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Article

Design and selection of solid urban waste collection routes using mixed integer linear programming under a practical approach

Diseño y selección de rutas de recolección de residuos urbanos solidos utilizando programación lineal entera mixta bajo un enfoque práctico

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Abstract

This research presents the design and selection of solid urban waste collection routes that may or may not be polluting depending on the area of application of the problem addressed, this design uses a mathematical model in mixed integer linear programming, in which the vehicle leaves from a collection center to each collection point located at strategic points of a city, through a fleet of homogeneous vehicles with limited known capacity in which it seeks to minimize the total distance traveled, the time used in collection as well as the reduction in the use of available resources to carry out this activity.

Design and selection of solid urban waste collection routes using mixed integer linear programming under a practical approach		
Objetive	Methodology	Contribution
Design of distribution or collection routes that contribute to the efficient management of available logistics resources.	Use operations research through mixed integer linear programming that provides a solution to the problem addressed	An optimization model in MIPL appropriate to the problem at hand. Optimal design of distribution or collection routes that makes efficient use of available logistics resources.

VRP, MILP, Transport

Resumen

En esta investigación se presenta el diseño y selección de rutas de recolección de desechos urbanos solidos que pueden ser contaminantes o no dependiendo del área de la aplicación del problema abordado, este diseño utiliza un modelo matemático en programación lineal entera mixta, en el cual se el vehículo parte desde un centro de acopio hacia cada punto de recolección ubicados en puntos estratégicos de una ciudad, mediante una flotilla de vehículos homogénea con capacidad conocida limitada en la que se busca minimizar la distancia total recorrida, el tiempo utilizado en la recolección así como también la reducción en el uso de los recursos disponibles para llevar a cabo esta actividad.

Diseño y selección de rutas de recolección de residuos urbanos solidos utilizando programación lineal entera mixta bajo un enfoque práctico		
Objetivo	Metodología	Contribuciones
Diseño de rutas de distribución o recolección que coadyuven al manejo eficiente de los recursos logísticos disponibles.	Utilizar la investigación de operaciones por medio de la programación lineal entera mixta que de solución al problema abordado.	Modelo en MIPL de optimización adecuado al problema abordado. Diseño optimo de rutas de distribución o recolección que eficiente los recursos logísticos disponibles.

VRP, MILP, Transporte

Area: Development of strategic leading-edge technologies and open innovation for social transformation

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Introduction

In the last decade, economic growth has favored the increase in the country's domestic consumption, thanks to this the emission of solid waste has also been increasing, coming to be considered a problem of vital importance since there are limited resources for its collection Fig. 1. In this type of problems the solution not only depends on the application of various methodologies, tools or knowledge in order to find a route to follow that satisfies the demand for collection of these wastes at minimum cost, it also depends largely on a change of the inhabitants and their governments which must work on a better analysis of the designs of collection systems that allow developing efficient and innovative solutions to this problem.

Within the problem of urban solid waste collection, a wide variety of tasks or activities can be found to be developed, ranging from the location of the containers where each user must deposit their waste, to studies of garbage generation, passing through the definition or calculation of both the fleet size and the personnel necessary to cover the demand, the design of the collection routes to follow, and thus reaching the waste treatment site.

Box 1

Figure 1
Urban solid wast

The general urban solid waste collection system can be divided into three stages. The first identifies the different sources of waste emissions and estimates their quantities. In the second stage, the waste collection system is established, leading to the third and final stage, which is waste treatment or disposal Mula et al. (2010).

The application of new tools and technologies to problems such as vehicle routing has increased in recent years. The goal is to find a solution that provides decision-makers with sufficient information about the different variables that make up the distribution system to ensure efficient use and allocation of available logistics resources.

Within these transportation or collection problems we find the vehicle routing problem (VRP) for its acronym in English, in Elatar et al, (2023) where it describes a series of special and general considerations intrinsic to the VRP among them it is mentioned that this problem belongs to a special type of combinatorial optimization, NP-Hard and that it was referred to for the first time in 1959 by Dantzing, whose primary objective is to design the different optimal distribution and / or collection routes that each of the vehicles must use starting from a distribution center, warehouse or plant and that through the visit scheduling provided by the solution to the problem, carry out the collection and / or delivery activities in order to meet the demand established at each point (client) and then return to the central warehouse (to the starting point), research on this problem has grown very rapidly causing multiple variants of this VRP.

