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Precision analysis of the integration rules used for transient simulation in electric networks

Análisis de la precisión de las reglas de integración utilizadas en la simulación de transitorios eléctricos

GALVÁN-SÁNCHEZ, Verónica Adriana†*, BAÑUELOS-CABRAL, Eduardo Salvador, GARCÍA-SÁNCHEZ, Jorge Luis and SOTELO-CASTAÑÓN, Julián

Universidad de Guadalajara, Departamento de Ingeniería Mecánica Eléctrica del Centro Universitario de Ciencias Exactas e Ingenierías

ID 1st Author: *Verónica Adriana, Galván-Sánchez* / **ORC ID**: 0000-0002-5462-2361

ID 1st Co-author: *Eduardo Salvador, Bañuelos-Cabral* / **ORC ID**: 0000-0002-6004-5898

ID 2nd Co-author: *Jorge Luis, García-Sánchez* / **ORC ID**: 0000-0003-2919-186X

ID 3rd Co-author: *Julián, Sotelo-Castañón* / **ORC ID**: 0000-0002-9776-5303

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Abstract

The adequate selection of the integration rule and time step is crucial to ensure accurate results in the simulation of an electric network. For this reason, this work has two main objectives: to analyze the distortion of the waves due to the integration rule and integration time step, and to provide guidelines for new circuit simulators' users on how to properly choose an integration rule and a time step according to a desired precision. When discretizing a continuous system, the frequency response of the discrete system can be used to evaluate the precision of the mentioned rule, since a discrete system can be viewed as a filter. However, the frequency response provides the accuracy of the discrete system during the steady state; the transient-state accuracy can be analysed by evaluating the pole distortion due the discretization. This work focuses on the analysis of the steady-state accuracy and the transient-state accuracy of two discrete systems: an RL branch used as a fist order system and an RLC branch used as a second order system. For this analysis, two commonly used discretization rules are considered, backward Euler and trapezoidal rules.

Frequency response, Integration rule, time step, backward Euler, Trapezoidal rule

Resumen

La selección de regla y del paso de integración (Δt) es primordial para asegurar resultados precisos en la simulación de una red eléctrica. Por esta razón, este trabajo tiene como objetivos analizar la distorsión que sufren las formas de onda obtenidas debida a la regla y al paso de integración, y proporcionar directrices que ayuden a los usuarios de simuladores de redes eléctricas en la selección adecuada de estos elementos acuerdo a cierta precisión deseada. Al aplicar una regla de integración a un sistema, se puede utilizar su respuesta en frecuencia para evaluar la precisión de dicha regla, ya que un sistema discreto se puede analizar como un filtro. Sin embargo, la respuesta en frecuencia proporciona solamente la precisión de la respuesta de estado estable; la precisión de la respuesta transitoria se puede analizar mediante la evaluación de la distorsión que sufren los polos de los sistemas continuos cuando se convierten a discretos. Este trabajo se centra en los análisis de las precisiones de estado estable y estado transitorio de dos sistemas discretos: una rama RL serie como sistema de primer orden y una rama RLC serie como sistema de segundo orden. Para este análisis se utilizan la regla trapezoidal y Euler hacia atrás, dos de las reglas de integración más utilizadas para la simulación de redes eléctricas.

Respuesta en frecuencia, Regla de integración, Euler hacia atrás, Regla trapezoidal

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* Author Correspondence (e-mail: veronica.galvan@academicos.udg.mx)
† Researcher contributing as first author.

Introduction

Transient analysis calculates the response of a circuit as a function of time by numerically integrating the set of differential equations that model the system or by modeling the voltage and current relationships of each element. These relationships are represented by equivalent circuits consisting of combinations of resistors and sources whose values depend on the integration rules used. This is the approach used in electromagnetic transient simulators such as EMTP and SPICE, which are widely used computer programs for the study of transients in power systems [1].

The accuracy of the integration rules used to simulate transients has been studied extensively using the frequency response of an inductor or capacitor in [2]-[3]; however, a power network can be viewed as a combination of multiple first- and second-order branches. This paper focuses on the analysis of the steady-state and transient-state accuracies of two discrete systems: a series RL branch as a first-order system and a series RLC branch as a second-order system. The trapezoidal and backward Euler rules, two of the most widely used integration rules for the simulation of electrical networks, are used for this analysis.

Integration Rules for Electromagnetic Transient Simulation

Two of the most commonly used integration rules in electrical network simulation are backward Euler (BE) and trapezoidal (TRA). These rules are A-stable in that, independent of the accuracy of the solution, they always yield stable results. It is important to note that the accuracy of the response or solution depends on three factors [2]: the inherent properties of the integration rule, the size of the integration step, and the characteristics of the circuit being simulated.

Discretization procedure

The discrete equivalent *H[z]* of a continuous transfer function *H(s)* can be obtained by mapping from the s-domain to the z-domain. For the cases of the integration rules used in this paper, the mapping is performed by the following formulas [3]:

$$s_{TRA} = \frac{2}{\Delta t}\frac{z-1}{z+1},\ s_{BE} = \frac{1}{\Delta t}\frac{z-1}{z}, \qquad \text{(1a), (1b)}$$

where Δt is the time increment or integration step.

Accuracy of integration rules

The frequency response of a system provides the steady-state response to sinusoidal inputs over a frequency range. When the system is in steady state, the output differs from the input only in magnitude and phase. For this reason, the accuracy of a discrete system *H[z]* can be evaluated by comparing its frequency response with the frequency response of the continuous system *H(s).* To obtain the frequency response of *H(s),* the complex frequency is sampled as

$$s = j2\pi k\Delta f, \quad k = 0, 1, 2, \ldots, N-1 \qquad (2)$$

where Δf is the frequency increment and N is the number of samples. Note that (N-1) Δf is the maximum frequency or Nyquist frequency f_{Ny}. To obtain the frequency response of H[z], $z = e^{s\Delta t}$ is used, where s is sampled as in (2) and $\Delta t = 1/(2f_{Ny})$. The frequency responses in this article are plotted using a normalized frequency

$$\text{Normalized frequency} = \frac{s}{j2\pi F_S}(\text{pu}), \qquad (3)$$

which is defined as the frequency per unit of the sampling frequency $F_s = 2f_{Ny}$.. The normalized frequency has a range from 0 to 0.5, where 0.5 is the normalized f_{Ny}.

