

## Metacognition in Sebastian`s Door: mathematics, education and art

### Metacognición en La Puerta de Sebastián: matemáticas, educación y arte

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#### Abstract

The aim is to show metacognition in how mathematics and culture are related in the sculptural work of Master Sebastián; with a deep playful character from which it emerges. Constituting a kind of artistic syncretism of apparently divergent beliefs between the plastic world, pure mathematics, abstract geometry and computational cybernetics. Methodology, study with a qualitative approach, the variables to consider were: Bach's baroque music with the canon and fugue, Gödel's theorem with the incompleteness of the axiomatic system; Eschery lithographs the reiteration of forms; Borje's literary-mathematical paradoxes; recursive programming routines in artificial intelligence systems (expert systems, mathematics) and Sebastián's sculpture. In conclusion, the convergence between Bach's baroque music can be identified, through the generation of the canon and the fugue; Gödel's theorem on the incompleteness of the axiomatic system, the reiteration of forms that invent themselves in Escher's lithographs, the literary-mathematical paradoxes in the stories of Jorge Luis Borges, and the use of recursive routines in the programming of expert systems or artificial intelligence. This game of plastic metamorphosis with which Sebastián surprised us a few years ago, turns out to predate the concept of the transformers, long before Hollywood made them a commercial success. As in the most diverse efforts of syncretism, Sebastián has traveled close to profanation as the staunch conservatives of the elite of scientists and artists can describe it.

#### Resumen

El objetivo es mostrar la metacognición en cómo se relacionan las matemáticas y la cultura en la obra escultórica del maestro Sebastián; con un profundo carácter lúdico del que surge. Constituyendo una especie de sincretismo artístico de las creencias aparentemente divergentes entre el mundo plástico, las matemáticas puras, la geometría abstracta y la cibernética computacional. Metodología, estudio con enfoque cualitativo, las variables a considerar fueron: la música barroca de Bach con el canon y la fuga, el teorema de Gödel con la incompletud del sistema axiomático; litografías de Eschery la reiteración de las formas; paradojas literario-matemáticas de Borje; rutinas recursivas de programación en sistemas de inteligencia artificial (sistemas expertos, matemáticas) y la escultura de Sebastián. En conclusión, se puede identificar la convergencia entre la música barroca de Bach, mediante la generación del canon y la fuga; el teorema de Gödel sobre la incompletitud del sistema axiomático, la reiteración de las formas que se inventan a sí mismas en las litografías de Escher, las paradojas literario-matemáticas en los cuentos de Jorge Luis Borges, y el uso de las rutinas recursivas en la programación de los sistemas expertos o de inteligencia artificial. Este juego de metamorfosis plástica con el que Sebastián sorprendió hace algunos años, resulta que antecede al concepto de los transformers, mucho tiempo antes de que Hollywood los volviera un éxito comercial. Como en los más diversos esfuerzos de sincretismo, Sebastián ha transitado cerca de la profanación como lo pueden calificar los acérrimos conservadores de la élite de científicos y artistas.

#### Metacognition, Mathematics, Art

#### Metacognición, Matemáticas, Arte

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## Introduction

In times when the reduction of thought that hovers through specialty and subspecialty prevents the human being in general but the educator in particular, not to be able to observe what is in other areas of knowledge and how they come together in the borders that are indivisible for those who have a teacher training, it is for this reason that metacognition is to go beyond (*meta*), from what goes together (*co*) to what is known (*gnosis*) and that what is known may depend on the subjective source of God (subjectivity), of man himself (subjectivity) or of the object for what it is (objectivity); This is why this study is relevant to know how topics that are of science, education, culture and society are immersed in the simple fact that they arise from the human being *per se*.

The added value of this study lies in the fact that the variables of the study are compared, focusing on the characteristics that each one of them has, the problem to be solved is that it is required that the human being has a perspective beyond what he knows in an area of knowledge, being able to observe their relationships, going beyond the reductionism of education that prevails in the specialty and subspecialty.

The following are the elements analyzed, what each one of them means, the methodology of analysis and variables based on a qualitative approach; the results and conclusions reached; the line of research derived from the study and, finally, the proposal of the study.