The problem addressed arises from the need for proper management of the use of various resources, including materials, equipment, and human resources available to carry out solid waste collection in a city in the southern region of Sonora, using a mathematical model that contributes to a better allocation of these resources, respecting applicable regulations and standards.

Method Description

Currently, the importance of proper management of solid waste collection has been increasing due to the major health threats and environmental impact this represents. Hence, the need for adequate and efficient procedures for managing this waste that help optimize the use of available resources.

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Among the many studies on this problem, one can find Dotoli and Epicoco (2017), who present a technique for solving the collection routing and scheduling problem based on limiting the total distance traveled by road along the route in order to save operating costs and particulate pollutant emissions into the environment, considering service times, vehicle availability, and time windows.

In the literature, multiple techniques can be found to find solutions to the problem addressed. One of them is proposed by Hnaien et al (2016), who uses a mixed-integer linear programming (MILP) mathematical model whose objective is to minimize the sum of the total costs. Likewise, Kechmane (2015) used a mathematical model with the objective of minimizing total transportation costs, also seeking to optimize the allocation of vehicles to trips, thus reducing the distances traveled. This work considers the capacity restrictions of each vehicle and each distribution center, as well as the fulfillment of the demand in each client.

Having efficient collection routes can help reduce operating costs and environmental emissions by reducing the route travel distance. It is critical to apply new technologies and optimization techniques to this problem, such as the one proposed by Akhtar et al (2017). He presents a modified search algorithm for the capacitated vehicle routing problem (CVRP). This algorithm uses the concept of a smart container to find the best waste collection route. This smart container provides a signal of the waste level it has, sending this signal to a center, which then maps the collection route to only those containers that need to be emptied, finding an optimal range, thus minimizing the distance traveled.

Other authors such as Hassan Sayed et al (2018) propose in their study the use of two different models, the first designed to minimize the installations of the containers and the second to maximize the coverage by seeking to find the optimal locations of these collection centers, in their study they consider four main or effective factors, including the total coverage of the service, the residential commitment, the capacity of the containers in each location and the relationship between the standard deviation and the arithmetic mean of the volume of solid waste in each station.

The results obtained from the comparison of these two methods demonstrated the levels of service offered, the second one being the one that showed the best performance, managing to obtain locations of the containers between 0 to 100 meters away from the end users.

In Guerrini Andrea et al. (2017), a nonparametric method was established to solve the solid waste collection problem. This method is based on calculating the system's performance to identify the environmental and operational variables that affect the efficiency and quality of the services provided.

This method used variables related to customer characteristics (population size by area, population density, tourist flow), those related to housing characteristics (house measurements per inhabitant per house), and operational characteristics such as waste collected in tons per route or per trip. They found that all the variables involved affect efficiency to varying degrees and directions, and therefore also the cost of collection.

The problem of designing collection routes for vehicles with capacity constraints is considered in the literature to be difficult to solve due to the multiple combinations that can exist, making it difficult to find a solution that satisfies all the optimality conditions of the problem.

In Veer et al, (2023) one can observe the implementation of a metaheristics based on a Tabu search to face this problem where a collection route must be built and selected using vehicles with defined and limited capacity without time limitations, the results obtained with the application of this algorithm surpass other techniques with instance sizes considered as large, but for small instance sizes, the local search with the TABU search produces results comparable to exact optimization in a shorter period of time, in this way the idea of applying exact and/or heuristic algorithms for medium to small sized problems in the exploration of optimal solutions is fortified.

Among the numerous variants of the Vehicle Routing Problem (VRP), we can identify applications where the route in a network consists of a different origin than the final point or destination.

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The solution to this type of problem can be obtained using shortest path algorithms between points, etc. VRPs are typically an exact description of many multi-company distribution systems, where delivery or collection situations are the core of their service. For example, garbage collection, personnel transportation, and school transportation systems can be modeled in a similar way due to their similar characteristics.