This paper also analyzes the errors of the natural response of discrete systems. To perform this analysis, the poles and residues of discrete systems are compared with those of continuous systems; the comparison is performed in the continuous plane. For this, the discrete parameter in question is converted to a distorted parameter that exists in the continuous domain; for example, a discrete $p_{discrete}$ pole is converted to a distorted $p_{distorted}$ pole using the formula [4].

$$p_{distorsion\,ado} = \ln\left(p_{discreto}\right)/\Delta t \qquad (4)$$

First Order System

The first-order system considered in this article is an RL branch. Taking voltage as input and current as output, *H(s)* is the transfer function of the continuous system, which is

$$H(s) = \frac{1/L}{s + R/L}\,. \qquad (5)$$

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Substituting (1) into (5) gives the following discrete transfer functions:

$$H_{TRA}[z] = \frac{\Delta t}{R\Delta t + 2L}\ \frac{z+1}{z + \left(R\Delta t - 2L\right)/\left(R\Delta t + 2L\right)}, \qquad (6a)$$

$$H_{BE}[z] = \frac{1}{R + L/\Delta t}\ \frac{z}{z - 1/(R\Delta t/L + 1)} \qquad (6b)$$

The impulse response of the first-order system has the form $h(t) = re^{pt}$, where r and p are respectively the residue and pole of (5). From (6) we obtain the impulse responses as in [5], which have the form

$$h[m] = \left\{ \left(r_{discrete}\right)\left(p_{discrete}\right)^{m\Delta t} + k_{discrete}[0] \right\} / \Delta t \qquad (7)$$

$m = 0, 1, 2, ..., N-1$, m

where $r_{discreto}$, $p_{discreto}$ and $k_{discreto}$ are respectively the residue, the pole and the constant term of (6); note that the values of these parameters are different for (6a) and for (6b).

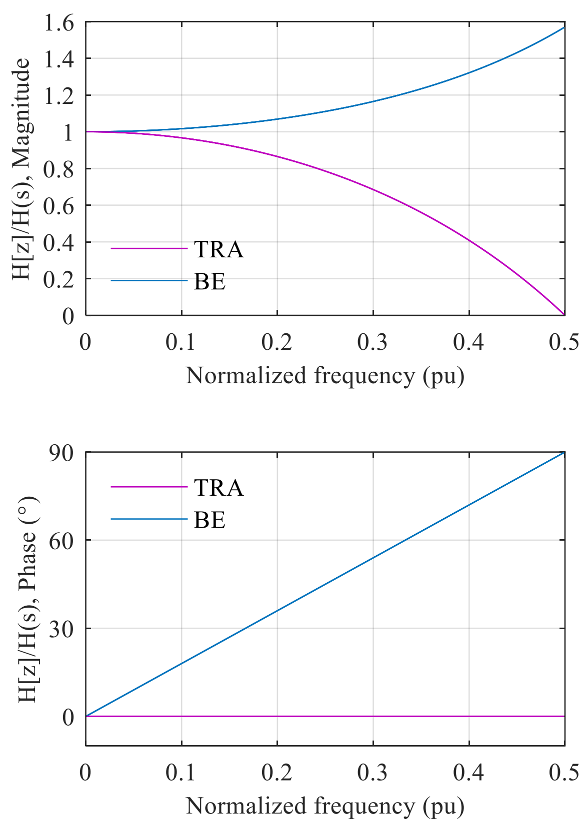
Accuracy of the steady state solution

The accuracy of some integration rules is presented in [1]. In this reference, the frequency response of a pure inductance is considered, which is reproduced in graph 1. The curves in graph 1 depend only on the Δt and the integration rule, which means that they do not depend on the circuit parameters (L in this case). However, this is not the case for circuits that are composed by a variety of elements as shown below. In the case of the RL branch, different plots are obtained for different values of R, L and Δt. To obtain standard shape curves as in graph 1, the Δt is defined in a general way, i.e., for any value of R and L. This time increment can be specified by the bandwidth of the system represented by (5). Although this system is not band-limited, most of the spectrum energy is contained in a band from f = 0Hz to f = BHz. For example, the bandwidth B containing 90% of the spectrum energy can be obtained by the energy criterion [6].

$$B = \frac{|p|\tan\left(0.90\pi/2\right)}{2\pi}. \qquad (8)$$

The criterion for selecting B depends on the tolerance error of each specific application [6]. It is worth mentioning that to select the Δt according to B, $\Delta t = 1/(2B)$ is used, i.e., $B = f_{Ny}$.Graph 2 shows the effect of energy content on the normalized Δt; the time constant of the system ($\tau = L/R$) is used as the time base.

Graph 3a presents the frequency response of the first order discrete system if f_{Ny} (B) is selected to contain 90% of the spectrum energy. Unlike the case of a pure inductance, it can be noticed in this graph that the RL system discretized by TRA presents error in the phase; however, this discrete system has less than 5% error in amplitude and only one degree of error at a frequency equal to one fifth of f_{Ny}, while the errors of the system discretized by BE are considerably larger at that same frequency. If f_{Ny} is selected so that it contains 99% of the energy of the spectrum, the frequency responses are as shown in graph 3b. Similar to the case of pure inductance, the phase error for TRA is negligible over the entire frequency range.



Graph 1 Comparison of the frequency response of the discrete systems versus the continuous system

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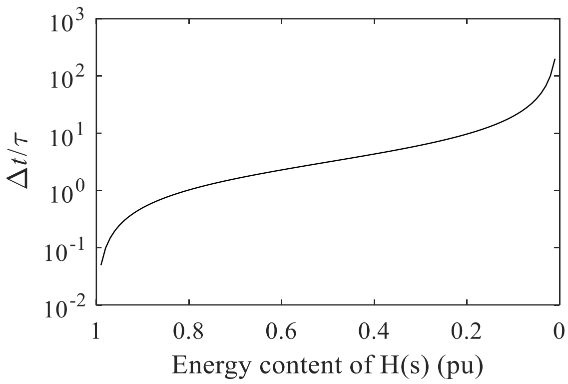
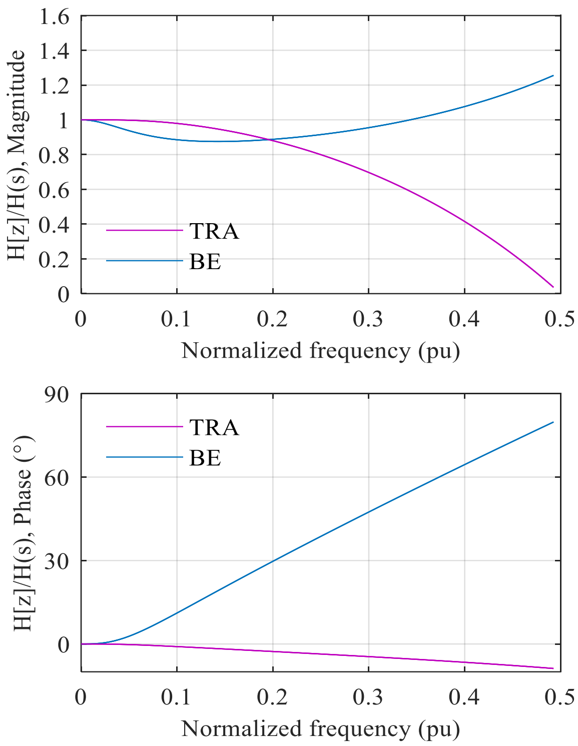
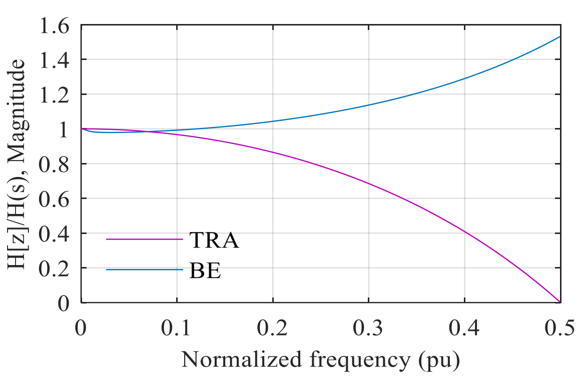
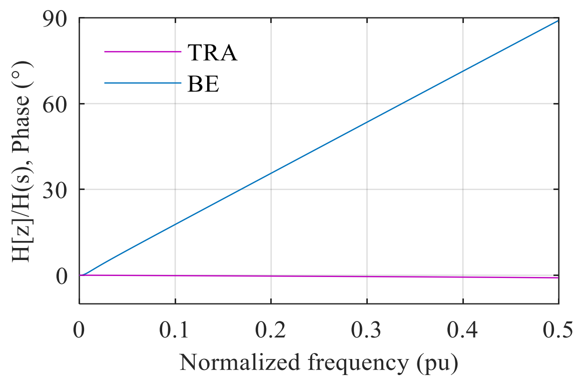


