the curve shown in Figure 1.1, which is typical of an experimental high vacuum thermionic diode.

It can be reproduced from the fundamental equations explaining thermionic emission.

Figure 1.2 (a) shows the electrostatic potential profile between the electrodes of the thermionic diode. Here, $E_F(\varepsilon)$ is the Fermi level of the emitter, $E_F(c)$ that of the collector and ϕE and ϕC their corresponding work functions. V is the voltage

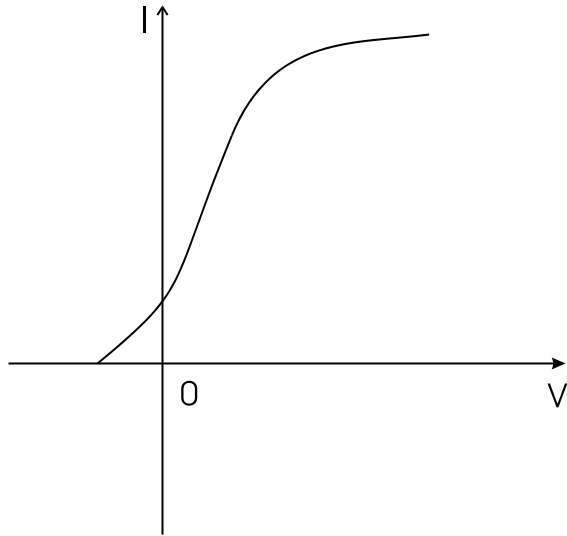


Figure 1.1 Characteristic curve of an experimental thermionic diode

Of charge generated by the energy difference in the Fermi levels. Region (1) denotes the case of the ideal diode model that neglects the space charge effect which generates a potential barrier at the electrode surfaces that substantially affects the transit of the emitted electrons.

This effect is reduced by considering reduced spaces between emitter and collector. Region (2) represents the actual potential profile considering space charge effects. In (b), the thermionic current density versus output voltage is plotted for the ideal (1) and real (2) case of the diode. At V equal to zero is the closed-loop case (3), and at (4) the open-loop case ($J=0$). There are other methods to reduce the space charge effect that are not of interest to study for now. In practice it has not been possible to operate thermionic diodes at spacings smaller than 0.01 mm. For this range of values, the output power density is less than 1.0 W/cm². Using cathodes for the emitter (1538 K) and collector (810 K), G. N. Hatsopoulos obtained a current density of 1.0A/cm² at an output potential of 0.7V.

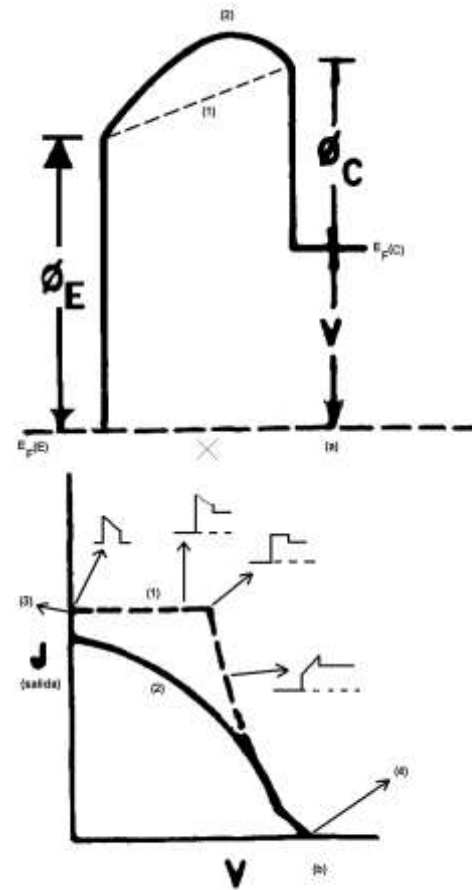


Figure 1.2 (a) Variation of the electrostatic potential profile (b) Thermionic current vs. output voltage

1.2 Description of electron emission on a hot metal surface (Richardson - Dushman Equation)

When the temperature of a metal increases, some of the electrons in the solid gain enough thermal energy to overcome the binding forces to free themselves from the solid, this process is called thermionic emission. As the temperature increases, the number of electrons emitted increases. Thermionic emission depends on the physical parameters of the solid, especially the work function ϕ and the temperature. Figure 1.3 (b) shows the energy distribution of electrons in a solid. When the x-component of the velocity of the electrons impinging on the surface is greater than,

$$v_{xc} = \frac{p_{xc}}{m} = \sqrt{\frac{2(E_F + \phi)}{m}}, \quad (1.1)$$

(where $E_F + \phi$ is the minimum energy required to be released from the solid).

The electrons will overcome the surface forces along the normal to the surface and will be emitted.

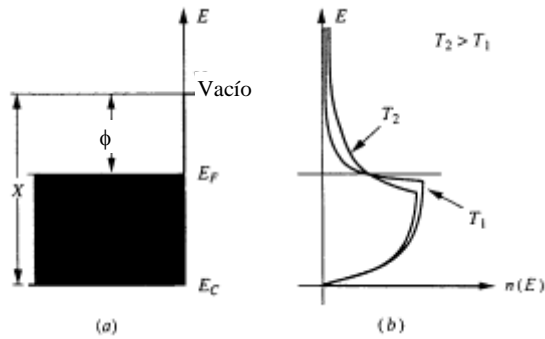


Figure 1.3 (a) Band diagrams of energies and (b) Electron energy distribution in a solid