## The canon and the fugue

The canon is a musical theme so wonderfully shaped that it can serve itself as an accompaniment when played in another key, so successively up to seven times, an expression that constitutes a masterpiece. Such efforts in the musical realm are compared to beating 75 opponents at chess, playing all of them at the same time and blindfolded (Batlisti, 2013).

The work of Johann Sebastian Bach (1685-1750) the Art of Fugue, is one of the best examples of metalanguage in the musical realm. Fourteen fugues and four canons are based on the same theme in D minor that serves as accompaniment, as well as in its execution in another key. That is to say, it makes reference to itself.



**Figure 1** Joahan Sebastian Bach

Source:

<https://www.nationalgeographic.es/historia/2019/07/la-anatomia-de-bach-podria-haber-potenciado-su-excelencia-musical>

## Gödel's theorem

The theorem of the Russian Kurt Gödel is the proof that there are mathematical truths that cannot be proved from any set of axiom systems or basic truths. He would say that they *constitute formally undecidable propositions* of the *Principia Mathematica and other appendices*, referring to the magnificent, but unfortunately incomplete effort of the construction of the axiomatic edifice elaborated by the philosophers Alfred Whithead and Bertrand Russell.

The work "*Principia Mathematica*" constitutes the major mathematical edifice with which Alfred Whithead and Bertrand Russell attempted to form an axiomatic system from which all the theorems of first-order logic could be proved (Russell, 1913; Basti, 2019).

Gödel's great contribution was to demonstrate with the use of metalinguistic statements that there are propositions that are false and true at the same time.

To understand the mechanism of statement formulation, we can turn to the axiomatic MIU system developed by Douglas Hofstadter in his "*Golden Braid*".

The MIU system consists of only three letters M, I, U, so that the statements or propositions are formed with a sequence between these three characters.

The mechanisms of demonstration of the MIU system are based on 4 axioms:

- 1. If a string ends with an I, a U can be added to it ( $xI \dot{=} xIU$ ).
- 2. Any string after an M can be fully duplicated ( $Mx \dot{=} Mxx$ ).
- 3. Where there are three consecutive I's (III) they can be replaced by a U ( $xIII \dot{=} xU$ ).
- 4. Two consecutive U can be eliminated or cancelled ( $xUU \dot{=} xy$ ).

According to the conception of metacognition, the MIU System is a synthetic example of a propositional system where each sequence is a statement, which can have a false or true sense, characteristic of first order logic, also known as predicate logic, predicate logic or predicate calculus. There are other forms of logic with statements that can have degrees of truth or falsity.

The issue in the MIU System is to generate the MU chain from MI. When attempting to enunciate, in succession, the subsequent chains on which MU is inferred, it is confirmed that its demonstration or enunciation is impossible. That is to say that MU is a formally undecidable proposition in the MIU system. Come on, it cannot be demonstrated. Then a jump out of the system is required to deduce that MU exists, but it is indemonstrable or formally undecidable.

In the human mind, knowledge is in essence a large set of statements which can be demonstrated from axioms; that is, from the rules of statement generation. The set of statements makes up our knowledge and the way of constructing them constitute the mechanisms of argumentation of the criteria of verification.

The axioms and the chains of demonstration make up the procedural system. Memory activates the ways of generating sequences to infer successive chains of statements. These statements can make sense and can refer to themselves in concepts that express their own logical value. In this way, knowledge defines itself.

In the same way, although with more complex processes, Gödel showed that there are mathematical truths that cannot be proved from the set of axioms of the *Principia* (Russell, 1913, Basti 2019).

The first idea that arises in the face of this paradox is to include those truths to the axiomatic system. However, Gödel also showed that as soon as more axioms are integrated, other statements appear that are true in practice but are also impossible to prove from the axioms, so that the first-order axiomatic system will always be incomplete.