Graph 2 Variation of Δt as a function of the energy content of the first-order system spectrum



Graph 3 A



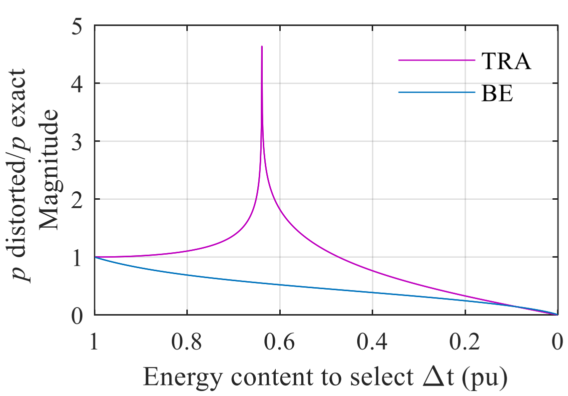


Graph 3 B

Graphic 3 Frequency response of first order systems if the Δt is calculated with the energy criterion; with (a) 90% of the spectrum energy, (b) 99% of the spectrum energy.

Accuracy of the transient solution

To discretize the continuous system, the *Δt* is varied according to a variation of the energy content of the spectrum of (5) from 0.1% to 99.9% (0.001pu to 0.999pu). The poles and distorted residuals are compared with the exact values in graph 4. This plot is standard for any first-order system if the *Δt* is chosen according to the energy criterion. *At* the *Δt* corresponding to approximately 64% (0.64 pu) of the energy there is a resonance in the plot. This resonance is due to the fact that from this *Δt* the pole and the discrete residue become negative; in the continuous plane the distorted magnitudes become complex. Graph 4a shows that with a Δt corresponding to 90% of the energy, the error of the TRA distorted pole is 2.1% while the BE distorted pole is 18.8%. From graph 4b it can be seen that TRA always overestimates the magnitudes of the residues when the distorted pole is real, while BE underestimates them. With a Δt corresponding to 90% of the energy, the error of the residue distorted by TRA is less than 10%, while the error of the one distorted by BE is greater than 30%.



Graph 4 A

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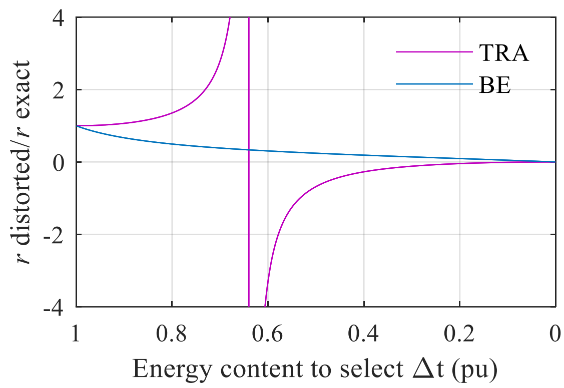

Graph 4 B

Graphic 4 Comparison of the distorted poles and residuals of the first order system against the exact values. (a) Comparison of the distorted poles, (b) comparison of the distorted residuals

Second order system

A series RLC circuit is used to evaluate the steady state response and transient response of a second order system. To obtain different damping factors ζ, the resistance *R* is varied in the following equation:

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}, \quad (9)$$

Since when simulating low ζ transients, the frequency and damping of the natural response strongly depend on the time increment and the integration rule. Transfer function *H(s)* of the *RLC* branch is used as a reference to compare the different numerical methods. If the source voltage is taken as input and the voltage on the capacitor as output, the transfer function of the *RLC* branch is

$$H(s) = \frac{1/(LC)}{s^2 + (R/L)s + 1/(LC)}. \quad (10)$$

Substituting (1) into (10), we obtain the discrete transfer functions

$$H_{TRA}[z] = \frac{1/(LC)}{\left(\frac{2}{\Delta t}\frac{z-1}{z+1}\right)^2 + \frac{R}{L}\left(\frac{2}{\Delta t}\frac{z-1}{z+1}\right) + \frac{1}{LC}} \quad (11a)$$

$$H_{BE}[z] = \frac{1/(LC)}{\left(\frac{1}{\Delta t}\frac{z-1}{z}\right)^2 + \frac{R}{L}\left(\frac{1}{\Delta t}\frac{z-1}{z}\right) + \frac{1}{LC}} \quad (11b)$$

The impulse response h(t) of the second-order system in (10) has the form $h(t) = r_1 e^{p1t} + r_2 e^{p2t}$, where the variables r and p are respectively the residues and poles of (10). From (11) we obtain the impulse responses of the discrete systems, which have the form

$$h[m] = \left\{ \left(r_{discrete,1}\right)\left(p_{discrete,1}\right)^{m\Delta t} + \left(r_{discrete,2}\right)\left(p_{discrete,2}\right)^{m\Delta t} + k_{discrete}[0] \right\} / \Delta t$$
$$m = 0, 1, 2, ..., N-1 \quad (12)$$

where the variables $r_{discrete}$, $p_{discrete}$ and $k_{discreto}$ are respectively the residues, the poles and the constant term of (11); note that the values of these parameters are different for (11a) and for (11b).