Based on the state-of-the-art review, the decision was made to use an exact solution method, which is based on the modification and adaptation of a mathematical model proposed by Akhtar Mahmuda et al. (2017) for the vehicle routing problem. Likewise, AMPL software was used with the CPLEX version 12.7 optimizer. The use of an exact solution technique can be considered risky since the computational time for large problems is usually very long. For the practical case addressed, this was not a problem since the problem size was considered small, as the application was to a pilot test group, which guarantees that we can find the optimal solution in a reasonably short time.

The following section shows the mathematical model based on mixed integer linear programming (MIPL), which was proposed taking into account the different characteristics of the real-world problem of route design for the vehicle routing problem.

Mathematical Model

Model Assumptions

The following conditions are considered so that the model adequately represents the real-world problem:

- All vehicles depart from and return to the final depot.
- All vehicles start their journeys from the depot at the same time.
- Each waste container is visited by only one vehicle.
- The cumulative waste capacity of all containers on a route must not exceed the maximum vehicle capacity.
- The vehicle fleet and waste containers are homogeneous.

Indexes and Parameters:

$V: Total\ number\ of\ container\ locations$
$to\ be\ collected.\ n \in |V|.$
$N: Total\ number\ of\ container\ locations\ to$
$be\ collected\ including\ the\ final\ deposit$
$N \cup V.$
$K: Total\ number\ of\ vehicles\ available$
$k \in |K|.$
$C_k: Vehicle\ capacity\ k, k \in K$
$i, j: indexes\ corresponding\ to\ the\ location\ o$
$containers\ i, j.\ \ i, j \in |V|$
$L_{ij}: Distance\ between\ the\ location\ of$
$container$
$i\ and\ container\ j.$
$Q_{ij}^{k}: Vehicle\ load\ k\ between\ container$
$locations\ i, j.$

Decision Variables of the Model

$$X_{ij}^{k} = \begin{cases} 1, If\ the\ vehicle\ visits\ the\ container \\ \quad locations\ ij. \\ 0, in\ any\ other\ case. \end{cases}$$

$$W_{ij} \begin{cases} 1, if\ the\ location\ of\ the\ container\ i \\ \quad belongs\ to\ the\ vehicle's\ route\ k. \\ 0, in\ any\ other\ case. \end{cases}$$

Objective Function

$$F.O\ Min\ Z = \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K} L_{ij}X_{ij}^{k} \qquad [1]$$

Sujeto a:

$$\sum_{j=1}^{n}\sum_{k=1}^{K} X_{0jk} = 1 \qquad [2]$$

$$\sum_{j=1}^{n} Q_{0jk} = 1\ \ \forall\ k = \{1,2,\dots,K\} \qquad [3]$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} X_{ij}^{k} = 1\ \ \forall\ j = \{1,2,\dots,n\} \qquad [4]$$

$$\sum_{j=1}^{n} X_{ij}^{k} - \sum_{j=1}^{n} X_{ji}^{k} = W_{ik}\ \ \forall\ i = \{1,2,\dots,n\}, \quad k = \{1,2,\dots,K\} \qquad [5]$$

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$$\sum_{i=0}^{n}\sum_{h=1}^{K} Q_{ji}^{k} - \sum_{i=0}^{n}\sum_{k=1}^{K} Q_{ij}^{k} = C_j \,, \quad \forall\, j = \{1,2,\dots,n\}\,, \qquad [6]$$

$$\sum_{j=1}^{n} C_i X_{ij}^{n} \le C_k, \forall\, j = \{1,2,\dots,n\}, \quad k = \{1,2,\dots,K\} \qquad [7]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} X_{i0}^{k} = 1 \qquad [8]$$

The equation (1) represents the objective function which seeks to minimize the total sum of the distances traveled by the vehicle on each route. The equations (2) and (3) specify that each vehicle k must start the trip from the depot without cargo. According to equation (4), each location of each container is visited by only one vehicle. The equation (5) ensures the continuity condition, that is, if vehicle k enters a location, it must also leave that location. The equation (6) ensures that every vehicle must empty the contents of the containers visited. The equation (7) ensures that the maximum capacity of the vehicle is not exceeded by stating that this capacity must be less than or equal to the total waste collected from all containers visited on the trip. Finally, the equation (8) ensures that each vehicle must return to the depot after the trip.

