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Fuzzy Modeling of DC Motors using the stimulus-response method

Modelado Difuso de Motores de CD usando el método de estímulo-respuesta

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Abstract

In this paper gives a brief review of the traditional modeling of *direct current* (DC) motors using the *stimulus response* method is made and a method is proposed to obtain an alternative model, using fuzzy logic theory. The two models capture the behavior of the DC motor in an acceptable manner. The connotation of acceptable is because some characteristics of the DC motor are disregarded, which do not have repercussions, in practical terms, as error factors in its study. The first model relates the analysis to the classical forms of control theory, through a first-order differential equation. The second model relates the analysis to the principles of generalization of necessary knowledge in reasoning and learning within the area of Artificial Intelligence where fuzzy sets and a series of if-then type rules are used. The fuzzy DC motor model is very convenient, within the control context, when using Artificial Intelligence paradigms but it not limited to this area.

Fuzzy Model, Control, DC motor

Resumen

En este artículo se hace una breve revisión del modelado tradicional de motores de *corriente directa* (CD) utilizando el método de estímulo respuesta y se propone un método para obtener un modelo alternativo, utilizando la teoría de lógica difusa. Los dos modelos capturan de forma *aceptable* el comportamiento del motor de CD. La connotación de *aceptable* obedece al hecho que, se desprecian algunas características del motor que no repercuten, en términos prácticos, como factores de error en su estudio. El primer modelo relaciona el análisis con las formas clásicas de la teoría de control, mediante una ecuación diferencial de primer orden. El segundo modelo, relaciona el análisis con los principios de generalización del conocimiento necesarios en el razonamiento y aprendizaje dentro del área de la Inteligencia Artificial, donde se utilizan conjuntos difusos y una serie de reglas del tipo si-entonces. El modelo difuso del motor es muy conveniente, dentro del contexto de control, cuando se utilizan paradigmas de Inteligencia Artificial pero no se limita a esta área.

Modelado difuso, Control, motor CD

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Introduction

DC motors have wide popularity because they are very versatile in industrial (Kumar, Surya, Tabrez, Afida, & Molla Shahadat, 2022), domestic (Khan, *et al*., 2021), and laboratory (Said, *et al*., 2022) mechanical applications. In many industries, DC motors are used as elements that perform essential tasks within their processes. The wide uses of DC motors motivate many laboratory projects to address some control problems (Goolak, *et al*., 2023) without the need to disassemble the motor to bring it to the laboratory. In control projects of design and/or redesign of control loops, it is necessary to have mathematical models of the elements involved and, within context, of the DC motor (Cero, Vázquez-Espinoza, & Aquino-Díaz, 2017).

It is possible to obtain the motor structure representation of the motor behavior using the stimulus-response method (Nasimba Medina & Nasimba de Janon, 2018), that is, by analyzing motor response to a disturbance (Fang, *et a*l., 2023). Properly speaking about the transfer function. In general, the model with a first-order differential equation equivalence can be obtained (Haro Martínez, 1998).

In the existing bibliography, to analyze and design control loops in industrial processes or control projects, mathematical modeling of motors is performed based on time constants and motor torque (Valtchev, Meshcheryakov, Gracheva, Sinyukov, & Sinyukova, 2023). Practical implementation of a control system involves knowledge of these values, having as objectives, position, or speed control. An example of the usefulness of this work is in robotic systems (Zhao & Seung-Hoon, 2023), where there is a need to know the rotational position of joints in automotive assembly lines using representation models that are easily obtained without detriment to simple operation and/or understanding.

This paper is organized as follows: Section II overviews control and fuzzy logic theory. Section III presents the procedure to obtain the parameters of a first-order differential equation, represented in the complex plane, which models the DC motor. The process obtains the fuzzy rules modeling the motor is presented in Section IV. Finally, the conclusions are shown in Section V.