Accuracy of the steady state solution

For the underdamped case, systems with damping factors 0.01, 0.2, 0.5 and 0.8 are analyzed, and for the overdamped case, systems with damping factors 1.1, 5 and 10 are analyzed. The critically damped case, i.e., the system with ζ=1, is also included. *Underdamped system*. If $f_{Ny} = 10 f_0$ ($\Delta t = 0.05T_0$), is chosen, the comparisons of the frequency responses are as found in graphs 5a and 5b. It is pertinent to clarify that the error in magnitude of TRA around the resonance frequency ($f_{Ny}/10$) is due to the phase shift of the frequency response [7].

What happens in the case of BE discretizations is different: the magnitude of the frequency response is significantly attenuated around f0 for all damping factors; the phase of the response is also highly distorted around the resonance frequency. If f_{Ny} is increased so that it is 100 f_0 ($\Delta t = 0.005T_0$), the frequency responses are as in graphs 5c and 5d. In this case, the error in magnitude around the resonant frequency for all considered values of ζ is negligible for TRA, but is considerable for BE when ζ = 0.01. The error in the TRA phase is less than 1° for the whole frequency range, and for BE the error has an almost linear behavior (from 0 to 180°) over the whole frequency range, except around f_0.

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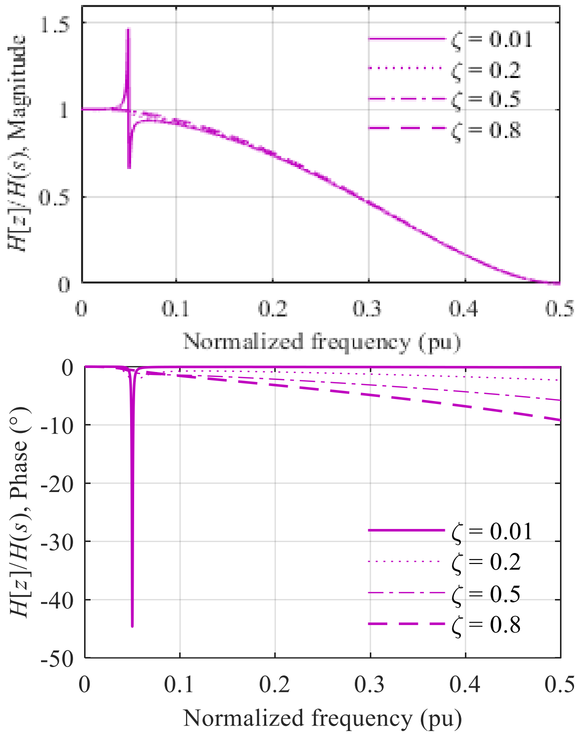
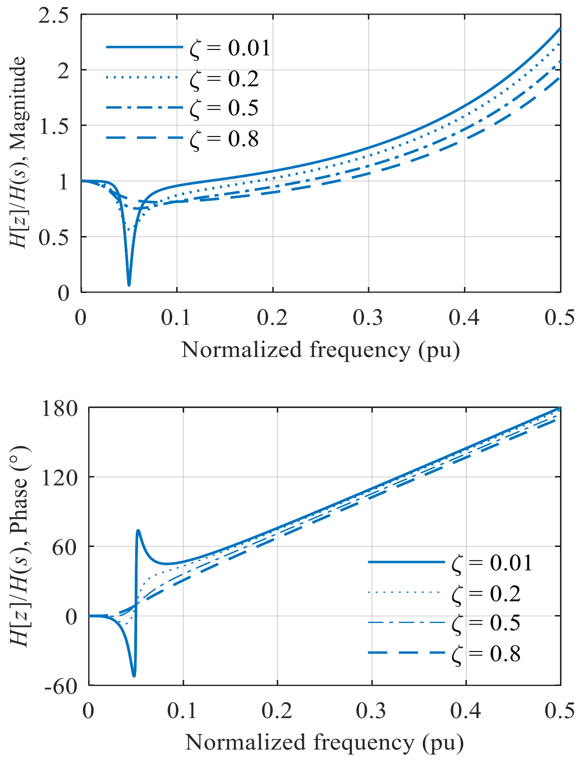
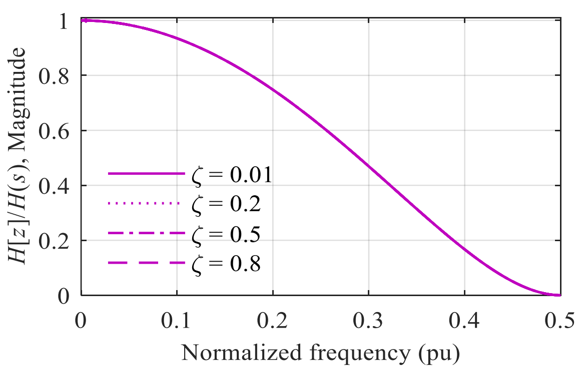
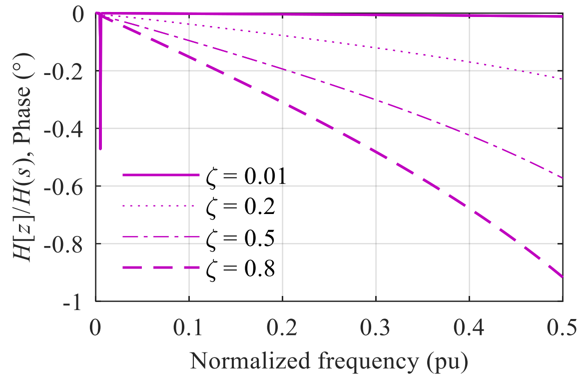