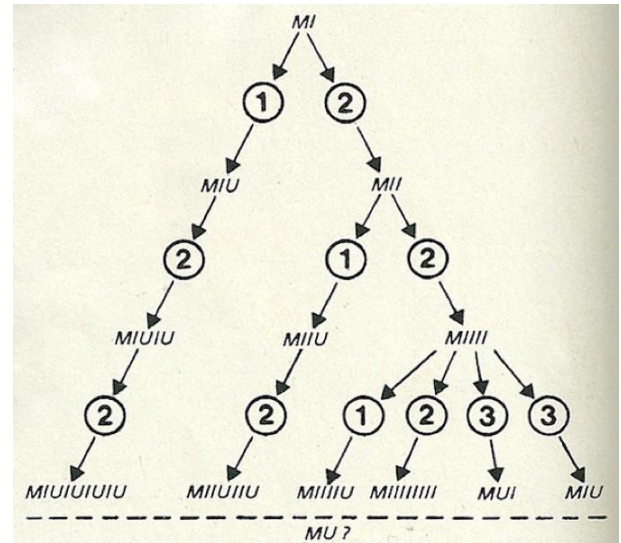


Figure 2 Gödel's theorem  
Source: <http://adolescentesyamas.com/el-sistema-miu-de-hofstadter-del-libro-godel-escher-y-bach-un-eterno-y-gracil-bucle/>

### Expert systems

For its part, the theoretical underpinning of the programming of expert computational systems is based on self-referencing routines, which call themselves or recursive would say the specialists in systems (Russel, 1913, Basti 2019). That is, the construction of cybernetic metacognition is possible from the formulation of metalinguistic statements and recursion in programming.

The classic example of metalinguistic subroutines is the definition of the factorial. The factorial of a positive integer n equal to or greater than 1!, expressed as n!, is the multiplication of all the positive integers less than or equal to n.

This means that,  $n! = 1 \times 2 \times 3 \times 4 \times \dots (n-1) \times n$ . That is, the factorial defines itself metalinguistically as:  $n! = n \times (n - 1)!$

Artificial intelligence is supported by the integration of subroutines that call themselves as in this example of the factorial (Russell, 1913, Basti 2019). In this way, it is possible for a computational system to increase its metacognitive pool or acquire cybernautical expertise. For example, traditional chess computer game programs are based on move probabilities. In that case, it would not be possible to build a perfect program because the analysis of probabilities would take unlimited sequences in infinite time.



The solution is to build an expert system based on metacognition in such a way that it learns to play with the skills of each opponent. Thus, as a system plays, it incorporates into its metacognitive pool a greater number of statements that make it more efficient. This mechanism is supposed to be similar to the way metacognition is built in the human mind.

In the same way, the programming of the mechanisms for demonstrating the MIU System's statement-strings is the principle for building an expert system. Suppose, for example, that an expert system is built to identify the operation problem of a nuclear power plant.

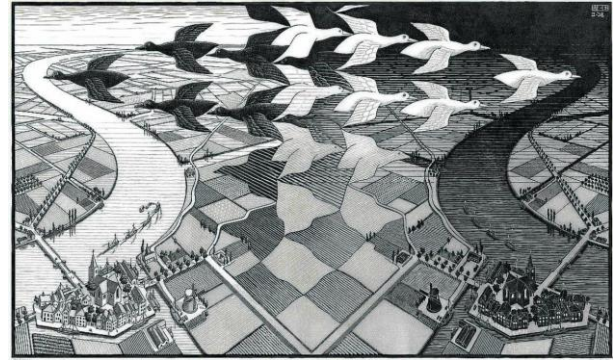
This type of plant requires immediate responses to an outage situation. The problem is stated and at the moment of developing the chains to reach it, it represents for the expert system the mechanism to identify the origin of the failure and to be able to act accordingly.

The greatest potential of fourth generation programming languages lies precisely in the possibility they have to build self-referential routines as is the case of C (Russel, 1913, Basti 2019) which has an enormous potential in the construction of self-referential subroutines (Brian, Dennis & Kate, 1991; Rodríguez Alvira, 2012; Guio, Navarro & Lukin, 2017).

### Escher's lithographs

The inexhaustible play between figure, background and form in the chains of metamorphosis in Escher's lithographs, whose efforts in drawing summon the imaginary of endless circles, like the constant fall of water or the inexhaustible walk in a labyrinthine castle, are examples of metacognition in the plastic arts.