The current density, thermally emitted in the x-direction is by definition equal to

$$dJ_{th} = ev_x n(v) dv \quad (1.2)$$

With $n(v)$ the electron velocity distribution. The total current density due to the contribution of all electrons is given by

$$J_{th} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{2m^3}{h^3} \frac{v_x dv_x dv_y dv_z}{1 + \exp\left[\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT} - \left(\frac{E_F}{kT}\right)\right]} \quad (1.3)$$

Since the energy required for the electrons to be emitted is greater than E_F , one can disregard the unit in the denominator and write equation (1.3) as

$$J_{th} \approx e \frac{2m^3}{h^3} \exp\left(\frac{E_F}{kT}\right) \int_{v_{x0}}^{\infty} \exp\left(-\frac{mv_x^2}{2kT}\right) v_x dv_x \int_{-\infty}^{\infty} \exp\left(-\frac{mv_y^2}{2kT}\right) dv_y \int_{-\infty}^{\infty} \exp\left(-\frac{mv_z^2}{2kT}\right) dv_z \quad (1.4)$$

Finally, equation (1.4) becomes:

$$J_{th} = \frac{4em\pi k^2 T^2}{h^3} \exp\left(-\frac{\phi}{kT}\right), \quad (1.5)$$

Yes A_0 is defined as:

$$A_0 = \frac{4\pi emk^2}{h^3} = 1.2 \times 10^6 \text{ A} / \text{m}^2 \text{K}^2 \\ = 120 \text{ A} / \text{cm}^2 \text{K}^2$$

Then J_{th} can be written as:

$$J_{th} = 120 T^2 e^{-\phi/kT} \text{ A} / \text{cm}^2 \quad (1.6)$$

Expression 1.6 is known as the Richardson-Dushman equation.

It is observed that, at a given temperature, a solid with a lower work function will emit more electrons than one with a higher one, there is also a strong dependence of the emission current density on temperature.

The constant $A_0 = 1.2 \times 10^6 \text{ A/cm}^2 \text{K}$, in general is different for real solids. The surface forces, the type of emitting material and the surface roughness of the material modify the value of A_0 . The temperature behavior given by equation 1.6 has been verified experimentally in many materials.

1.3 Behavior of electron flow subject to an electric field (Langmuir - Child Eq.)

The analysis of a diode is particularly simple when the cathode and anode are flat, parallel surfaces, with very little separation between them. In the following study, it is assumed that the electric field is perpendicular to the electrodes at all their points (thus neglecting edge scattering). It can be seen that the relationship between the current and the potential to be obtained also describes the behavior of devices whose geometrical shapes are more complex. Figure 1.4 shows schematically the geometrical shape of the diode to be analyzed.

Next, the behavior of electrons subjected to an external electric field is examined. Consider the cathode at a temperature T , such that there is an appreciable emission of electrons generating a charge density in the interelectronic space. The equation relating the electron density to the potential at any point in the interelectronic space is Poisson's equation,

$$\frac{d^2 V}{dx^2} = -\frac{\rho}{\epsilon_0} \quad (1.7)$$

Where V satisfies that at $x=0$, $V(0)=0$ and at $x=d$, $V(d)=V_a$.

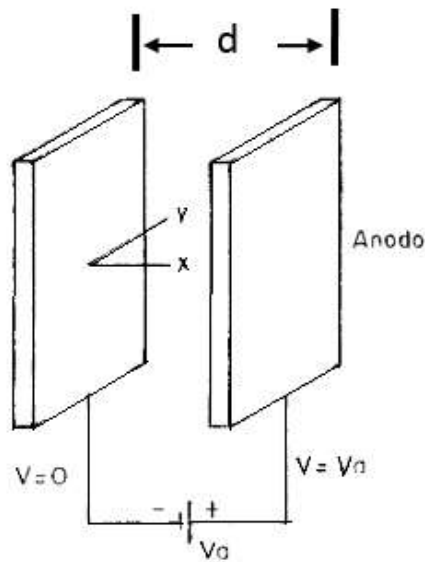


Figure 1.4 Potentials and geometrical aspects of the diode

To find a relationship between the current density flowing between cathode and anode, with the applied potential and the separation distance d , the basic relationship is evoked,

$$J = -Nev = -\rho v \quad (1.8)$$

On the other hand, at any point in the interelectronic space is satisfied:

$$eV(x) = \frac{1}{2}m[v(x)]^2 \quad (1.9)$$

Where $V(x)$ is the potential and $v(x)$ is the velocity. From equations (1.7), (1.8) and (1.9) we obtain

$$\frac{d^2V(x)}{dx^2} = \frac{-JV(x)^{-1/2}}{\left(2\frac{e}{m}\right)^{1/2}\epsilon_0} \quad (1.10)$$

Solving equation 1.10 for the current density J , it follows that:

$$J = 2.33 \times 10^{-6} \frac{V_a^{3/2}}{d^2} \quad (1.11)$$

Note that equation 1.11 depends only on the applied potential and the separation distance, ignoring the cathode temperature and work function. This result is known as the Langmuir - Child law or three-medium power law.

In practice, the current density values may be lower than those predicted by equation (1.11) because the electron emission at the cathode is limited by the cathode temperature mainly when the cathode temperature is low.

1.4 Schottky effect

The derivation of the thermionic emission equation is based only on the electrons that overcome the work function of the solid due to the thermal energy they have associated with them at a given temperature T . Some of the electrons that become free are forced out of the cathode and towards the anode by the application of an external potential between these two electrodes. At low potentials the characteristic electron emission does not change, but at high potentials, i.e. high electric fields, the current density by thermionic emission from the cathode changes significantly. This phenomenon is known as the Schottky effect.