Graph 5 a

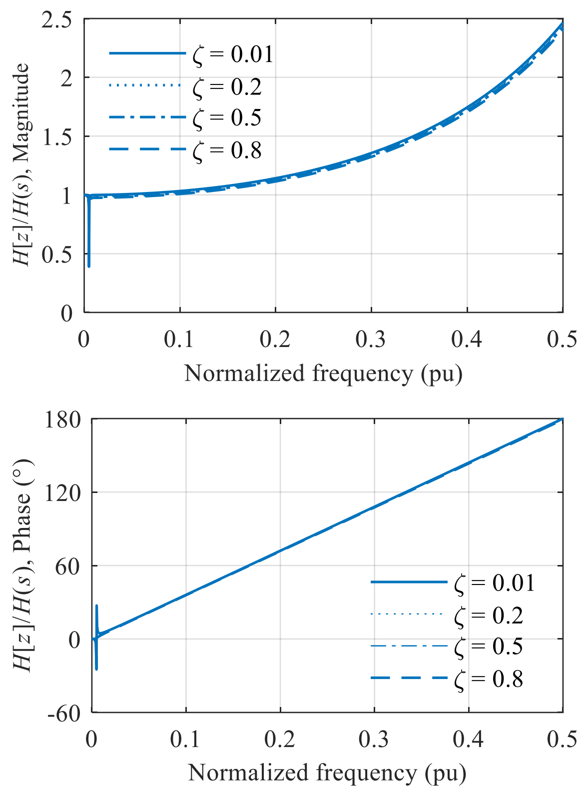


Graph 5 b





Graph 5 c

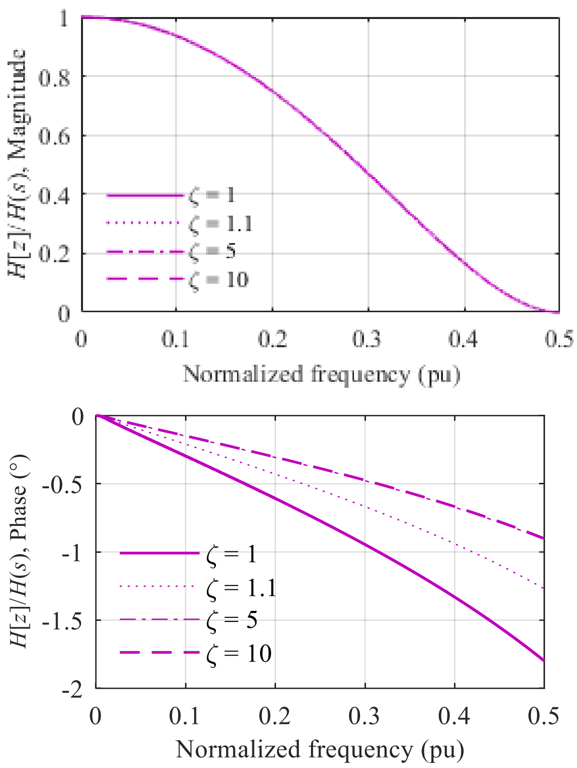


Graph 5 d

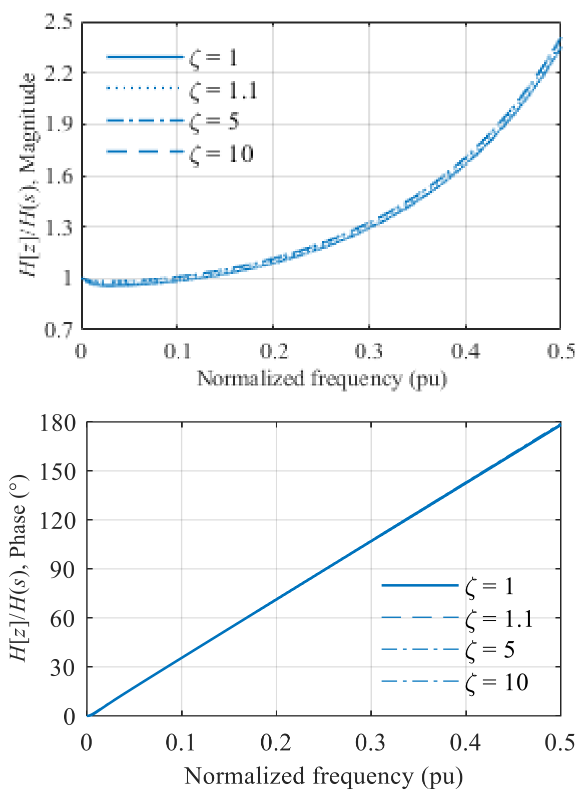
Graphic 5 Comparison of frequency response of underdamped second order systems. (a) TRA discretized systems with $f_{Ny} = 10f_0$, (b) BE discretized systems with $f_{Ny} = 10f_0$., (c) TRA discretized systems with $f_{Ny} = 100f_0$., (d) BE discretized systems with $f_{Ny} = 100f_0$.

Critically damped and overdamped systems

The comparison of the frequency response of discrete and analytical systems is shown in graph 6. f_{Ny} used for the analysis corresponds to 99% of the energy. For all the systems discretized by TRA, the error in the magnitude of the response is less than 2% for sources whose frequency is up to one tenth of f_{Ny}; in the same frequency range, the error of the steady state response in the phase is less than half a degree. It can be seen that with BE the error in the response amplitude is smaller than with TRA, however, the error in the phase of BE is very large; for example, for a source frequency of one-fifth of f_{Ny}, the error is greater than 30°.



Graph 6 a



Graph 6 b

Graphic 6 Comparison of the frequency response of critically damped and overdamped second order systems for a Δt corresponding to 99% of the energy. (a) Discrete system using TRA, (b) Discrete system using BE.

Accuracy of the transient state solution

To make comparisons of the discrete system parameters with the analytical system parameters, the damping factor is varied from 0.01 to 0.99 and the Δt to discretize the continuous systems is varied from 0.1 to 0.0017 times the period of the resonance frequency T_0; in other words, the T_0; is varied from 5 to 300 times the resonance frequency f_0.

Underdamped system

The impulse response $h(t)$, with $p_1, p_2 = \sigma \pm j\omega_n$ and $r_1, r_2 = A/2\ e^{\pm j\theta}$ has the form of a damped cosine and can be expressed in the following ways.

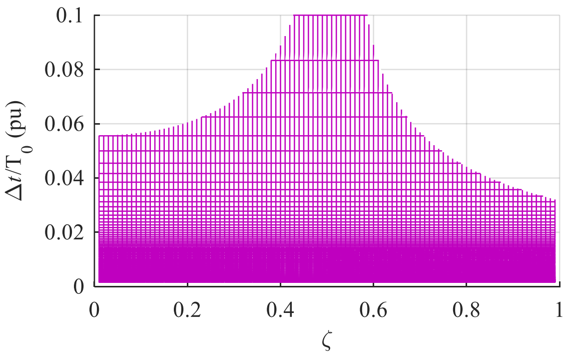
$$h(t) = r_1 e^{p_1 t} + r_2 e^{p_2 t} = Ae^{\sigma t}\cos(\omega_n t + \theta) \\ = 2|r_1| e^{\Re\{p_1\}t} \cos\left(\Im\{p_1\}t + \angle r_1\right) \quad (13)$$

In the underdamped case, r_1 and r_2 as well as p_1 and p_2 are complex conjugates respectively. The imaginary part of p1 is considered to be positive and therefore r_1 contains θ, the cosine phase. The mapping of s to the z-plane distorts the real part and the imaginary part of a complex number differently [4]. For this reason, the accuracy analysis is performed on the following parameters of the impulse response: $\omega_n = \Im\{p_1\}$, $\sigma = \Re\{p_1\}$, $A = 2|r_1|$, $\theta = \angle r_1$. The graphs are left as a sheet and a cut is made to show minimum precisions.

