

## Total harmonic distortion optimization in a seven level multilevel inverter by the random search heuristic algorithm

### Optimización de la distorsión armónica total en un inversor multinivel de siete niveles empleando un algoritmo heurístico de búsqueda aleatoria

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#### Abstract

In this article, the random search heuristic optimization algorithm is proposed to guarantee that the voltage synthesized by a seven-level multilevel inverter contains the lowest possible total harmonic distortion. The algorithm in this work is presented as an alternative, simple, and easy-to-implement method for solving a set of transcendental mathematical equations of multilevel inverters. The obtained results demonstrate the algorithm's capability to solve the mathematical formulation that minimizes the total harmonic distortion. It is important to highlight that the algorithm was implemented in Matlab®, and the obtained results were validated in a simulation conducted in Simulink®. Equally and not less important, the results were physically implemented in a laboratory prototype of three integrated circuits with H-bridges containing insulated gate bipolar transistors.

**Algorithm, Optimization, Multilevel-Inverter, Total Harmonic Distortion**

#### Resumen

En este artículo se propone el uso de un algoritmo de optimización heurístico de búsqueda aleatoria para que permita garantizar que la tensión sintetizada por un inversor multinivel de siete niveles contenga la menor distorsión armónica total posible. El algoritmo expuesto en este trabajo se propone como un método alternativo, simple y de fácil implementación para la solución de ecuaciones matemáticas trascendentales de un inversor multinivel. Los resultados expuestos demuestran el correcto funcionamiento del algoritmo al minimizar la forma distorsión armónica total de la onda de salida. Es importante destacar que el algoritmo fue implementado en el software Matlab® y los resultados obtenidos fueron validados por medio de simulación en el software Simulink®. De igual y no menos importante, los resultados fueron implementados de manera física en un prototipo de laboratorio que consta de tres circuitos integrados con puentes H, los cuales contienen transistores bipolares de compuerta aislada.

**Algoritmo, Optimización, Inversor Multinivel, Distorsión Armónica Total**

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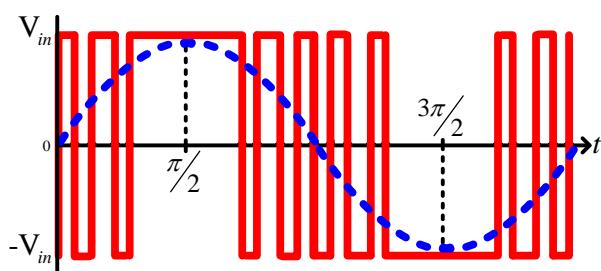
**Introduction**

Nowadays, there is an increase in the use of alternative or renewable energy sources such as photovoltaic cells, wind generators, fuel cells, among others in the industrial, commercial and residential sectors (Vaiaso & Jack, 2021). This is because these energy sources offer direct benefits for the environment and their cost has been reduced thanks to current technological advances. While such energy sources are becoming more widely used and more studied every day, the use of various specialised electronic circuits capable of synthesising a voltage and current that meets the guidelines stipulated by the electrical or electronic products that are to be used with renewable energy sources is indispensable.

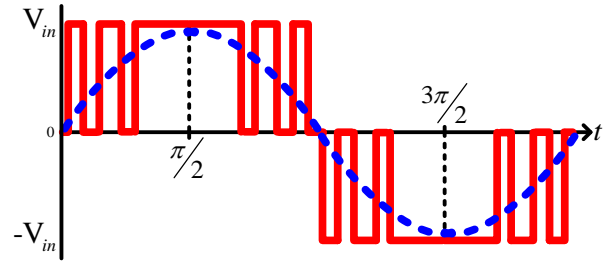
Among the most common electronic circuits used in the area of alternating energies are the DC-DC converters, also known as switched-mode power supplies, and the DC-AC converters, also known as inverters, the latter being the one studied in this article.

Inverters are circuits capable of synthesising a voltage and current in the form of a modulated alternating wave, with variable amplitude and frequency, from direct current sources. This makes them indispensable in applications related to electricity generation with renewable energy sources (Colak et al., 2011).