The idealized potential barrier seen by an electron in the solid is shown by the dotted line in Figure 1.5.

When the electrons are free they move a distance x away from the solid generating a positive charge on the solid (image charge effect). The resulting Coulomb force between these two charges separated by a distance of $2x$ is

$$F_x = -\frac{e^2}{4\pi\epsilon_0(2x)^2} \quad (1.12)$$

and the associated potential energy is:

$$\Phi_s(x) = -\frac{e^2}{16\pi\epsilon_0 x} \quad (1.13)$$

If an external electric field directed towards the surface of the emitter is now considered, then the electron experiences an additional potential due to this applied field ϵ . The resulting potential at a distance x away from the surface is

$$\Phi_e(x) = -e\epsilon x \quad (1.14)$$

The externally applied field effectively reduces the equivalent potential barrier for the electron in the solid as shown in Figure 1.4.

The total potential energy is

$$\Phi_t(x) = \Phi_e(x) + \Phi_s(x) \tag{1.15}$$

The effect of the external field reduces the work function W to a value $W' < W$. The thermally emitted electrons now experience an equivalent work function W' , this results in a current density given by:

$$J_e = A_o T^2 e^{-W'/kT} \tag{1.16}$$

Which in terms of the electric field is written as:

$$J_e = J_{th} \exp\left[\frac{e(e\mathcal{E} / 4\pi \epsilon_o)^{1/2}}{kT}\right] \tag{1.17}$$

The externally applied electric field then increases the density of the emitted electrons.

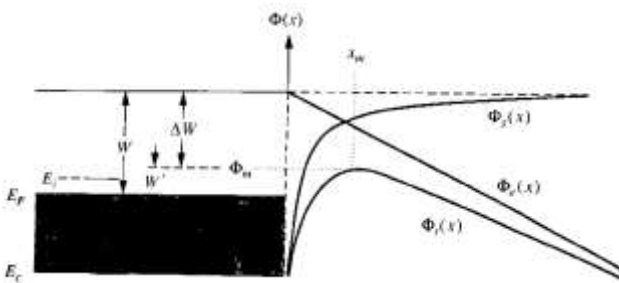


Figure 1.5 Modification of the idealized potential barrier (dashed line) for a solid with image potential $\phi_e(x)$ and additional potential $\phi_s(x)$ due to the applied external electric field

1.5 Hatsopoulos' Ideal Diode Model

The ideal model of a thermionic converter corresponds to a diode in which the emitter and collector are placed very close to each other. In order to reduce the complexity of the equations, the effects due to ion emission are also neglected. Although these assumptions do not strictly correspond to some thermionic converter, they approximate a close-spaced diode operating in the vacuum mode. The ideal diode model defines the limit of the development that can be expected for a thermionic converter and provides a basis for comparison of practical converters.

In the Hatsopoulos model, the current density J across the load was considered to consist of JEC flowing from the emitter to the collector minus Jce flowing from the collector to the emitter, thus

$$J = AT_E^2 \exp\left(-\frac{V + \phi_c}{KT_E}\right) - AT_C^2 \exp\left(-\frac{\phi_c}{KT_C}\right) \tag{1.18}$$

For $V + \phi_c > \phi_E$ and V voltage at the output load.

When the output voltage is reduced such that: $V + \phi_c < \phi_E$ then:

$$J = AT_E^2 \exp\left(-\frac{\phi_E}{KT_E}\right) - AT_C^2 \exp\left(-\frac{\phi_E - V}{KT_C}\right) \tag{1.19}$$

Equations (1.17) and (1.18) can be used for the construction of the $J - V$ curve, for different values of ϕ_e , ϕ_c , T_e and T_c . Usually $1600 < T_e < 2000K$, $600 < T_c < 1200K$, $2.4 < \phi_e < 2.8eV$ and $\phi_c \cong 1.6 eV$

Thermionic electric power production. It exploits the difference in the electron fluxes of the two surfaces facing each other at different temperatures, thus the net current is stable. Figure 1.6 shows a cell with the electrodes facing each other at different temperatures. The electrode at the higher temperature generally emits a higher current than the cold electrode, so a net current is obtained.

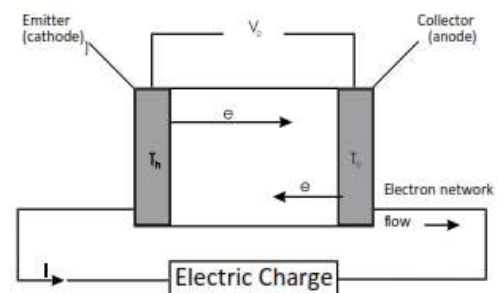


Figure 1.6 The thermionic diode as an energy converter

2. Description of the algorithms used for the theoretical study of natural and forced emission

The mathematical algorithm used to determine the thermionic emission, by means of the Richardson - Dushman equation, is described by means of flow diagrams; for this, the temperature is varied while the work function is kept constant. To calculate the thermionic emission between two electrodes separated by a constant distance, the equation developed by Hatsopoulos for the vacuum diode is used. The forced emission or field emission is calculated from the Langmuir - Child equation. Finally, the algorithm to evaluate the high field emission (Schottky effect) is developed.

2.1 Algorithm to determine the current density for different temperatures in the emitter

The behavior of the current density generated for different temperatures in the emitter of a thermionic diode while keeping the work function of the material fixed is determined by the Richardson - Dushman equation, *i.e.*:

$$J_{th} = A_o T^2 \exp\left(-\frac{\phi}{kT}\right) \quad \frac{A}{m^2} \quad (1.6)$$

$$A_o = 1.2 \times 10^6 \quad \frac{A}{m^2 K^2}$$

With

And further where:

k = Boltzmann's constant

ϕ = Work function

T = Absolute temperature (K)

To calculate the electron emission at a hot electrode by means of the Richardson - Dushman equation for different work functions, the evaluation algorithm was first defined by means of the flow diagram in Figure 2.1.