II. Background

A. Control theory

The analog control loop is represented by block diagrams which contain an algebraic mathematical expression (differential equations in the complex plane) (Dorf & Bishop, 2017). The equations contained in the blocks represent the behavior of the defined elements (plant, controller, transducer, etc.) as a function of the input-output. The relation between output and input is known as the *transfer function* (TF). The TF of the DC motor involves a couple of constants based on the physical characteristics of the motor. Knowing the value of these constants allows precise knowledge of the value of the angular position of the shaft, the applied speed-voltage relationship, and the motor torque.

B. Fuzzy logic theory

In possibility theory, a fuzzy set (FS) $\tilde{a}$ is used to delimit poorly known values and/or to represent values characterized by symbolic expressions (Kaufmann & Gupta, 1991). The set is defined as $\tilde{a} = (a_1, a_2, a_3, a_4)$ such that $a_1, a_2, a_3, a_4 \in \mathbb{R}$, $a_1 \leq a_2$ y $a_3 \leq a_4$ (Klir & Yuan, 1995) (Fig. 1).

Definition 1: A FS $\tilde{a}$ of a universe of discourse X={x} is defined as a mapping $\alpha_{\tilde{a}}: X \rightarrow [0, \gamma]$ where each $x \in X$ is assigned to a number in the range between $[0, \gamma]$.

FS $\tilde{a}$ is a membership $\alpha_{\tilde{a}}(x)$ of x elements, that are compatible with the $\tilde{a}$ concept.

Definition 2: FS is normal when membership function is normalized $\alpha_{\tilde{a}}: X \rightarrow [0,1]$.

A FS $\tilde{a}$ is referred to indistinctly by function $\alpha_{\tilde{a}}(\tau)$ or characterization (a_1, a_2, a_3, a_4), (Fig. 1).

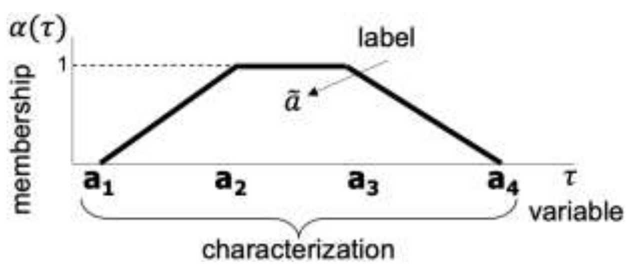

Figure 1 Fuzzy set representation
Source (González Castolo & López Mellado, 2009)

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In this work, normal and trapezoidal FS will be used where $(a_2 = a_3)$, that is, triangular FS, $\tilde{a} = (a_1, a_2, a_4)$.

Definition 3: A *Fuzzy Rule* (FR) is identified as Rx with the form *if-then* where exists the antecedent and consequent represented by fuzzy sets;

Rx: if *antecedent* then *consequent*

III. DC motor model with the classical method

The physical system under study consists of a DC motor with a tachometer coupled to know when the motor reaches its maximum operating speed, (Fig. 2).

In most DC motors, the effect of the motor inductance L_a can be ignored because it is negligible with respect to the other values of the system. Under this consideration, simplifying the block diagram as Fig. 3(a), which indicates the ratio of the position $\theta(s)$ with respect to the supply voltage $E_{a_i}(s)$. Using this TF, the ratio of the speed and the supply voltage can be obtained, $W_o(s)/E_{a_i}(s)$. It is known that $w_o(t) = d(\theta)/dt$ in the complex plane remains as $W_o(s) = s\theta(s)$, (Fig. 3(b)).

