## Rotational vibrations absorber analysis for damped oscillatory systems

# Análisis de un absorbedor de vibraciones tipo rotacional para sistemas con amortiguamiento

VÁZQUEZ-GONZÁLEZ, Benjamín<sup>†\*</sup>, JIMÉNEZ-RABIELA, Homero, RAMÍREZ-CRUZ, José Luis and GARCÍA-SEGURA, Pedro

Universidad Autónoma Metropolitana, Unidad Azcapotzalco, División de Ciencias Básicas e Ingeniería, Departamento de Energía, Av. San Pablo 180 Colonia Reynosa Tamaulipas, Azcapotzalco C. P. 02200 Ciudad de México, México.

ID 1<sup>st</sup> Author: *Benjamín, Vázquez-González /* **ORC ID:** 0000-0002-9030-5662, **Researcher ID Thomson:** S-2417-2018, **CVU CONACYT ID:** 25749

ID 1<sup>st</sup> Co-author: *Homero, Jiménez-Rabiela /* **ORC ID:** 0000-0002-1549-0853, **Researcher ID Thomson:** S-2299-2018, **CVU CONACYT ID:** 123386

ID 2<sup>nd</sup> Co-author: *José Luis, Ramírez-Cruz /* **ORC ID:** 0000-0003-0762-2630, **Researcher ID Thomson:** G-3405-2019, **CVU CONACYT ID:** 921268

ID 3<sup>rd</sup> Co-author: *Pedro, García-Segura /* **ORC ID:** 0000-0003-4947-084X, **Researcher ID Thomson:** S-2360-2018, **CVU CONACYT ID:** 371233

#### Received July 15, 2022; Accepted December 30, 2022

#### Abstract

The phenomenon of vibration absorption is an energy exchange mechanism, which can be used in mechanical engineering applications to solve problems of attenuation or reduction of high amplitudes that a moving body or system can reach. There are basically two types of vibration absorbers; passive and active. Active vibrations absorbers are composed of servomechanisms that are capable of modifying structural conditions, to produce a specific required performance. Passive vibrations absorbers are not made up of elements that directly modify the structure of the mechanical system as a whole, thus a passive system is designed to operate under conditions that will not change over time, however, active systems will require the addition of some type of external energy, passive systems do not consume extra energy and this makes them attractive from the point of view of their accessibility. In this work, the performance of a rotationaltype passive vibration absorber for a primary system with harmonic excitation and viscous-type damping is studied.

#### Vibrations, Passive absorption, Viscous damping

#### Resumen

El fenómeno de la absorción de vibraciones es un mecanismo de intercambio de energético, el cual puede ser utilizado en aplicaciones de la ingeniería mecánica para resolver problemas de la atenuación o reducción de amplitudes elevadas que un cuerpo o sistema en movimiento puede alcanzar. Existen fundamentalmente dos tipos de absorbedores de vibraciones; los pasivos y los activos. Los absorbedores activos se componen de servomecanismos que son capaces de modificar condiciones estructurales, para producir un desempeño especifico requerido. Los absorbedores pasivos no se constituyen por elementos que modifiquen de forma directa la estructura del sistema mecánico en su conjunto, de esta forma un sistema pasivo está diseñado para operar bajo condiciones que no cambiarán en el tiempo, sin embargo, los sistemas activos requerirán de la adición de algún tipo de energía externa, los sistemas pasivos no consumen energía extra y esto los hace atractivos desde el punto de vista de su accesibilidad. En este trabajo se estudia el desempeño de un absorbedor de vibraciones pasivo tipo rotacional, para un sistema primario con excitación armónica y amortiguamiento de tipo viscoso.

Vibraciones, Absorción pasiva, Amortiguamiento viscoso

<sup>\*</sup> Author correspondence (E-mail: bvg@correo.azc.uam.mx)

<sup>&</sup>lt;sup>†</sup> Researcher contributing as first author.