In the first block, the Richardson - Dushman equation is introduced to the plot, and then the value of the work function is chosen for different values between 2.0 and 5.5, with increments of 0.5 eV. In the next block, the evaluation of the equation for temperature variations between 0 and 4000K is done to obtain the corresponding graph.

If it is desired to obtain another graph for another work function, it is returned to the point where this one is chosen. Once the evaluations of the different graphs are finished, the process is printed and the process is finished.

2.2 Algorithm to determine the behavior of the electron flow in a thermionic diode

If a vacuum thermionic converter has emission and collection surfaces with dimensions much larger than the interelectronic space d , the net electron flow in the interelectronic space is almost unidirectional. Therefore, the output current characteristic can be presented in terms of the output current density. If the reverse emission current density is not negligible, then:

$$J = AT_E^2 \exp\left(-\frac{\phi_E}{kT_E}\right) - AT_C^2 \exp\left(-\frac{\phi_C}{kT_C}\right) \quad (1.19)$$

Where:

ϕ_E = Work function at emitter.

ϕ_C = Work function at the collector

TE = Emitter temperature

TC = collector temperature

k = Boltzmann constant

To evaluate the behavior of the electron flow in a thermionic diode according to the mathematical model proposed by Hatsopoulos, the sequence defined by flow diagrams 2.2 and 2.3 was used.

Flowchart 2.2 operates as follows; first the equation of the ideal diode model proposed by Hatsopoulos is introduced, then a work function between 2.4 eV and 4.0 eV with increments of 0.4 eV is chosen. The equation is then evaluated by varying the emitter temperature between 1000 and 3000K. For each evaluation developed, the emitter temperature was considered equal to 800 K and its work function equal to 3 eV. If it is desired to obtain another graph for another work function, it is returned to the point where this one is chosen, after evaluating the emission for different work functions, the graph is printed and the process is finished. The flow chart in Figure 2.3 was developed to evaluate the emission of the ideal diode when the collector temperature is varied. For this, the emitter conditions are first defined and then a work function is chosen for the collector between 1 and 1.8 eV with 0.2 eV increments. The collector temperature is varied in the range between 300 K and 2000K.

If it is desired to obtain another graph for another work function, it is returned to the point where this one is chosen, finally, the graph is printed and the process is finished.

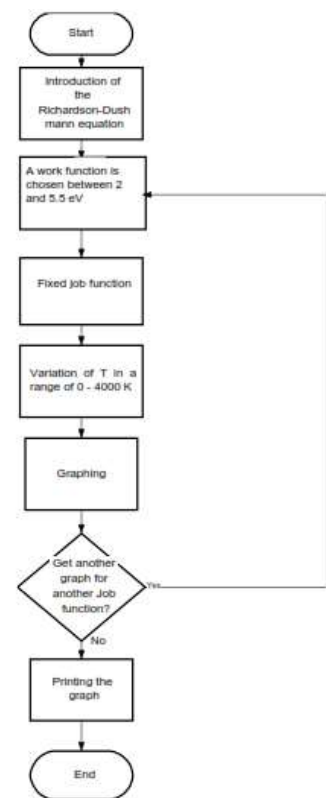


Figure 2.1 Flow chart for calculating electron emission by means of the Richardson - Dushman equation

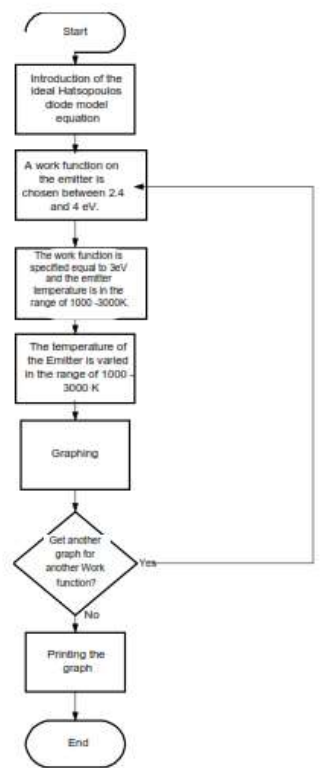


Figure 2.2 Flow chart to calculate the electron emission in a thermionic diode when the temperature of the emitter is varied

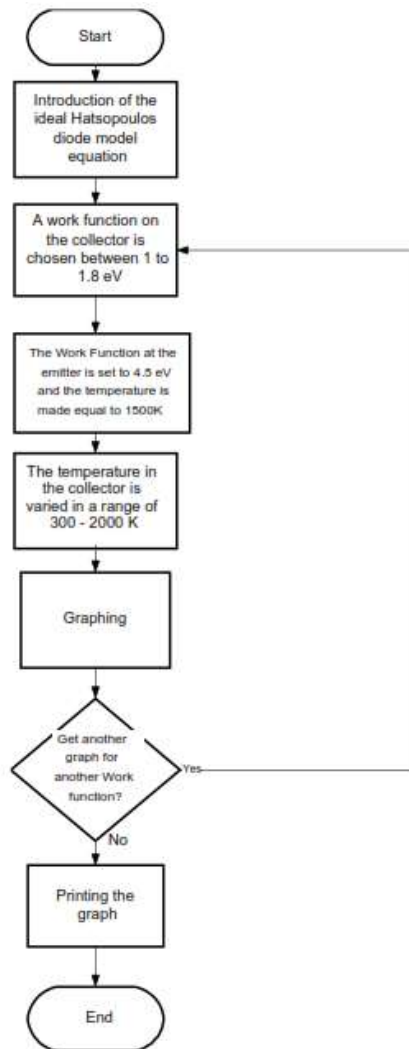


Figure 2.3 Flow chart to calculate the electron emission in a thermionic diode when the collector temperature is varied