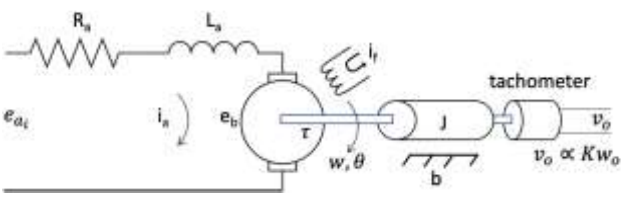

Figure 2 Motor physical system
Source: (Haro Martínez, 1998)

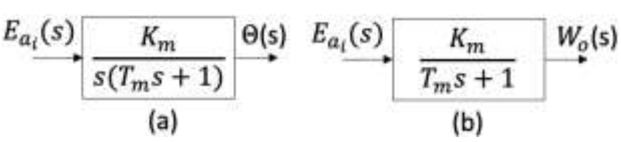

Figure 3 Motor transfer function
Source: (Haro Martínez, 1998)

The (1) is obtained by manipulating the *speed-supply voltage* relationship.

$$W_o(s)\left(\frac{K_m}{T_m}\right)\frac{1}{s+\frac{1}{T_m}}E_{a_i}(s) \tag{1}$$

The (2) is obtained considering that a unit step input is applied.

$$e_{a_i}(t) = \mu(t) \Longrightarrow E_{a_i}(s) = \frac{1}{s} \tag{2}$$

the values of K_m, (3), and T_m, (4), can be obtained.

$$K_m = c_1 \tag{3}$$

$$T_m = \frac{t_d}{\ln(2)} \tag{4}$$

With (3) and (4), the motor TF $W_o(s)/E_{a_i}(s)$ can be obtained.

This procedure boils down to measuring the motor response. The terminals of an oscilloscope are placed at the output of the tachometer and the nominal voltage applied $e_{a_i}(t)$ to the motor to obtain the graph of Fig. 4(a). The vertical axis has two variables with different scales because $v_o(t)$ is proportional to $w_o(t)$. The TF parameters of the motor (1) are calculate using (3) and (4). For example, the speed motor (7) with $t_d = 10$ time units and $c_1 = 1$ speed units the correspondent point is shows in Fig. 4(b) using (10).

$$w_o(t) = 1 - e^{-\frac{t}{14.4}} \tag{10}$$

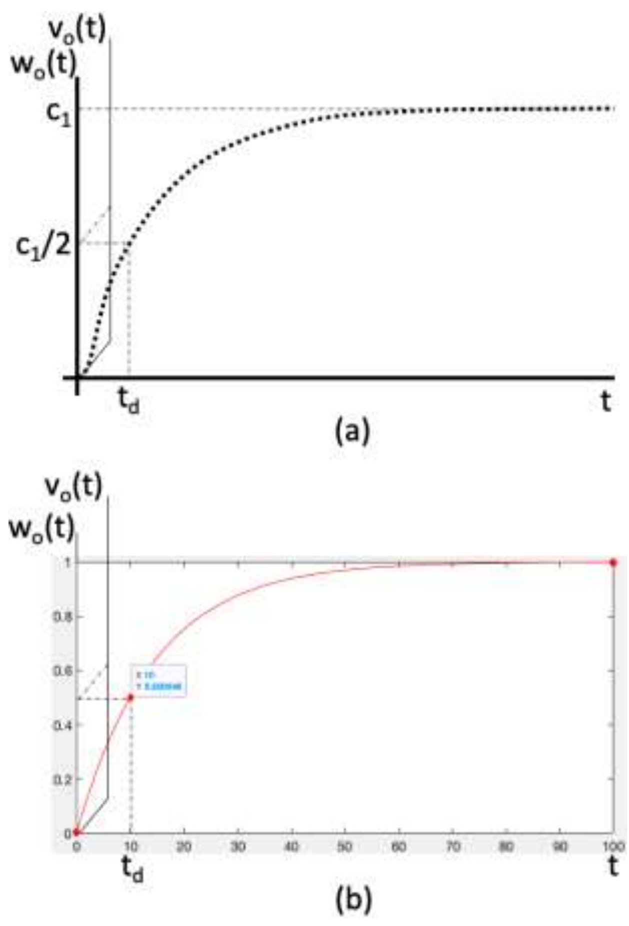

Figure 4 a) Voltage normalized of tachometer in oscilloscope. b) Mathematic $w(t)$ curve
Source: (Haro Martínez, 1998)

IV. Fuzzy logic DC motor model

A fuzzy version of the motor model is useful in control and automation applications using elements of Artificial Intelligence theory.