### Introduction

In the disciplines of mechanical engineering science, the study of the vibration absorption phenomenon has been cultivated since its incipience, when Den Hartog managed to establish the direct interaction between the primary system and its absorber. Subsequent studies and developments have been directed towards expanding the absorption range of the absorber, these developments have led to the innovation of control techniques, an active absorber has as its fundamental reference a passive absorber, at the end of its performance the active absorber converges to passive. Novel techniques are presented in Cheung and Wong, work, where non-traditional designs of vibration absorbers are made, the technique presented in said work focuses on modifying the velocity amplitude of the primary system. For this reason, the analysis of passive vibration absorbers can be directed towards the development of active absorbers. reducing sophisticated control actions, which may require a long processing time, and may reduce their efficiency. Other techniques include damping modification along with tuning, Abdel-Hafiz and Hassaan they report results with these techniques. In this work, the analysis of a rotational type vibration absorber is carried out, for which the primary system is damped, Vázquez et. al. conducted a study for the undamped case.

The results obtained establish relations of amplitude of the vibration of the primary system in relation to the magnitude of the excitation force, likewise, it is determined that the absorption is defined by the tuning of the undamped frequency between the primary system and its absorber, in such a way that the damping does not have a direct influence, which allows modifying this parameter arbitrarily, however, the damping establishes a stable state regime.

## The rotational absorber for a primary system with damping

The rotational absorber consists of a cylindrical mass  $m_2$  which rolls without slipping on the body of the primary system  $m_1$ . This configuration establishes a geometric coupling, apart from the dynamic coupling defined by the spring  $k_2$ . The geometric coupling allows the absorber to be compact in the sense of being contained within the primary system. Figure 1 illustrates the above.

ISSN 2524-2121 ECORFAN® All rights reserved. The damping, that is considered is present in the primary system and including its presence is necessary because the damping can produce phase shifts between the forcing action, as well as attenuation in the amplitude of the response of the primary system.



Figure 1 Rotational absorber  $m_2$  coupled to the primary system  $m_1$ , due to elastic geometric and dynamic restriction

The rotational absorber is an interesting system to study, because it develops two types of movement that participate in the absorption of energy, these movements are the rotation of the primary system and the translation of the center of mass of the same absorber, unlike the single mass displacement absorber. The primary system consists of a spring-mass system, with viscous-type damping c, harmonic-type excitation with the frequency  $\omega$  and magnitude F.

### Dynamic equations of the rotational absorber

The dynamic equations of the system as a whole are determined using the Euler-Lagrange formulation, in Figure 2 the kinematic coordinates are illustrated.

The absorber has radius r and its displacement ratio is relative to the primary system motion.

Let D be the relative displacement between the center of mass of the primary system and its absorber, thus, in terms of the motion coordinates, it is true that,



Figure 2 A scheme that identifies the kinematic coordinates of the primary system and its absorber

$$D = x_2 - x_1 \tag{1}$$

Figure 3 illustrates the relative displacement between both bodies.



**Figure 3** Determination of the relative displacement of the absorber in relation to the primary system

but,

$$D = r\theta = \mathbf{x}_2 - \mathbf{x}_1 \tag{2}$$

so,

$$\theta = \frac{\mathbf{x}_2 - \mathbf{x}_1}{r} \tag{3}$$

The variables in time are the position coordinates and the angular displacement, therefore, when differentiating equation (3) in time, its results,

$$\dot{\theta} = \frac{\dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1}{r} \tag{4}$$

The expression of the kinetic energy of the system requires determining the forms of the kinetic energy of each element of the system in motion, therefore, the total kinetic energy is,

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}I\dot{\theta}^2$$
(5)

ISSN 2524-2121 ECORFAN® All rights reserved. and in terms of the displacement variables, it turns out

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}I\left(\frac{\dot{x}_2 - \dot{x}_1}{r}\right)^2 \tag{6}$$

where *I* represents the moment of inertia of the cylinder or absorber,

$$I = \frac{1}{2}m_2r^2\tag{7}$$

The potential energy of the system is manifested in the elastic elements, that is,