2.3 Algorithm to determine the electron flow behavior for field variations

The behavior of the current density when a potential is applied to the anode and there is a distance d between the electrodes is determined by the Langmuir - Child equation.

$$J = 2.33 \times 10^{-6} \frac{V_a^{3/2}}{d^2} \tag{1.11}$$

Where:
 J = Is the current density between the electrodes, in A/m².
 Va = Voltage applied between the plates, in Volts.
 d = is the separation distance between the electrodes, in meters.

In the algorithm shown in Figure 2.4 the Langmuir - Child equation is first introduced and then evaluated for different interelectronic separation distances.

The values chosen for these distances are: 0.01, 0.02, 0.05, 0.1, 0.5 and 1 μm . After a distance is chosen, the electric field is varied in the range of 0 to 0.016 V/ μm . If you want to obtain another graph for another distance, go back to the point where this one was chosen, when the sequence is finished, print the graphs and end the process.

2.4 Algorithm to determine the current density for field variation while the emitter temperature is kept constant

To calculate the current density for electric field variations when the emitter temperature is kept constant, the emitting electrode is brought to saturation, and the Langmuir - Child and Richardson - Dushman law equations are combined, for this the algorithm shown by the flow chart in Figure 2.5 was defined. First, the Langmuir - Child equation is introduced and calculated for an interelectronic distance of 0.1 mm. Subsequently, the electric field is varied in the range of 0 to 0.16 V/mm. On the other hand, to determine the saturation current of the emitter, it is evaluated with the Richardson - Dushman equation for a work function of 4.5 eV. Emitter temperatures of 2700, 2800, 2900, 3000, 3100 and 3200 K were chosen. Finally, the obtained results were combined and the process is finished with the printing.

2.5 Algorithm to evaluate the behavior of the electron flow due to the Schottky effect

The behavior of the current density for different temperatures, while keeping the work function of the emitter fixed, is determined by the Schottky equation, ie:

$$J_e = J_{th} \exp\left[\frac{e(e\mathcal{E} / 4\pi \epsilon_0)^{1/2}}{kT}\right] \quad (1.17)$$

Where

J_{th} = Richardson - Dushman Equation

e = Electron charge (C)

\mathcal{E} = Electric field (V/m)

ϵ_0 = Permittivity in vacuum (C²/Nm²)

k = Boltzmann constant

T = Absolute temperature (K)

To calculate the current density when an electric field is applied, at a fixed temperature and constant work function at the emitting electrode, the Schottky equation is used, the algorithm defined to perform these calculations is shown in Figure 2.6.

In the first block, the Schottky equation is introduced and then a value is chosen for the emitter temperature in the range of 800 to 1500 K with 100 K increments. In the next step, the electric field in the range 0 to 1E6 V/m is varied. If another graph is desired, for another temperature, return to the point where a new temperature is chosen. Finally, the graphs are printed and the process is finished.

Algorithm to determine the I vs V behavior of an ideal thermionic converter.

To determine the current with respect to voltage in an ideal thermionic converter, the emission current equation 2.11 is used.

$$J = AT_E^2 \exp\left(-\frac{\phi_E}{kT_E}\right) - AT_C^2 \exp\left(-\frac{\phi_C}{kT_C}\right) \quad (1.19)$$

and $\phi_E = \phi_C + eV$

Where:

ϕ_E = Emitter work function.

ϕ_C = Work function at the collector

T_E = Emitter temperature

T_C = collector temperature

k = Boltzmann constant

V = Voltage at the load

To calculate the current, the flow diagram in Figure 2.7 was used for different emitter work functions in the range of 1.8 eV to 5 eV. In this case, the emitter work function is substituted by the sum of the collector work function and the load voltage. In this way, the behavior of the emitter current can be determined for values of expected load voltage even larger than its own work function. This adequacy of the equation simulates the fraction of electrons generated with higher kinetic energy than specified by the work function.

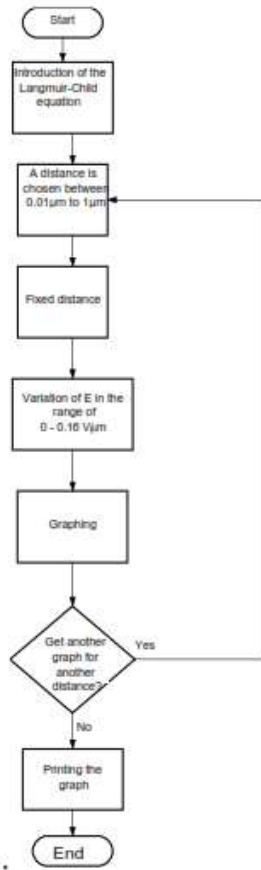


Figure 2.4 Flow diagram to determine the electron flow behavior according to the Langmuir - Child law.