Or in his case, the construction of endless worlds in the literary imaginary of the "*Fictions*" of Jorge Luis Borges as in that Universe, which others call the library and which is composed of an indefinite or perhaps unlimited number of hexagonal galleries, enclosed by very low railings and from where each hexagon the upper and lower floors can be seen endlessly (Escher, 2000; Russel, 1913, Basti 2019). Likewise, the logic and self-reference games that appear in the book "*Alice in Wonderland*" by Lewis Carroll (1832-1898) (Carroll, 2011; Rodríguez Alvira, 2012; Hofstadter's, 2013).



**Figure 3** "Birds": Background and form in Escher's work  
Source:  
<https://webpages.ciencias.ulisboa.pt/~ommartins/seminario/escher/obra2.html/>

### The door of Torreón

It is precisely in the field of plastic art that one can glimpse the masterful quality of Sebastián's sculptural work (Enrique Carbajal González, November 16, 1947) and some of its keys, given that it is integrated with forms that repeat themselves or self-generate in such a way that they invite infinite reiteration (Escher, 2000; Batistri, 2013; Hofstadter's, 2013).

Hence, one can identify in the sebastian work plastic loops that reproduce themselves, but also jump out of the system, or the subject of a specific form that as in the baroque canon, which serves as background and form at the same time. That is to say that a three-dimensional body is repeated per se while accompanying itself as a figure, to later manifest itself as a background. The path of geometric fish, whose scales metamorphose into spherical bodies.

As all the magicians of the world have known, Merlin in the British and Celtic tradition, Gandalf in the Scandinavian and evocative heritage of Tolkien; Mephistopheles in the Germanic tradition; Houdini in the theater or George Méliès the magician of light in the language of cinema; Sebastián knows perfectly well that the authentic and true magic is created from the optical illusion and the traps set by the human being's own visual perception, which is always discontinuous. A defect, between quotation marks, of the eyes that have even allowed the invention of the cinematograph.

Strictly speaking, the relationship between sculpture and the mathematical principles of geometry has been a very old amasiato, since the classical world where even the search for aesthetics, for beauty, was based on the investigation of symmetrical forms.

However, it is in the Renaissance where this symbiosis, sculpture, mathematics and geometry has a full expression in the work of Filippo Brunelleschi (1377-1446) (Escher, 2000; Batsitri, 2013; Hofstadter's, 2013), for example, by creating those wonderful arches, he also explores in the field of mathematical demonstration.

For the same reason, the resonance of the baroque is evidently identified in Sebastian's work, due to his constant eagerness in the exacerbation of forms. However, there is a capacity to maintain the Renaissance spirit and heritage, enriched with the symbolic character of surrealism and cubism. Sebastián's work emerges at a precise moment in the framework of a cybernetic society that recreates itself in the visual imaginary and draws from the geometric world as an extension of the experiential territory of the human being. A fusion of visual beauty and mathematical truth, where the pre-Columbian tradition, the Baroque and the Renaissance soul meet (Hofstadter's, 2013; Rodríguez Alvira, 2012, Nagel & Roy, 2008).

For the same reason, he celebrated the splendid fabrication of the Torreón Gate that has been integrated in such a way with the character and vocation of the laguneros; that it is already identified with the city itself, in an indivisible way as it constitutes for example Pisa and its Tower, which are not known one without the other.

Much has been said about the enormous symbolic potential of master Sebastián's sculptures; however, little is explored in the exercise of interpreting the meaning of these symbols. The invitation in general is to appreciate that Sebastián's work is closely related to the act of interpreting it, because doing so is also a way of belonging and redefining oneself with it (Escher, 2000; Nagel & Roy, 2008; Rodríguez Alvira, 2012; Hofstadter's, 2013).

In the initial project of this magnificent Puerta de Torreón, the spikes and the chain of the 38 municipalities that make up the state of Coahuila de Zaragoza were proposed, in addition to being the entrance and welcome to its border with Durango.