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2 \tag{8}$$

The Lagrangian function L, is determined by the difference between the total kinetic and potential energy,

$$L = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{4}m_2(\dot{x}_2 - \dot{x}_1)^2 - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2(x_2 - x_1)^2$$
(7)

The damper is a dissipative element of the system, it interacts directly with the primary system and its energy is quantified by the dissipation function,

$$\Delta = \frac{1}{2} c \dot{x}_1^2 \tag{8}$$

The Euler-Lagrange equations for this system, including the dissipative terms due to damping and with external excitation, are,

$$\frac{\mathrm{d}}{\mathrm{dt}} \left( \frac{\partial \mathrm{L}}{\partial \dot{\mathrm{x}}_1} \right) - \left( \frac{\partial \mathrm{L}}{\partial \mathrm{x}_1} \right) = \frac{\partial \Pi}{\partial \dot{\mathrm{x}}_1} - \frac{\partial \Delta}{\partial \dot{\mathrm{x}}_1} \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} \left( \frac{\partial \mathrm{L}}{\partial \dot{\mathrm{x}}_2} \right) - \left( \frac{\partial \mathrm{L}}{\partial \mathrm{x}_2} \right) = 0 \tag{10}$$

where,

$$\Pi = (F\cos\omega t)\dot{x}_1 \tag{11}$$

Expression (11) is known as the supplied power function, from this expression the excitation term is evaluated,

$$\frac{\partial \Pi}{\partial \dot{x}_1} = \text{Fcos}\omega t \tag{12}$$

By developing the partial and total derivatives of equations (9) and (10), together with the excitation expression, the following differential equations result,

$$\left(m_1 + \frac{1}{2}m_2\right)\ddot{x}_1 - \frac{1}{2}m_2\ddot{x}_1 + c\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = Fcos\omega t$$
(12)

$$\frac{3}{2}m_2\ddot{x}_2 - \frac{1}{2}m_2\ddot{x}_1 + k_2x_2 - k_2x_1 = 0$$
(13)

Finally, defining  $M = m_1 + \frac{1}{2}m_2$ , we have the following system of equations,

$$M\ddot{x}_1 - \frac{1}{2}m_2\ddot{x}_1 + c\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = Fcos\omega t \quad (14)$$

 $\frac{3}{2}m_2\ddot{x}_2 - \frac{1}{2}m_2\ddot{x}_1 + k_2x_2 - k_2x_1 = 0$ (15)

## Determination of dynamic absorption conditions

The system of equations (14)-(15) corresponds to a coupled linear system of differential equations, to determine which performance conditions are met, the following set of solutions is proposed,

$$x_{1}(t) = X_{11}sen(\omega t) + X_{12}cos(\omega t)$$
(16)

$$x_{2}(t) = X_{21}sen(\omega t) + X_{22}cos(\omega t)$$
(17)

Where  $X_{11}$ ,  $X_{12}$ ,  $X_{21}$  and  $X_{22}$  they are associated with the amplitude of the movement that each element or body of the system can develop in time.

Differentiating in time equations (16)-(17) and substituting equations (14)-(15), the following set of four equations results,

$$-\omega^2 M X_{11} + \frac{1}{2} m_2 \omega^2 X_{21} - \omega c X_{12} + (k_1 + k_2) X_{11} - k_2 X_{21} = 0 \quad (18)$$

$$-\omega^2 M X_{12} + \frac{1}{2} m_2 \omega^2 X_{22} + \omega c X_{11} + (k_1 + k_2) X_{12} - k_2 X_{22} = F \quad (19)$$

$$-\omega^2 \frac{3}{2}m_2 X_{21} + \frac{1}{2}m_2 \omega^2 X_{11} + k_2 X_{21} - k_2 X_{12} = 0$$
(20)

$$-\omega^2 \frac{3}{2}m_2 X_{22} + \frac{1}{2}m_2 \omega^2 X_{12} + k_2 X_{22} - k_2 X_{12} = 0$$
(21)