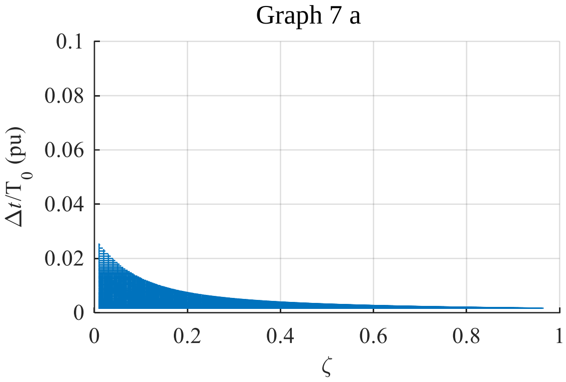
Graph 7 shows comparisons of the natural frequency ω_n and damping σ with their exact equivalents at 99% minimum accuracy. Graphs 7a and 7c illustrate that when discretizing by TRA and using a $\Delta t = 0.03T_0$, the accuracy of both ω_n and σ is at least 99% for all damping factors; in contrast, Graphs 7b shows that when discretizing by BE and using a $\Delta t = 0.01\ T_0$, only damping factors smaller than 0.1 can be represented with 99% minimum accuracy. Using BE, Graph 7c shows that the smallest damping factor that can be used such that the 99% minimum accuracy of σ is obtained is $\zeta = 0.37$; to achieve this accuracy the smallest Δt considered in this work, 0.0017 s, must be used.

Graph 8 shows the comparisons for the amplitude of the response A and the phase of the discrete systems θ. The sheets present a cutoff so that a minimum accuracy of 99% is shown.

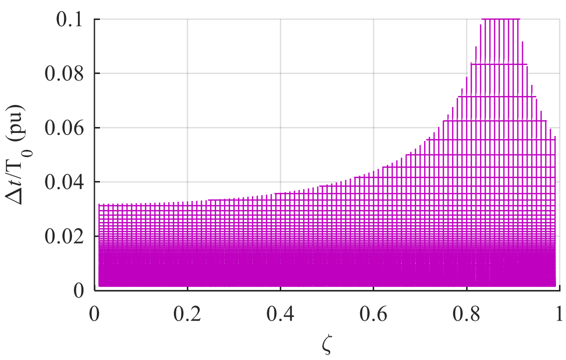
With TRA, similar to the behavior of ω_n and σ, when using a maximum time increment $\Delta t = 0.03T_0$, the accuracy in A is at least 99% for all damping factors. With BE the time increments must be much smaller to reach 99% accuracy for all damping factors.



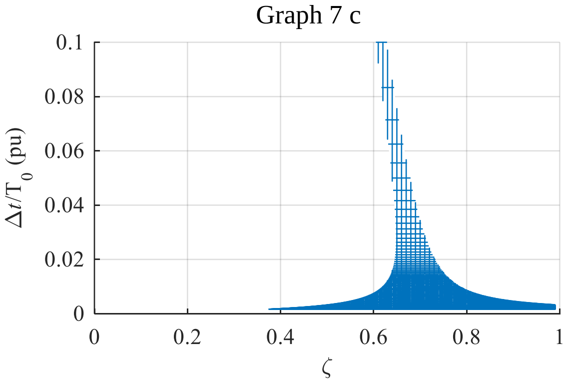
Graph 7 a



Graph 7 b



Graph 7 c

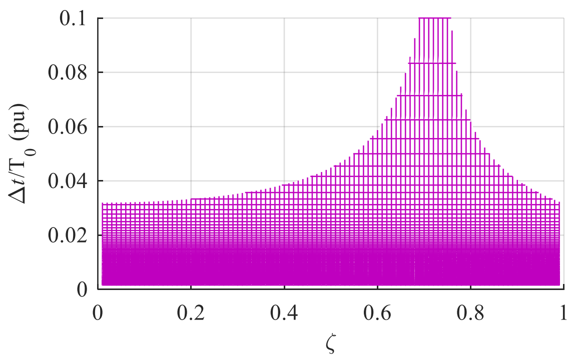


Graph 7 d

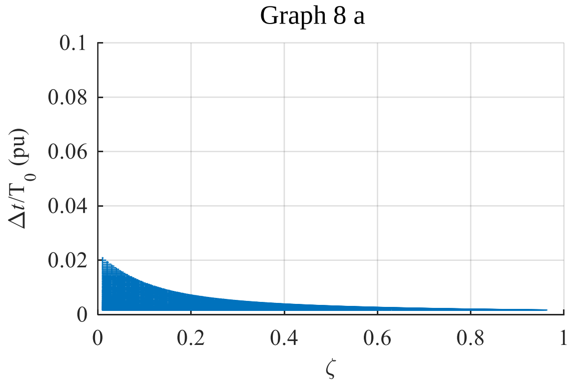
Graph 7 *(a) $\omega_{n,\ distorted}/\omega_{n,\ exact}$, TRA; (b) $\omega_{n,\ distorted}/\omega_{n,\ exact}$, BE; (c) $\sigma_{distorsted}/\sigma_{exact}$, TRA; (d) $\sigma_{distorted}/\sigma_{exact}$, BE.*

For TRA, with an approximate maximum time increment $\Delta t = 0.04T_0$, the accuracy θ is within the mentioned accuracy range for all damping factors considered. It is very interesting to note that for BE the error with all time increments for all damping factors considered is zero, i.e., from this it can be concluded that the phase of the natural response is not affected by the application of BE.

Overdamped system. The two real poles of the second order system have different time constants. To choose the appropriate time step using the energy criterion, the appropriate bandwidth B is selected according to the faster pole (the one with the larger absolute value). The behavior of the faster pole is seen as in the Graph of the first-order system in Graph 5. The other pole, whose bandwidth is smaller, possesses higher accuracy for the specified Δt.

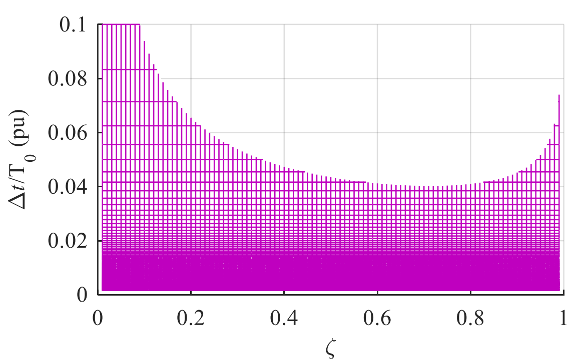


Graph 8 a



Graph 8 b

Graph 8 (a) $A_{distorted}/A_{exact}$, TRA; (b) $A_{distorted}/A_{exact}$, BE; (c) $\theta_{distorted}/\theta_{exact}$, TRA; (d) $\theta_{distorted}/\theta_{exact}$ BE



Graph 9 a

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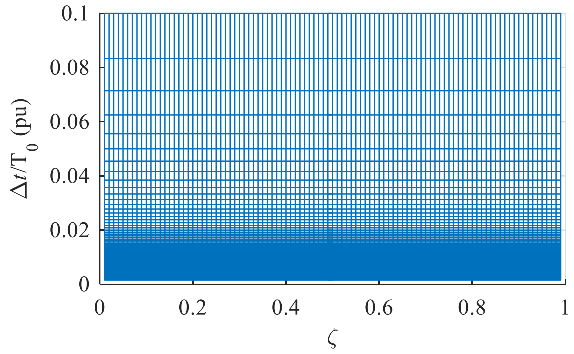