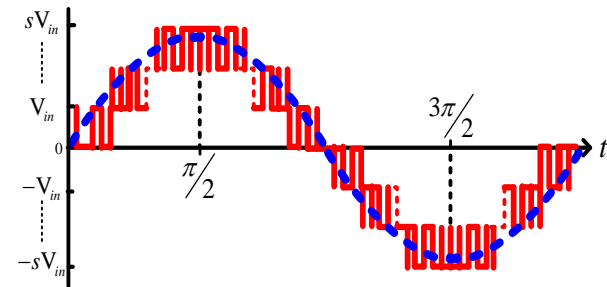
Within the literature, inverters can be classified into two types, constant voltage source inverters and constant current source inverters, the former being the most widely used due to its better performance and practicality (El-Hosainy et al., 2017). This paper focuses on the use of a constant voltage source inverter. Similarly, constant voltage source inverters can be subclassified into two-level, three-level and multilevel inverters. The following figures show the waveforms that can be synthesised by the above-mentioned inverters.



**Figure 1** Two-level alternating voltage

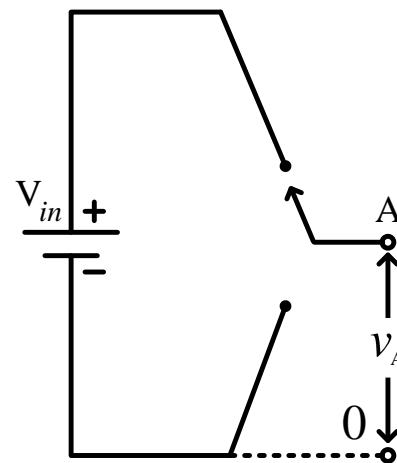


**Figure 2** Three-level alternating voltage

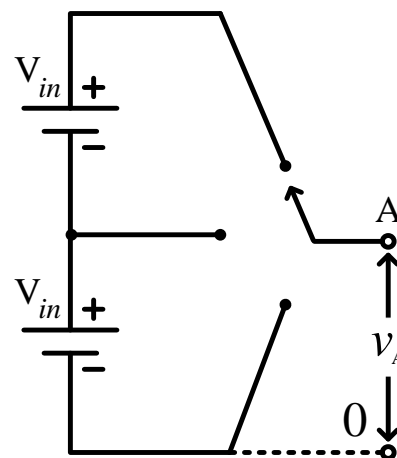


**Figure 3** Multilevel AC voltage

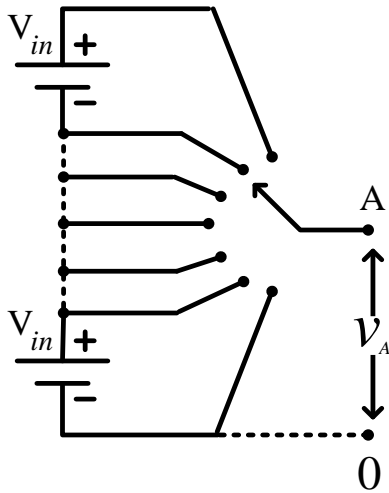
Similarly, Figures 4, 5 and 6 show the general schematics of constant voltage source inverter types, where switch "A" is usually a power switching device such as a MOSFET or IGBT.



**Figure 4** Two-level inverter



**Figure 5** Inversor de tres niveles



**Figure 6** Multilevel AC voltage

Since its inception, multiple studies have reported the multilevel inverter as the best alternative to conventional two- and three-level inverters. This is mainly because they are able to supply higher power levels with lower cost switching devices, in addition, the synthesised voltage has a lower harmonic content (Reddy & Narayana, 2020).

### Modulation techniques

Modulation techniques are used in inverters to control the on and off times of the switching devices that compose them, being these mechanisms that allow to give particular characteristics to the voltage synthesised by the inverter. Among its most important features is the possibility of manipulating the amplitude, phase and frequency (Vijeh et al., 2019). Globally, modulation techniques can be classified into high and low switching frequency and the use of these is subject to the type of application. Based on the above, this paper will focus on the use of a low switching frequency modulation technique.

Low switching frequency modulation techniques are techniques designed with the purpose of making inverters achieve the best possible efficiency and prolonging the lifetime of the switching devices. These techniques are used in applications demanding very high power levels.