But Sebastian goes much further. For example, in the light of the interpretation of the symbols of the cabala and the tarot, what connotation could be inferred from this work. First, in its indivisible integrity, it speaks of the need for unity of the work of art, of its indivisible character, of its individuality; that is, it summons to speak one's inner self, to the inner capacity to make magic (Tajonar, 2017).

Likewise, the Gate is erected from the earth, as the mother that gives the creative force, it is built from yellow columns, where the color evokes the internal fire that must be sustained by firm traditional values and knowledge that constitute the columns themselves, same that end up uniting; that is to say that all efforts and knowledge must be oriented towards the same end (Tajonar, 2017).

For its part, the direction towards the sky, this wonderful blue lagoon sky, summons to always keep the mind open towards the new, so open that one can even go beyond the frame in which they are immersed or imprisoned, that frame of the lower door that must be crossed to transcend towards the highest ideals.



**Figure 4** Sebastian's gate

Source:

<https://www.mexicoenfotos.com/estados/coahuila/torreon/la-puerta-de-torreon-MX12182383097189>

### Methodology to develop

The objective is to show the metacognition in how mathematics and culture are related in the sculptural work of master Sebastian.

It is a research with a qualitative approach in the content analysis of what is referred to in the theoretical foundations, in addition to the relationship of the variables to be considered, which were: Bach's baroque music with the canon and the fugue, Gödel's theorem with the incompleteness of the axiomatic system; Escher's lithographs and the reiteration of the forms of expert systems; Borges' literary-mathematical paradoxes; recursive programming routines in artificial intelligence systems and Sebastian's sculpture.

## Results

In the world, there are known things and unknown things, between the two are the doors. Between the order of the concrete and the spiritual ascension lies this Sebastian's Gate.

In these adverse times where materialism has taken root in the hearts of modern man, through the Gate of Torreon the dearest yearnings of unity rise and the dearest of our gods of the past pass through every day in silence.

To construct a geometric form of self-reference is to generate a mental structure of chains of statements that allow us to generate new knowledge from basic principles or axioms.

The more solid the axiomatic bases of the cognitive systems, the greater the capacity to generate and demonstrate existing truths.

The most direct and effective application of metacognition is in the field of cybernetics, in the creation of expert systems and in the programming of artificial intelligence where the key is the use of self-referential subroutines. The greatest expression of human intelligence is sustained in the generation of self-referential statements, i.e., in metalanguage and metacognition, i.e., going beyond specialty and subspecialty by interrelating the different subjects of culture and science in the development of the areas of knowledge.

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## Conclusions

Known and unknown things are identified, the relationship between the two is where the doors between the areas of knowledge are found.

Between the order of the concrete and the spiritual ascension is the Gate of Sebastian.

The Gate of Torreon elevates the dearest longings of unity and transits every day in silence, the dearest of the gods of the past.

To build a geometric form of self-reference by generating a mental structure of chains of statements that allow generating new knowledge from basic principles or axioms.

The solidity of the axiomatic bases of the cognitive systems allows to increase the capacity of generation and demonstration of existing truths.

The use of self-referential subroutines allows the direct and effective application of metacognition in the field of cybernetics, in the creation of expert systems and in the programming of artificial intelligence.

Metacognition with metalanguage favors the expression of human intelligence, based on the generation of self-referential statements.

Two are the lines of research that emerge from this study; the first is the possibility, as confirmed by the geometry of the Hebrew alephbeth where each word has a phonetic value, a signifier and also a numerical value that links it with the meaning of the words that have the same numerical value; the second, lies in the similarity that the attractors are the set of points towards which a dynamic system tends, after a high number (infinite, ideally), to achieve from the expert systems the variation in initial conditions and where its obtained values are never repeated with exactitude, given the great variety of initial conditions of a system.

It is proposed that, in order to open the mind (open mind) of the learner, to seek the relationship and integration of phenomena in different areas of knowledge, allowing to permeate, filter, learn to learn, be self-taught and go beyond the limits imposed by the specialty and subspecialty, in short, go beyond the general culture, learn to relate the various topics of the different areas of knowledge, expanding their horizons in knowledge.

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