Results:

The presented work considers the vehicle routing problem applied to the urban solid waste collection problem in a given area. For this purpose, the work presented by Akhtar Mahmuda et al. (2017) was used as a reference. This work considers waste collection using vehicles with homogeneous capacities. This work was modified to adapt to the real-world problem addressed, which considers different capacities for various types of vehicles. The results obtained establish the different values of the decision variables, as well as compliance with the different constraints that together define the optimal solution to the problem addressed.

For the practical problem, a set of nine container locations in a defined area were considered. Table 1 shows the different distances between the collection locations, where M represents a very large value.

The objective is to minimize the total distance traveled by the vehicle while considering compliance with the set of model constraints.

Box 2

Table 1

Table of distances of container locations in kilometers

	1	2	3	4	5	6	7	8	9
1	M	2.2	2.5	3.8	2.5	3	4.1	1	34.4
2	2.2	M	1.6	1.4	2.4	3	4.1	2.4	37.8
3	2.5	1.6	M	1.3	1	1.6	2.6	2.2	38.7
4	3.8	1.4	1.3	M	1.7	2.2	3.2	2.9	37.5
5	2.5	2.4	1	1.7	M	0.6	1.8	2	36.1
6	3	3	1.6	2.2	0.6	M	2.1	2	35.9
7	4.1	4.1	2.6	3.2	1.8	2.1	M	4.4	37.4
8	1	2.4	2.2	2.9	2	2	4.4	M	37.8
9	34.4	37.8	38.7	37.5	36.1	35.9	37.4	37.8	M

The results obtained with the implementation of AMPL software with the CPLEX optimizer version 12.10 are presented below, obtaining very favorable results in the search for minimizing the collection and transportation costs included in the trips of each vehicle used. Table 2 shows the route designed for these vehicles, which meets the requirements agreed with the client. A comparison is also presented with a heuristic method in which a genetic algorithm was applied, finding the following results (see Table 2).

Box 3

Table 2

Results of both solution methods

Method Used	Selected Route	Traveled Distance (Kilometers)
Exact Method	1-8-2-4-3-5-7-6-9-1	81.3
Heuristic Algorithm	1-9-6-7-5-3-4-2-8-1	81.3

The results show that both methods give us the same total distance traveled for the pilot case, which guarantees that the alternative (heuristic) method of using a genetic algorithm provides a solution that is equally good as the exact method. It can be seen that the solutions reported in terms of the collection route are the same, with only a small difference: they are reported in reverse regarding the start and end order of each route.

The implementation of the genetic algorithm programmed in Microsoft Excel software was sought because an economical option was sought that the company could access without a large investment, which is why it was decided to use this tool.

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Conclusions

The application of exact optimization techniques to the problem of solid waste collection in a network represented for the company a better use of the resources available for the design of collection routes, due to the establishment of a collection route that minimizes travel time while respecting the characteristics of the practical problem addressed. The results helped the company establish a methodology that systematically solves the problem of collection route design. In the same way, it demonstrated that optimization techniques can represent an interesting option to address small to medium-sized problems, offering results that can increase the efficiency of its different distribution logistics systems, allowing the company to minimize operating costs and increase its profitability.

Recommendations

Consideration should be given to expanding the scope of the research by considering different areas where collection routes should be carried out, as well as having trained human resources that can efficiently operate the methodology in case adjustments need to be made or to adequately interpret the solution. Likewise, it is of utmost importance to consider establishing controls that help better monitor the implementation and start-up of the solution to ensure that the results are as expected by the company.

Declarations

Conflict of interest

The authors declare no interest conflict. They have no known competing financial interests or personal relationships that could have appeared to influence the article reported in this article.

Author Contributions

Conceptualization, J.V., M.L.; methodology, J.V., A.C.; software, J.V., A.C.; resources, J.V., M.L. E.R.; writing—original draft, J.V., A.C., M.L., E.R.; writing—review and editing, J.V., M.L., A.C., E.R.; visualization, J.V.; project administration, J.V., M.L.; funding acquisition, J.V., M.L., A.C. All authors have read and agreed to the published version of the manuscript.

Availability of data and materials

The data presented in this study are available on request from the corresponding authors

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Abbreviations

AMPL	A Modeling Language for Mathematical Problems
CVRP	Capacited Vehicle Routing Problem
MIPL	Mixed Integer Linear Programming
VRP	Vehicle Routing Problem

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Antecedents

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