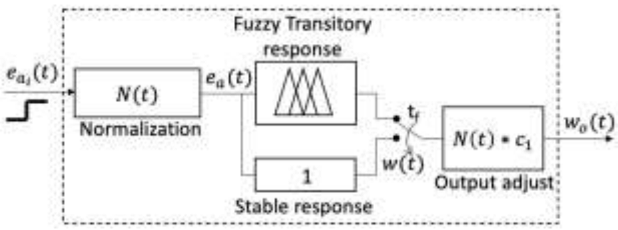

Figure 5 Fuzzy DC motor model
Own source

The fuzzy model has some blocks inside it, (Fig. 5). First, a *Normalization block* $N(t)$ guarantees that the variation of $e_a(t)$ will be in $[0,1]$ range as is shown in (11).

$$N(t) = \frac{e_{a_i}(t)}{\max(e_{a_i}(t))} \quad (11)$$

The *Fuzzy transitory response block* (FTR(t)) is the output at time $[0, t_f]$, and this consists of fuzzy sets and fuzzy rules.

The *Stable response block* (SR) is the output after the input does not change, and this is time t_f.

Finally, the *Output adjust block* gives the speed $w_o(t)$ as shown in (12).

$$w_o(t) = (FTR(t) \vee SR)N(t)c_1 \quad (12)$$

For FTR(t), the Mamdani fuzzy inference method is used, which is more common and intuitive, in this case, than the Takagi-Sugeno method. For defuzzification, the centroid method is used.

Examining the motor response curves, (Fig. 4), the following work point are obtained: (0,0), $(t_d, 0.5c_1)$, (t_f, c_1) where $t_d = 10$ and $t_f = 10t_d$. The t_f is the time when the output will be stable. In this example, $e_a(t) = e_{a_i}(t)$ because the input is the unit step.

To fuzzy model the behavior of DC motor, three fuzzy rules are proposed, (Table 1).

R1	**if** short-time **then** slow-speed
R2	**if** middle-time **then** average-speed
R3	**if** large-time **then** fast-speed

Table 1 Fuzzy Rules
Own Source

For the above rules, three FSs are proposed as antecedents and three FSs as consequents. The sets are defined in Table 2.

Antecedent (time)		Consequent (speed)	
short	$t_f(0,0,0.5)$	slow	(0,0,0.4)
middle	$t_f(0.03,1,1)$	average	(0,1,2)
large	$t_f(0.1,0.5,0.9)$	fast	(0.6,1,1.4)