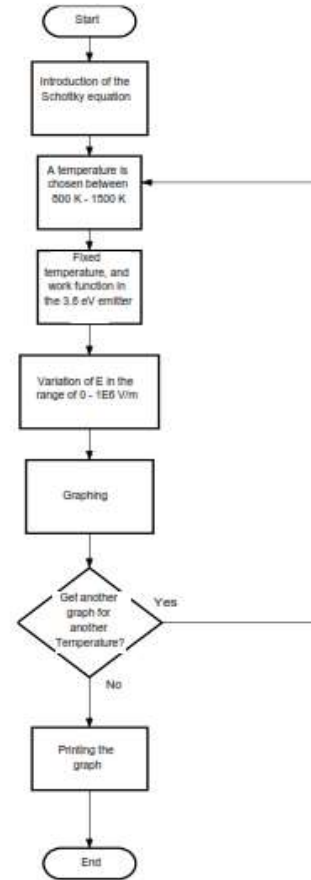


Figure 2.6 Flow diagram to determine the behavior of the electron flow due to the Schottky effect

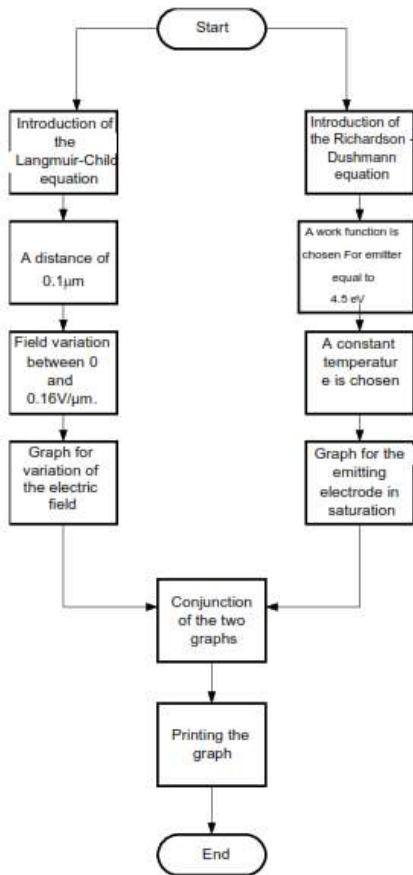


Figure 2.5 Flow diagram to determine the electron flow behavior by combining the Langmuir - Child and Richardson - Dushman equations

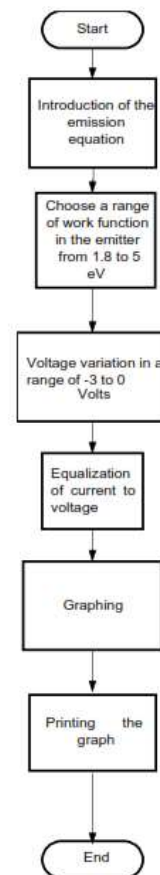


Figure 2.7 Flow chart to determine the behavior of an ideal diode using equation 1.11 developed by Hatsopoulos

3. Results and discussion

The graphical behavior of the current density for a thermionic diode when it operates naturally and when an external electric field is applied to it is shown below. Several exercises are shown for different work functions and temperature variations. The expected result for a thermionic diode with predefined characteristics is also shown.

3.1 Emitter current density behavior of a thermionic diode for different temperatures.

To determine the behavior of the current density generated for different temperatures, the temperature in the emitter was considered in a range from 300 to 3000 K, for a variation of the work function of the emitter from 2.5 to 5 eV with increments of 0.5 eV; with these conditions the behavior of the current density for different temperatures in the emitter was found as shown in Figure 3.1.

It can be seen in this Figure 3.1, that the onset of electron emission at the electrode requires a higher temperature for a high work function than for a low function.

For a work function of 2.0 eV, appreciable emission starts at around 1250 K.

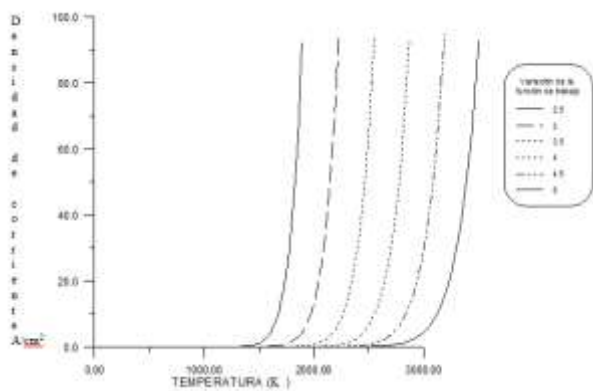


Figure 3.1 Behavior of current density at the emitting electrode as a function of temperature for different work functions

3.2 Vacuum thermionic converter

To determine the behavior of the current density in a vacuum diode, the following conditions were proposed:

T variations in the range of 1000 to 3000K, work functions at the emitter between 2.4 and 4 eV, constant collector temperature of 800K and constant collector work function equal to 2 eV.

Figure 3.2 shows the characteristics of the current density for different work functions in the emitter, it can be seen that for a lower work function a higher current density is reached for the same temperature.

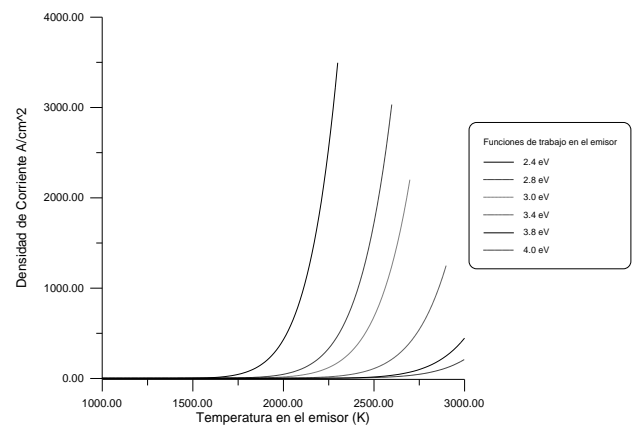


Figure 3.2 Current density behavior in a vacuum diode for temperature variations in the emitter and different work functions of the same

Figure 3.3 shows the current density for Tc variations between 300 and 1200K, collector work functions between 1 and 1.8eV at a constant emitter temperature equal to 1500K, and constant emitter work function equal to 2.5 eV. Figure 3.3 also shows the variation of the current density for different work functions at the collector, in this case, the inversion of the current in the diode is observed from 700K. The inversion of the current density is not appreciable for collector work functions higher than 1.4 eV.