The system of equations (18)-(21) can be written in matrix form for the four unknowns  $(X_{11}, X_{12}, X_{21}, X_{22})$ , the result is the following,

$$\begin{bmatrix} k_{1} + k_{2} - \omega^{2}M & -\frac{c\omega}{2} - \frac{1}{2}m_{2}\omega^{2} - k_{2} & \frac{0}{2}m_{2}\omega^{2} - k_{2} \\ \frac{1}{2}m_{2}\omega^{2} - k_{2} & \frac{1}{2}m_{2}\omega^{2} - k_{2} \\ 0 & \frac{1}{2}m_{2}\omega^{2} - k_{2} & \frac{0}{2}m_{2}\omega^{2} \\ 0 & \frac{1}{2}m_{2}\omega^{2} - k_{2} & \frac{0}{2}m_{2}\omega^{2} \\ \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \\ X_{21} \\ X_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{0} \\ 0 \end{bmatrix}$$
(22)

From the system of equations (22), it is possible to determine dynamic conditions for absorption, which is reflected as the performance for which the resulting amplitude of the primary system is the smallest possible or null. In the process of searching for such conditions, it is first necessary to determine the expression of each of the components of the amplitude. The solutions for the system of equations of system (22) are presented below.

$$X_{11} = (36cm_2^2\omega^5 - 48ck_2m_2\omega^3 + 16ck_2^2\omega)\frac{F}{4}$$
(23)

$$X_{12} = (a_1\omega^6 + b_2\omega^4 + c_2\omega^2 + f)\frac{F}{4}$$
(24)

where,

$$a_1 = 6m_2^3 - 36Mm_2^2 \tag{25}$$

$$b_2 = 36k_1m_2^2 + 8k_2m_2^2 + 48Mk_2m_2 \tag{26}$$

$$c_2 = -(48k_1k_2m_2 + 16Mk_2^2 + 8k_2^2m_2) \quad (27)$$

$$f = 16k_1^2k_2^2 \tag{28}$$

$$X_{21} = (12cm_2^2\omega^5 - 32ck_2m_2\omega^3 + 16ck_2^2\omega)\frac{F}{\Delta}$$
(29)

$$X_{22} = (a_3\omega^6 + b_3\omega^4 + c_3\omega^2 + f)\frac{F}{\Delta}$$
(30)

where,

$$a_3 = 2m_2^3 - 12Mm_2^2 \tag{31}$$

$$b_3 = 12k_1m_2^2 + 32Mk_2m_2 \tag{32}$$

$$c_3 = -(32k_1k_2m_2 + 16Mk_2^2 + 8k_2^2m_2) \quad (33)$$

The determinant of the system of equations is the following,

$$\Delta = a\omega^8 + b\omega^6 + d\omega^4 + e\omega^2 + f \tag{34}$$

where in turn,

$$a = m_2^4 - 12Mm_2^3 + 36M^2m_2^2 \tag{35}$$

 $b = 12k_1m_2^3 + 4k_2m_2^3 - 72Mk_1m_2^2 - 16Mk_2m_2^2 - 48M^2k_2m_2 + 36c^2m_2^2$ (36)

$$d = 96Mk_1k_2m_2 + 16Mk_2^2m_2 - 48c^2k_2m_2 + 16k_1k_2m_2^2 + 16M^2k_2^2 + 36k_1^2m_2^2 + 4k_2^2m_2^2$$
(37)

$$e = 16c^2k_2^2 - 16k_1k_2^2m_2 - 32Mk_1k_2^2 - 48k_1^2k_2m_2$$
(38)

The set of above expressions is very extensive as can be seen, however, it is possible to determine the parameters for the required performance.

It is sought that the amplitude of the primary system is as small as possible, which establishes that the amplitudes of the movement meet the following condition,  $X_{11} = X_{12} = 0$ .