Graph 9 b

Graph 9 $\theta_{distorted}/\theta_{exact}$, (a) TRA, (b) BE. Comparisons show 0.1° maximum error

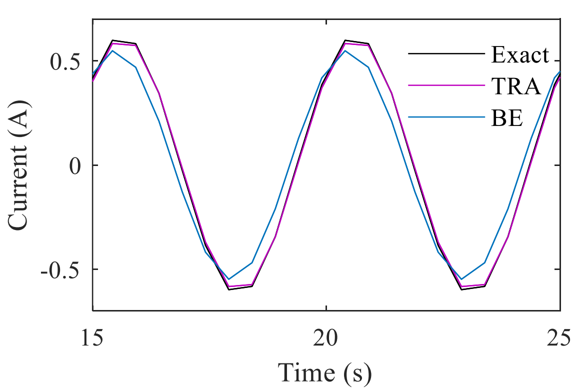
Results and discussions

In order to verify the accuracy analysis presented in the previous section, time domain simulations of the first and second order systems are performed.

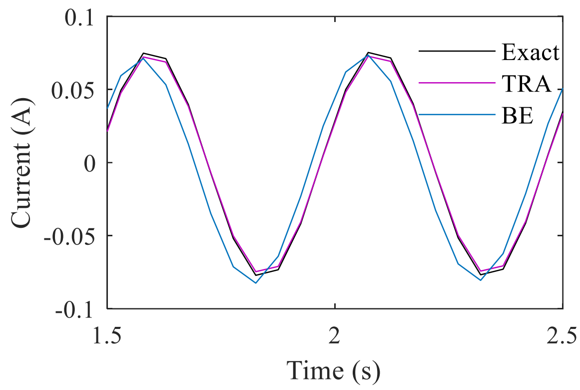
Series RL circuit

The impulse response and the response to a sinusoidal input of a series RL circuit are analyzed. R = 1Ω and L = 1H are used in this case, so that the time constant of the system is τ = L/R = 1 s.

To evaluate the accuracy during steady state for a sinusoidal input, a voltage source with frequency equal to one fifth of the f_{Ny} and amplitude of 1 V is used. The f_{Ny}, and hence the Δt, is calculated with two different bandwidths, corresponding to 90% and 99% of the energy of the system transfer function. When using the f_{Ny} corresponding to 90% of the energy, it can be seen in Graph 10a that the accuracy of BE is very poor, as predicted in Graph 3a, which presents the frequency response; in the case of Graph 10b, the accuracy in the magnitude of BE is higher than TRA, as shown in 3b, however, the response is leading by more than 15°.



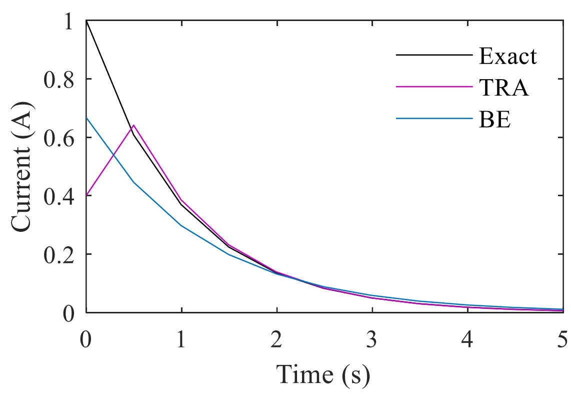
Graph 10 a



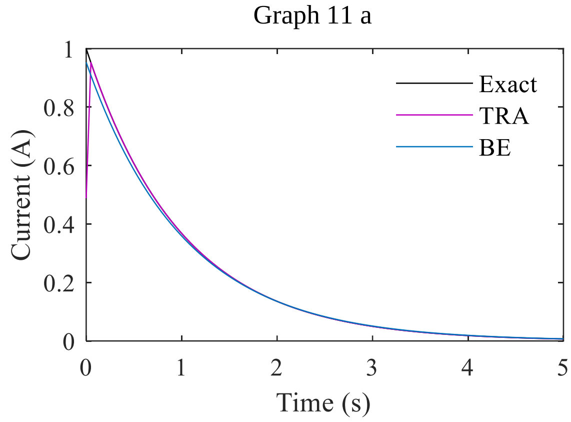
Graph 10 b

Graph 10 Comparison of the steady-state response of the first-order system for a source with frequency $f = f_{Ny}/5$; $f = f_{Ny}/5$ is calculated with (a) 90% of the signal energy, (b) 99% of the signal energy

The accuracy of the transient response of discrete systems is evaluated by their impulse response. The time increments used to apply the integration rules are obtained with 90% and 99% of the signal energy. In order to evaluate the behavior up to the steady state, the responses are plotted with a duration of 5τ, i.e. 5s. Graph 11 shows the solution of the system with the two integration rules.



Graph 11 a



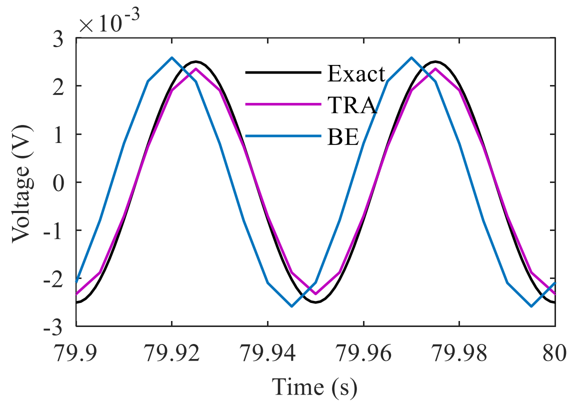
Graph 11 b

Graph 11 Comparison of impulse response of discrete systems with analytical. (a) Impulse responses with Δt corresponding to 90% of the signal energy, (b) Impulse responses with Δt corresponding to 99% of the signal energy

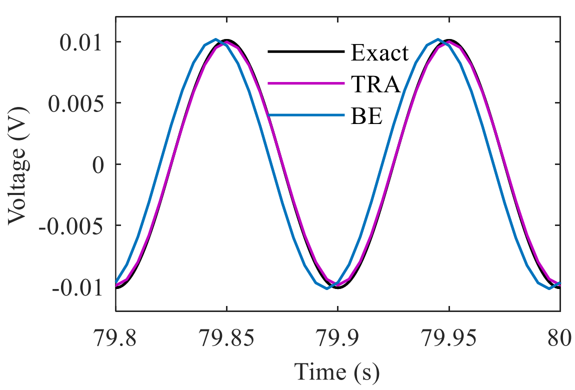
Series RLC circuit

The values of inductance L and capacitance C are taken as $0.5/\pi$ so that the circuit has a resonant frequency $f_0 = 1$ Hz or $\omega_0 = 2\pi$ rad/s. The period of the resonant frequency is $T_0 = 1/f_0 = 1$ s. *Underdamped case*: the most critical case considered in this work is used, which is the system with $\zeta = 0.01$. Graph 12 shows the steady state solution of the second order system if a time increment $\Delta t = T_0/200 = 0.005T_0 = 0.005$s is used or, in other words, if $f_{Ny} = 100f_0 = 100$Hz. Two solutions are presented corresponding to the use of two sources with frequencies of $f_{Ny} = 100f_0 = 100$Hz and $f_{Ny}/10$ (10 Hz). The Graph s show the solution at times close to the 5 time constants.