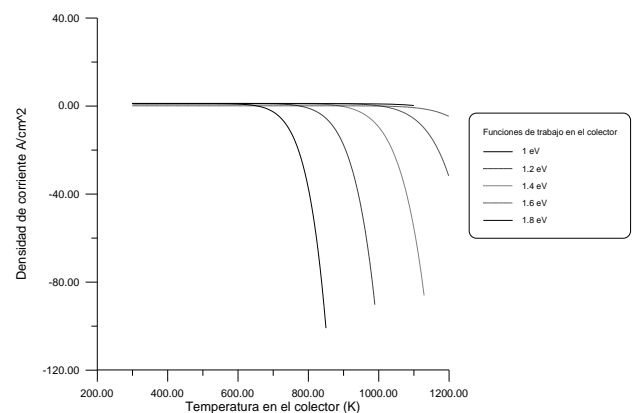


Figure 3.3 Current density behavior in a vacuum diode for temperature variations in the collector and different work functions of the diode

Behavior of the current density in a thermionic diode for field variation.

To determine the behavior of the current density generated for different temperatures, the following parameters were considered: the electric field was varied in a range from 0 to 0.16 V/m, varying the interelectronic distance from 0.01 to 1mm obtaining Figure 3.4.

Figure 3.4 shows the characteristics of the current density generated when the electric field is varied while the interelectronic distance is kept constant. It can be seen that, at greater distances, a more intense electric field is needed to obtain a current density equivalent to the density obtained for smaller distances.

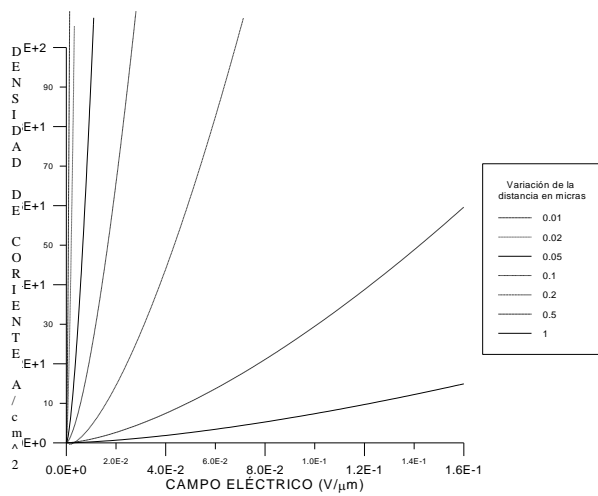


Figure 3.4 Expected behavior of the current density in a thermionic diode when the field is varied while the emitter temperature is kept constant

As can be seen from the graph in Figure 3.5, a conjunction of the two electronic emission equations leads the emitting electrode to saturation. The behavior of the increasing current density is given by the Langmuir - Child equation, and saturation is established by the Richardson - Dushman equation. The independent variable is the electric field.

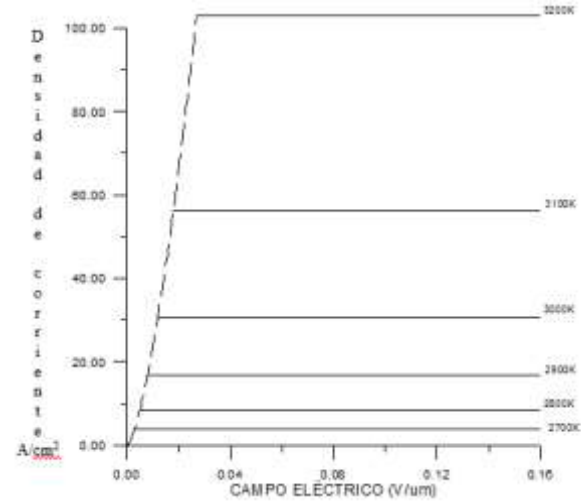


Figure 3.5 Current density behavior with respect to field variation for different temperatures. The interelectronic separation was kept constant and equal to 0.1um, the work function in the emitter was made equal to 4.5 eV

3.5 Current density behavior in a thermionic diode for field variation according to Schottky's equation

In the behavior of the current density for the thermionic diode according to the Schottky equation, an electric field in a range from 0 to 1.6E6 V/m is considered, being parametric in the temperature in the range from 1000 to 3000 K in the emitter, with increments of 500K, maintaining a work function in the same of 4.5 eV. Figure 3.6 shows the current density characteristics for different temperatures, when varying the electric field and constant work function at the emitter.

It can be seen that for the different temperature conditions, the electric field generates a further increase of the current density in contrast to the saturation point calculated with the Richardson - Dushman equation. This can be seen as a modulation of the saturation point generated by the external electric field.