### Harmonic distortion

Harmonics are defined as sine components of a periodic wave called the fundamental that have a frequency corresponding to an integer multiple of the fundamental frequency wave (Reddy & Narayana, 2020). Measuring the harmonic content of the voltage waveform synthesised by inverters is a fundamental part of determining their performance (Aguila-León et al., 2023) (Khan et al., 2023). Nowadays, norms and standards limit the harmonic content of the voltage and current synthesised by inverters, so ensuring the lowest possible harmonic content is necessary.

Total Harmonic Distortion (THD) is a parameter that quantifies the deviation between the waveform of a sinusoidal signal against any other type of periodic signal (usually voltage and/or current). Official Mexican standards such as CFE L-000045 recommend the quantification of this parameter up to the fiftieth harmonic order and is given by the following equation.

$$THD_V = \frac{V_{RMS}}{V_{1RMS}} = \frac{\sqrt{\sum_{h=2}^{50} V_h^2}}{V_1} 100\% \quad (1)$$

Where, the magnitude of the fundamental component (Aboadla et al., 2016) is given by  $V_{1RMS}$ , the magnitude of the h-th harmonic component will be  $V_h$  and the order of each harmonic will be h. While  $V_{RMS}$  is the rms value of the voltage function synthesised by the inverter.

### Optimisation algorithms

Optimisation algorithms (Houssein et al., 2021) are numerical methods, usually heuristics, capable of finding optimal values of a specific parameter in a problem that can be expressed by means of a mathematical function. This value or values minimise or maximise the objective function. It is important to note that the search for those optimal values is performed by means of population dynamics within an n-dimensional search space. This paper presents the use of the random search heuristic algorithm as an option to determine the firing angles of the semiconductor devices of a multilevel inverter to obtain its ladder output voltage.

The heuristic random search algorithm has the fundamental purpose of solving optimisation problems in a simple way. The dynamics of this algorithm is based on the iterative process of a population of elements, PS, in which each interaction starts from a feasible solution whose neighbourhood is analysed to subsequently locate better solutions. If the solution located by each interaction,  $j$ , improves the previous one,  $j-1$ , this will be maintained as the best global solution,  $G_b$ , and will be used in the next interaction,  $j+1$ . On the other hand, the mechanism used in the first interaction for the first possible solutions can be constructed randomly within a search window that follows constraints imposed by the problem to be solved.  $G_{b1}$

### THD minimisation at a seven level output voltage

A multilevel inverter is capable of generating an alternating waveform voltage varying in amplitude and frequency from multiple constant voltage levels. This voltage can come from renewable energy sources such as photovoltaic modules. Several works report that the voltage generated by multilevel inverters has a harmonic content that is inversely proportional to the number of levels. This work concentrates only on the case of a seven-level single-phase inverter. The following figure shows the possible voltage waveform to be synthesised by a 7-level multilevel inverter.

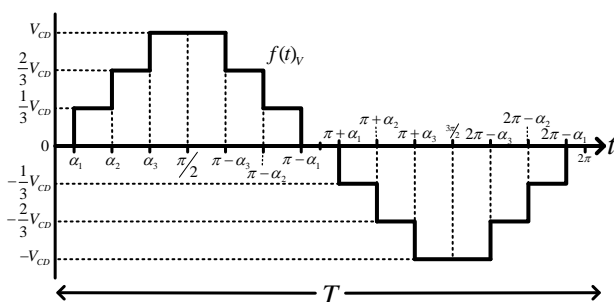


Figure 7 Seven-level alternating voltage

It is important to note that the synthesised voltage considers the levels of the voltage sources as constant and of equal magnitude. Because of this, and considering the odd symmetry in the waveform shown, it can be expressed in Fourier series as (Ajami et al., 2013):

$$f(t)_V = \frac{4V_{CD}}{3\pi} \sum_{n=1}^{\infty} V_h^2 [\cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3)] \frac{\sin(n\omega_0 t)}{n} \quad (2)$$

Similarly, it is possible to calculate its r.m.s. value with the following equation.

$$f(t)_{VRMS} = \frac{V_{CD}}{3} \sqrt{9 - \frac{\alpha_1 + 3\alpha_2 + 5\alpha_3}{90}} \quad (3)$$

Where  $\alpha_1 < \alpha_2 < \alpha_3 < 90^\circ$  determine the switching on and off of the switching devices.