From the numerator of equation (23) it can be seen that it can be reduced to the following expression,

$$X_{11} = (9m_2^2\omega^4 - 12k_2m_2\omega^2 + 4k_2^2)\omega c = 0$$
(39)

since the frequency is not null because it is the movement parameter and the case in which damping is present is being analyzed, it is concluded that the required parameters are found in the expression,

$$9m_2^2\omega^4 - 12k_2m_2\omega^2 + 4k_2^2 = 0 \tag{40}$$

where does it come from,

$$\omega^2 = \frac{2k_2}{3m_2} \tag{41}$$

Equation (29), which corresponds to the second term of the amplitude of the primary system, can also provide information to determine the absorption parameters. Substituting expression (41) in equation (29) it is found that

$$X_{12} = X_{12}(\omega^2) = 0. (42)$$

With which it is concluded that equations (23) and (24) are canceled for the tuning frequency (41), this allows choosing one of two design parameters for the vibration absorber, however, as observed, the damping c, is a parameter that does not participate in the absorption process, so it can be proposed arbitrarily. However, the damping c is present in the response of each element of the system, this is observed in the terms of the determinant (34), so the magnitude of the response is defined by the numerical value that it can reach.

A final stage of validation in determining the design parameters of the absorber consists of obtaining design parameters from equations (29) and (30). However, reevaluating equation (41) in equations (29) and (30), results in the following,

$$X_{21} = X_{21}(\omega^2) = 0. (43)$$

and

$$X_{22} = X_{22}(\omega^2) = 0. (44)$$

With these results it is concluded that there is only one design parameter and it corresponds to equation (41).

#### Determination of response in time

To obtain a representation of the performance over time of the system and its absorber, the system of differential equations (14)–(15) is expressed in the state space representation, for which the following changes of variables are made,

$$\dot{x}_1 = \dot{y}_1, \dot{y}_1 = y_2, \dot{x}_2 = \dot{y}_3, \dot{y}_3 = y_4$$
 (45)

which allows us to write the system (14)-15 as,

$$\begin{bmatrix} \dot{y}_{1} \\ \dot{y}_{2} \\ \dot{y}_{3} \\ \dot{y}_{4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{6k_{1} + 4k_{2}}{d_{1}} & \frac{6c}{d_{1}} & -\frac{4k_{2}}{d_{1}} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k_{2} - 2k_{1}}{d_{1}} - \frac{4Mk_{2}}{d_{2}} & \frac{2c}{d_{1}} & \frac{4Mk_{2}}{d_{2}} & 0 \end{bmatrix} \begin{bmatrix} y_{1} \\ y_{2} \\ y_{3} \\ y_{4} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{6F\cos\omega t}{d_{1}} \\ 0 \\ -\frac{2F\cos\omega t}{d_{1}} \end{bmatrix}$$
(46)

where,  $d_1 = m_2 - 6M$  and  $d_2 = m_2(m_2 - 6M)$ 

The representation in the space of states allows to obtain answers in time, from numerical simulations.

## Time responses of the primary system and its absorber

Simulation results are presented in the time response of the system from the state space representation. The simulations are performed in situations in which is tuned to the absorber for equation (41) and in its case different damping values, the above is the result of having determined that damping is a parameter that can be set arbitrarily, however, it is part of the system response. The values of force and frequency are arbitrary, however, they are significant, in the sense that a clear perception can be had about the effect that 15 N produces on a mass of 4 kg.

The frequency has been chosen to be 5 rad/sec., which also allows us to clearly observe the shape of the response over time.

Table 1 shows the data for a very massive primary system in relation to its absorber, the tuning is not performed and an arbitrary viscous damping is considered.