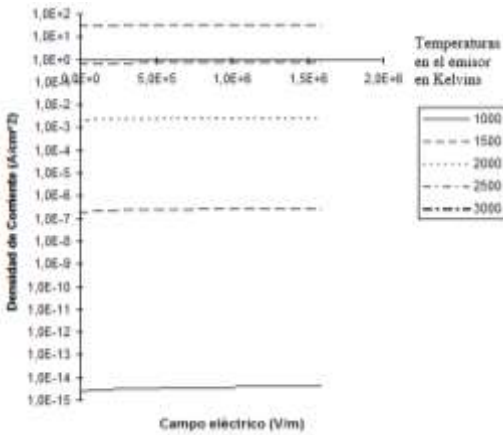


Figure 3.6 Behavior of the current density emitted by an emitter electrode for field variations. The work function of the emitter remains constant and equal to 4.5 eV.

3.6 I vs. V behavior of an ideal thermionic converter

To determine the expected current vs. voltage behavior of an ideal thermionic converter, a particular case was proposed. In this example, the work function at the emitter is assumed to be 2.8 eV, the collector work function is assumed to be 1.8 eV, the emitter temperature is assumed to be 1364 K, the collector temperature is assumed to be 600K, and an emission area at the emitter is assumed to be 2 cm².

As seen in the graph of Figure 3.7; the ideal diode has a growth with respect to the voltage, which starts at -1.8V and ends at the 20 mA point.

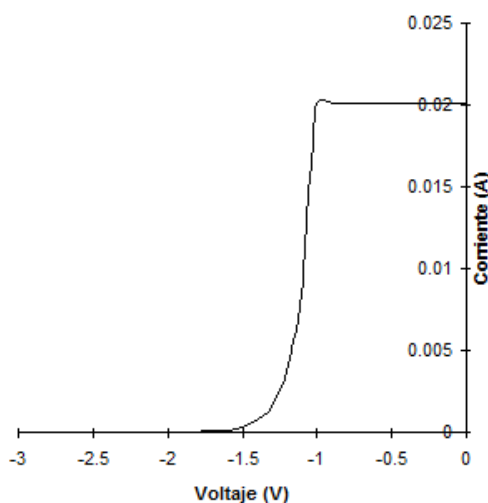


Figure 4.7 Comportamiento de la corriente generada por un convertidor termoiónico ideal; para funciones de trabajo en el emisor y colector de 2.8 eV, 1.8 eV. Respectivamente, la temperatura en el emisor es de 1364K y en el colector es de 600 K, además, el área efectiva de emisión es de 2 cm²

3.7 I vs. V behavior of a thermionic converter

To determine the expected current vs. voltage behavior of a thermionic converter, a particular case was proposed. In this example it is assumed that the emitter work function is 2.8 eV, the collector work function is equal to 1.8 eV. The interelectronic distance is 0.8 mm. The emitter temperature is 1364 K, and the collector temperature is considered to be 600 K, the emission area is 2 cm².

In the graph of Figure 3.8, a conjunction of the electronic emission equations defines the expected behavior of the current as a function of voltage for the proposed thermionic converter. The starting point corresponds to the open-circuit condition developed by Hatsopoulos. The initial growth of the current is given by the Langmuir - Child equation, and reaching the point of intersection with the electric field equation the Schottky equation begins to predominate. The flat behavior corresponds to the saturation of the diode and the slight lifting of the curve is due to the applied external electric field. Note that when the electric field is zero, the magnitude of the current coincides with the saturation current defined by the Richardson - Dushman equation. Figure 3.8 also shows the ideal behavior of the thermionic converter and the expected behavior when all emission conditions are considered.

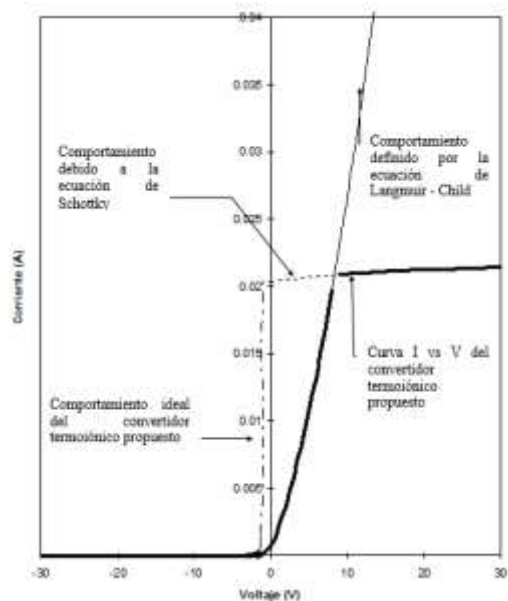


Figure 3.8 Superposition of the curves defined by the Langmuir - Child and Schottky equations for the particular case of a thermionic converter. In this example; $\phi_e = 2.8$ eV, $\phi_c = 1.8$ eV, the interelectronic distance is 0.8 mm, the temperature in the emitter is equal to 1364 K, and in the collector is 600K.

Conclusions

As main conclusions of this work, the following were obtained:

The theory that has been developed to explain electron emission between the electrodes of a thermionic converter was compiled and organized.

The algorithms that allow the use of the equations that predominantly explain the electron emission were developed.

The theory was organized in such a way that the behavior of the current density of a thermionic diode could be obtained independently as a function of: the work function, the temperature, the electric field and the interelectronic separation.

Practical cases related to the expected electronic emission for a thermionic converter were evaluated.

The importance of having low work functions in both the collector and the emitter, in order to work at low temperatures, was observed.

Interelectronic separation is a critical parameter to obtain high current densities.

It was feasible to simulate the behavior of a thermionic diode subjected to an electric field, from the birth of the curve to saturation.

A possible application of this work is to study the behavior of electron emission in nanometer thermionic structures. This study can be used to develop close-spaced thermionic converters, and also to study high-field emission; the latter can be useful for the development of high-vacuum microelectronics.

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