On the other hand, from equation (2) we can calculate its fundamental component, when  $n=1$ , with:

$$f(t)_{V1} = \frac{4V_{CD}}{3\pi} \sum_{n=1}^{\infty} V_h^2 [\cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3)] \sin(\omega_0 t) \quad (4)$$

Where its r.m.s. value would be given by:

$$f(t)_{V1RMS} = \frac{2\sqrt{2}V_{CD}}{3\pi} [\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3)] \quad (5)$$

Similarly, the THD can be calculated with:

$$THD_V = \frac{\sqrt{f(t)_{VRMS}^2 - f(t)_{V1RMS}^2}}{f(t)_{V1RMS}} 100\% \quad (6)$$

Substituting (3) and (5) into (6) we can obtain that the THD of such a waveform can be calculated with:

$$THD_V = \frac{25\sqrt{2}\sqrt{9\pi^2 - A - B - C}}{D} 100\% \quad (7)$$

Where:

$$A = \pi^2 \left( \frac{\alpha_1 + 3\alpha_2 + 5\alpha_3}{90} \right)$$

$$B = 8[\cos(\alpha_1)^2 + \cos(\alpha_2)^2 + \cos(\alpha_3)^2]$$

$$C = 16[\cos(\alpha_1)\cos(\alpha_2) + \cos(\alpha_1)\cos(\alpha_3) + \cos(\alpha_2)\cos(\alpha_3)]$$

$$D = \cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3)$$

Equation 7 will be used as the objective function to be minimised by the algorithm proposed in this article.

### Programming methodology of the algorithm

Because the proposed algorithm uses the population dynamics of a set of elements, calculating the position and velocity of the switching angle values is necessary.

For this purpose, the position and velocity of the vectors of each element,  $i$ th and the interaction,  $j$ th can be calculated with the following equations:

$$\alpha_i^j = (\alpha_{i1}^j, \alpha_{i2}^j, \alpha_{i3}^j) \quad (8)$$

$$\Delta\alpha_i^j = (\Delta\alpha_{i1}^j, \Delta\alpha_{i2}^j, \Delta\alpha_{i3}^j) \quad (9)$$

Where,  $\alpha_{ij}$  represents the possible optimal values to be used for switching the multilevel inverter switching devices,  $\Delta\alpha_{ij}$ , is the velocity and determines the change of the position of each  $\alpha_i$ . The objective function given by equation (7), evaluates the capabilities for each  $\alpha_{ij}$  fundamental process to recalculate future positions,  $\alpha_{ij+1}$  and velocities,  $\Delta\alpha_{ij+1}$ . The following equations show how to calculate these parameters:

$$\alpha_i^{j+1} = G_b + \Delta\alpha_i^j \quad (10)$$

$$\Delta\alpha_i^{j+1} = \left\| \text{ran}\left(\frac{1}{i}\right) - \frac{P_s}{i} \right\| \quad (11)$$

The following figure shows the scheme followed for the programming of the proposed algorithm.

### Algoritmo heurístico de búsqueda aleatoria

#### Procedimiento

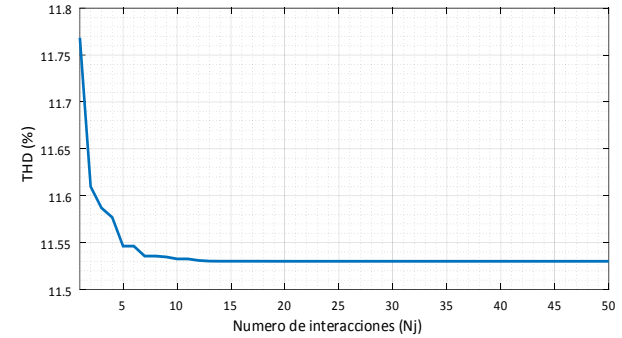
- 1: Datos de entrada:  $P_s$ , and  $N_j$ .
- 2:  $\alpha \leftarrow$  Soluciones iniciales  $\{\text{rand}(\pi/2)\}$   
Aquí asumimos que  $\alpha = \langle \alpha_1, \alpha_2, \alpha_3 \rangle$
- 3: Sort( $\alpha$ )
- 4: Evaluación de la función objetivo
- 5: Selección del mejor valor  $G_{b(0)}$
- 6: **for**( $j=1; j < N_j; j++$ ) **do**
- 7:     Calcular  $\Delta\alpha_i^{j+1}$
- 8:     Calcular  $\alpha_i^{j+1}$
- 9:     Evaluar la función objetivo
- 10:     Selección de  $G_{b(j)}$
- 11:      $j = j+1$
- 12: **end for**