$m_1 = 4 \text{ kg}$	$m_2 = 0.5 \text{ kg}$	$k_1 = 100 \text{ N/m}$
$k_2 = 300 \text{ N/m}$	F = 15 N	$\omega = 5 \text{ rad/s}$
$c = 10 \text{ N} \cdot \text{s/m}$		

**Table 1** Parameters of the vibrating mechanical systemcorresponding to Figures 4 and 5

Figure 4 shows the time response of the primary system  $x_1(t)$  for the data in Table 1. The magnitude of the amplitude is approximately  $x_1(t) = 0.13$  m, likewise, a uniform response is verified, according to the steady state. There is no tuning between both subsystems.

Figure 5 shows the time response of the secondary system  $x_2(t)$  for the parameters in Table 1, in which the tuning condition is not considered. The amplitude of the absorber reaches approximately  $x_2(t) = 0.14$  m.

Table 2 shows the set of parameters for the previous experiment, in this case there is the tuning condition between the primary and secondary systems.



**Figure 4** Time response of the primary system for the parameters in Table 1



**Figure 5** Response in time of the secondary system for the parameters in Table 1

$m_1 = 4 \text{ kg}$	$m_2 = 0.5 \text{ kg}$	$k_1 = 100 \text{ N/m}$
$k_2 = 18.75 N/m$	F = 15 N	$\omega = 5 \text{ rad/s}$
$c = 10 \text{ N} \cdot \text{s/m}$		

**Table 2** Parameters of the vibrating mechanical systemcorresponding to Figures 6 and 7

Figure 6 shows the time response of the primary system in tuning with its absorber, in accordance with the data in Table 2. The amplitude of the primary system reaches a value  $x_1(t) = 0.065$  m in the steady state, in relation to Figure 4, which corresponds to the primary system without tuning, a reduction of close to 50% is observed.



Figure 6 Time response of the primary system for the parameters in Table 2

Figure 7 shows the time response of the absorber for the parameters in Table 2, the developed amplitude reaches  $x_2(t) = 2.24$  m, this magnitude corresponds to a hypothetical case, in according to the proposed data, however, the effect of absorption can be observed, which requires a high amplitude to absorb and release the mechanical energy transferred from the primary to the secondary system.



**Figure 7** Time response of the primary system for the parameters in Table 2

Table 3 shows the same parameters as Table 2, with the exception that the damping value has been modified, remembering that this parameter can be modified arbitrarily, in this case its numerical value has been doubled and it follows keeping the tuning.

$m_1 = 4 \text{ kg}$	m <sub>2</sub> =0.5 kg	$k_1 = 100 \text{ N/m}$
$k_2 = 18.75 \text{N/m}$	F = 15 N	$\omega = 5 \text{ rad/s}$
$c = 20 \text{ N} \cdot \text{s/m}$		

**Table 3.** Parameters of the vibrating mechanical systemcorresponding to Figures 8 and 9.

Figure 8 shows the response in time for the primary system for the data in Table 3, in this case the tuning of the primary system with its absorber is preserved, but the damping value has been increased twice in relation to Table 2. A steady state amplitude of approximately  $x_1(t) =$ 0.062 m is identified, which means a small decrease in relation to the previous experiment.



**Figure 8** Time response of the primary system for the parameters in Table 3

Figure 9 shows the time response of the absorber for the data in Table 3. The amplitude recorded in the stable state reaches a value of  $x_2(t) = 2.089$  m, which represents a very small value in the decrease amplitude in relation to the parameters in Table 2.



**Figure 9** Time response of the secondary system for the parameters in Table 3

Table 4 shows the parameters in which the mass of the absorber has doubled in relation to the previous experiments, the value of viscous damping c has decreased by half, tuning is considered, for this reason, we have a value of  $k_2$ different from the previous ones.