Graph 12a shows that the BE accuracy is very poor in both magnitude and phase; the solution with TRA is in phase and only slightly attenuated. In Graph 12b it can be seen that the magnitude of the solution with BE is slightly closer to accurate than with TRA (as predicted in Graph 5b), however, the phase is leading.



Graph 12 a



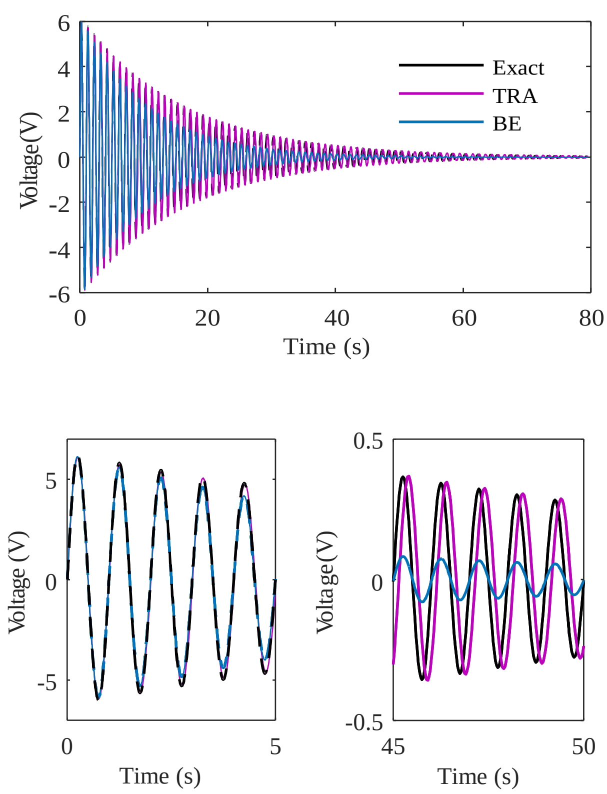
Graph 12 b

Graph 12 Steady state solution of a second order system with $\zeta = 0.01$, *(a)* $f_{Ny}/5$, *(b)* $f_{Ny}/10. = 100f_0$ and with the source at the following frequencies *(a)* $f_{Ny}/5$, *(b)* $f_{Ny}/10$.

To solve for the transient state of the system, the maximum Δt is chosen from Graphs 7-9 so that the natural response parameters are at least 99% accurate. For TRA, the value is $\Delta t \approx T_0/32 \approx 0.0313T_0 \approx 0.03125$s; for BE the minimum Δt considered, $\Delta t = T_0/600 \approx 0.00167$s is used; this Δt does not guarantee 99% accuracy in the σ parameter.

The exact and numerical impulse responses are presented in Graph 13. Regarding the solution with TRA, one can observe a shift of the solution with respect to the analytical one that looks like a phase shift; however, this error is due to the error in the natural frequency ω_n. and is not an error in the phase. It can be seen in Graph 9a that for the ζ and Δt used, the error in phase is less than 0.1°. What occurs in the BE solution is excessive attenuation; the oscillations remain in phase with the analytical solution.

This error is due to the fact that BE underestimates the value of σ (overestimates the absolute value), which causes the natural response to decay more rapidly.



Graph 13 Impulse response of the second-order system with $\zeta = 0.01$. For TRA a $\Delta t \approx 0.03125$ s is used and for BE a $\Delta t \approx 0.00167$ s is used

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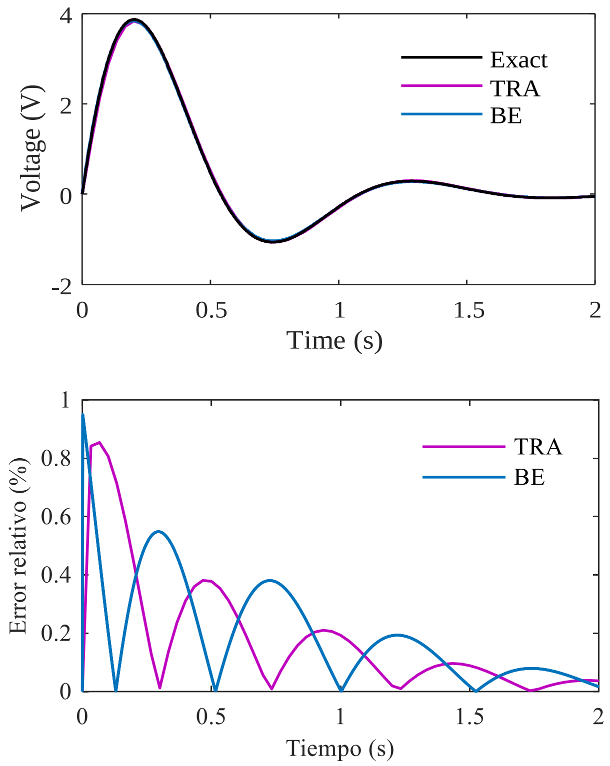

Graph 14 Impulse response and relative error of the second-order system with ζ = 0.01. For TRA *Δt ≈ 0.03333* (f_{Ny} = $15f_0$) a and for BE a *Δt ≈ 0.00167 s* (f_{Ny} = $300f_0$) is used

If BE is used and a Δt considered in this work (*Δt ≈ 0.00167s*), the system with ζ = 0.38 is the case with the smallest ζ that meets the 99% accuracy in all its parameters (see Graphs 7-9). To obtain similar accuracy with TRA, the chosen time increment is *Δt ≈ 0.0333s*. The impulse responses and relative error are presented in Graph 14 for both BE and TRA. The relative error is defined, with the magnitude of the analytical damped cosine (A_{exact}) instead of the maximum value of *h(t).*

Conclusions

This paper presents an analysis of the accuracy of a first order and a second order system when they are discretized by backward Euler and trapezoidal rule. This analysis is intended to guide new users of electromagnetic transient simulators in the proper choice of the integration step, since the size of the integration step affects the accuracy of the simulation. It was observed in this work that the appropriate time step size depends not only on the maximum frequency of the phenomenon to be observed as mentioned in [3], [8], but also on the damping factor of the systems.

As future work, based on this work it is proposed to automatically choose a suitable time increment according to the accuracy set by the user. This would be possible if the eigenvalues of the system to be simulated are known in advance; programs such as Simulink [9] can provide such eigenvalues.

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