Figure 8 Programming procedure

## Results

Following the methodology proposed in this article and evaluating the objective function with an initial population "Np" as well as a number of iterations "Nj" of 50, it was possible to minimise the objective function to a minimum value of minimum THDV=11.5303, with the switching angles  $\alpha_1=8.8827$ ,  $\alpha_2=27.5967$  and  $\alpha_3=50.5412$ .

The following graph shows the behaviour of the objective function.



Graph 1 Dynamics of the objective function

It can be observed that with the use of the proposed algorithm the objective function converges to a value of 11.55 from  $N_j=10$ . However, the algorithm from this interaction continues to minimise the function until it remains stable.

Based on the switching angles found, the following system implementation in the multilevel inverter was chosen.

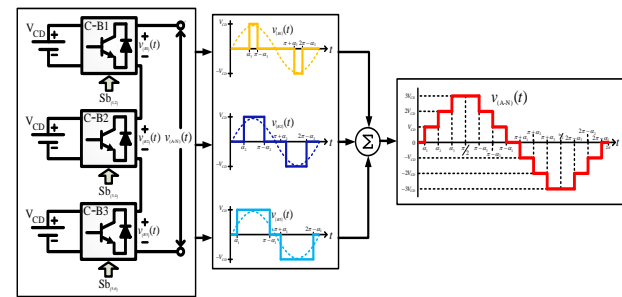


Figure 9 Implementation scheme

Figure 10 samples the seven-level waveform obtained through simulation from the scheme shown using the previously calculated optimisation angles.

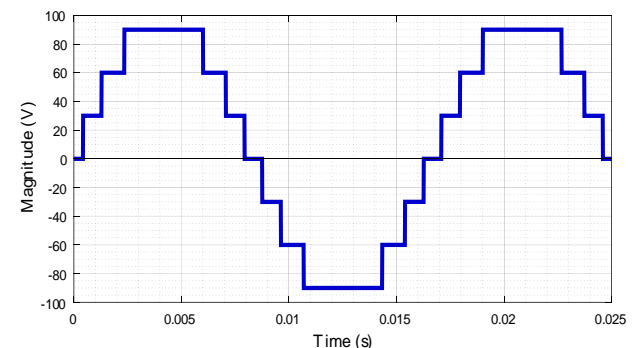
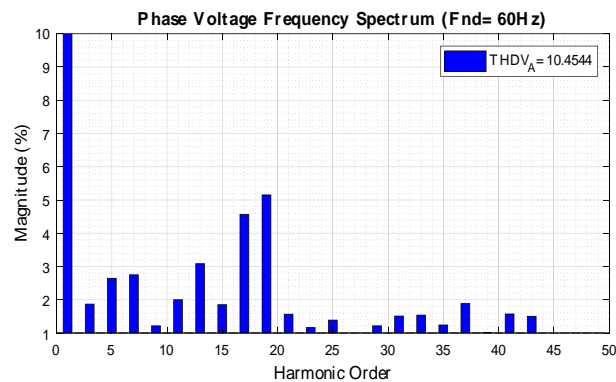


Figure 10 Seven-level waveform

Similarly, Figure 11 shows the harmonic spectrum corresponding to the staircase waveform previously shown in Figure 9. It is observed that the difference between the THDV obtained during optimisation and that determined through the waveform has a difference of about 1.0803 in the percentage of total harmonic distortion.



**Figure 11** Harmonic spectrum

## Conclusion

As could be demonstrated by the results obtained, the objective function proposed in this article was successfully minimised. The optimal switching angles were found by means of a simple and easy to implement algorithm. These results were validated by simulation through a spectral analysis on the voltage waveform obtained by a seven-level multilevel inverter. Based on the above, the use of heuristic random search algorithms for the solution of the total harmonic distortion minimisation problem in 7-level multilevel inverters is validated.

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