$m_1 = 4 \text{ kg}$	m <sub>2</sub> =1 kg	$k_1 = 100 \text{ N/m}$
$k_2 = 37.5 \text{N/m}$	F = 15 N	$\omega = 5 \text{ rad/s}$
$c = 10 \text{ N} \cdot \text{s/m}$		

**Table 4** Parameters of the vibrating mechanical systemcorresponding to Figures 10 and 11

Figure 10 shows the graph of the development over time of the primary system. The amplitude in the stable state reports a magnitude of  $x_1(t) = 0.061$  m, tuning is present and although the damping is the same as that of the parameters in Table 3, the mass of the absorber has increased, the amplitudes, between Figures 8 and 10 are very similar, however, it can be seen in Figure 11, that the amplitude of the absorber has decreased in a proportion close to half, this is due to the increase in energy consumption to put in motion a greater mass, the measured amplitude has an approximate value of  $x_2(t) = 1.09$  m. Table 5 shows the parameters for the last experiment, the mass of the absorber has doubled, as well as the viscous damping c, in relation to the parameters in Table 4.



**Figure 10** Time response of the primary system for the parameters in Table 4



**Figure 11** Time response of the secondary system for the parameters in Table 4

Tuning is considered in the experiment. It should be noted that in the cases of tuning, a different value is obtained when the value of the absorber mass is modified, which produces an effect of variable elastic stiffness.

$m_1 = 4 \text{ kg}$	m <sub>2</sub> =2 kg	$k_1 = 100 \text{ N/m}$
$k_2=75 \ N/m$	F = 15 N	$\omega = 5 \text{ rad/s}$
$c = 20 \text{ N} \cdot \text{s/m}$		

**Table 5** Parameters of the vibrating mechanical systemcorresponding to Figures 12 and 13

Figure 12 shows a greater attenuation of the amplitude of the movement of the primary system,  $x_1(t) = 0.053$  m, in relation to Figure 10.



**Figure 12** Time response of the primary system for the parameters in Table 5

Figure 13 shows the time response of the absorber, it is observed that the joint effect of increasing the mass of the absorber and increasing the damping, under the tuning condition and this produces a reduction in the amplitude of the primary system and significantly in that of the secondary,  $x_2(t) = 0.049$  m, again a value close to half the response of the secondary system in relation to the parameters in Table 4.



**Figure 13** Response in time of the secondary system for the parameters in Table 5

#### Acknowledgement

The authors appreciate the institutional support of the Universidad Autónoma Metropolitana and the Azcapotzalco Campus, in particular the Division of Basic Sciences and Engineering and the Department of Energy.

### Conclusions

Vibration absorption is a complex phenomenon of mechanical systems, its implementation in mechanical engineering requires the study of transfer mechanical energy phenomena. Knowing the effects of modifying some parameters is important to achieve efficient device design and development, even more so when simple devices are implemented in complex performances. In this work, the analysis of a rotational type vibration absorber has been developed for the case of a primary system with viscous type damping. The reported results present the possibility of openly choosing the damping value, which induces the behavior of the stable state in both bodies of the system as a whole and, in turn, knowing in advance the tuning condition, efficient results are obtained in the sense to reduce the amplitude of both the primary and secondary systems. The rotational absorber for a forced system with viscous damping allows adjustments in the parameters in such a way that the attenuation of the amplitude of the primary system, as well as of the secondary, is achieved.

### References

Den Hartog J P. Mechanical vibrations. Dover Publications Inc; 1985.

Mohammed Abdel-Hafiz and Galal Ali Hassaan. (2015) Tuning Condition Modification of Damped and Un-damped Adaptive Vibration Absorber. International Journal of Computer Techniques, Volume 2, Issue 2, 2015,170-175. http://www.ijctjournal.org/Volume2/Issue2/IJC T-V2I2P28.pdf

Vázquez González Benjamín, Jiménez Rabiela Homero, Ramírez Cruz José Luis, (2020) Design of a rotational type vibration absorber. Journal of Mechanical Engineering, ECORFAN-Spain. June 2020, Vol.4, No.13 16-22. DOI: 10.35429/JME.2020.13.4.16.22.

Y. L. Cheung and W. O. Wong. (2009) Design of a non-traditional dynamic vibration absorber (L). J. Acoust. Soc. Am. 126 (2), August 2009, 564-567. DOI: 10.1121/1.3158917.