Table 2 Fuzzy sets
Own Source

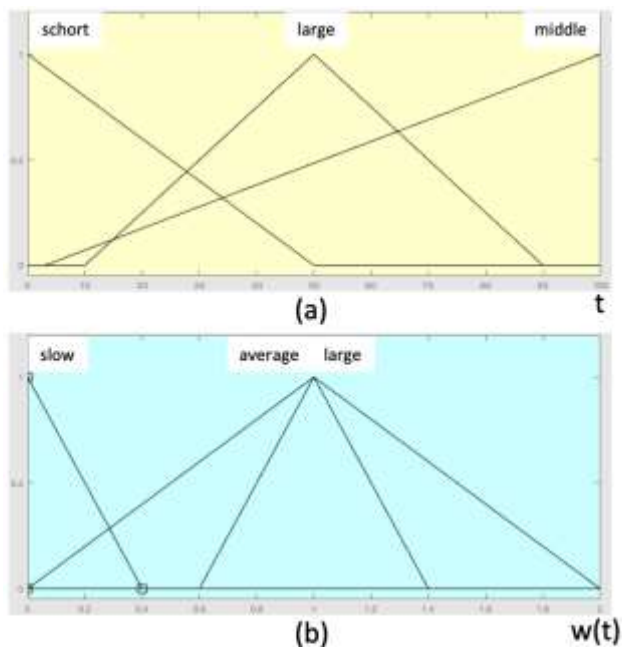

Figure 6 a) Antecedent fuzzy sets. b) Consequent fuzzy sets
Own Source

The range of variation of the antecedents and consequents can be from negative to greater than one. In this case, they are $(0, t_f)$, $(0,2)$, respectively. The sets used are shown graphically in Fig. 6.

Fig. 7 shows the fuzzy sets in the rules and the defuzzification (motor speed $w(t)$) for a specific time of *25* units. In this case, all antecedents in the rules have some degree of truth. Therefore, each has an answer that contributes to form the parallelogram whose centroid is the answer $w(25) = 0.845$. This calculus means that motor speed has *84.5%* of nominal speed c_1 at *25* time units.

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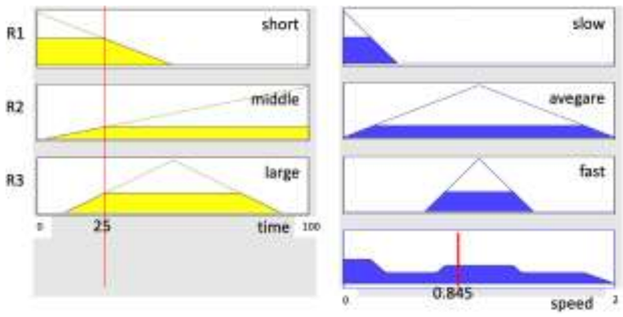

Figure 7 Defuzzification
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The motor response curve $w(t)$ is shown in Fig. 8. The point that corresponding to the calculation performed above for 25 time units is marked. After 100 time units, the response switches to *SR*.

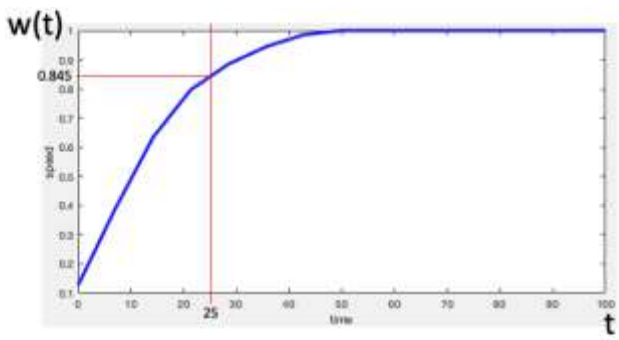

Figure 8 Motor response in time-speed relation
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Results comparison

The following table, Table 3, shows some results obtained (defuzzified values) and they are compared with the values of the motor response of the first model, (10). Here, $w_o(t) = w(t)$ because $c_1 = 1$.

Input (t) (time units)	Output of classical model $100(w_o(t))$	Output of fuzzy model $100(w(t))$
5	32	27.4
10	50	51
15	64	65.1
20	75	77.4
25	82	84.7
30	87	89.9
35	92	94

Table 3 Results comparison
Own Source

V. Conclusions

Some procedures for calculating the model of DC motors have been presented, and the results obtained are very reliable even though the base calculation process is approximate. It can be concluded that the fuzzy model is a good approximation of the motor behavior.

The importance of this procedure lies in the fact that it shows how a mechanical entity with intelligence features, based on experience, can improve the knowledge of its environment. If the robot controller is desired to be autonomous, adaptive, and causal, then an increasingly accurate reality can be reconstructed from the cause-effect relationship. In future work, a genetic algorithm will be used to find the improved fuzzy sets. In addition, based on the described procedure, thermal and hydraulic systems, among others, could be modeled with fuzzy sets using genetic algorithms to find these.

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