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Analysis of ways of thinking in the approach to systems of homogeneous linear equations

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Abstract

This research to analyze the modes of thinking synthetic-geometrical, analytic-arithmetic and analytic-structural in Linear Algebra Sierpinska (2000), and approach in solving problems Homogeneous Linear Equations Systems and their relationship with concepts of linear independence, as well as strategies and present difficulties by students of the first semester. The study was done by writings and interviews, questionnaires applied in two stages. The first exploratory stage questionnaire was applied as a group as two different groups who were in linear algebra, then a student selected were interviewed. In a second phase an interview was applied to three successful undergraduate students who had recently completed Linear Algebra. The answers to problems were recorded on paper and interviews students were videotaped and audio for analysis. The research analysis shows that there are difficulties with the student in the transition of different modes of thinking, we see that the concepts are fragmented, does not exist in the mind of the student relationship among these are concepts without significance was also observed you do not have a clear concept of homogeneity, and there is difficulty in the transition on modes of thinking.

Modes of thinking, Linear Equations Systems, Linear Algebra.

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Introduction

In recent decades they have developed research focused on the teaching of linear algebra by different groups of researchers in many countries, including a French group composed of Jean Luc Dorier, Aline Robert, Jacqueline Robinet, Marc Rogalski, Michele Artigue, Marlene Alves Dias; In Canada a group led by Anna Sierpinska, Joel Hillel and T. Dreyfus and the United States two groups, one composed of Gershon and the other by Ed Harel Dubinky with the theory (APOS). In the curricular part there is a group led by David C. Lay (Author of Linear Algebra and its Applications, Addison Wesley, 1994). In Mexico a group of researchers led by CINVESTAV-IPN Asuman Oktaç.

As a result of these studies it was concluded that any approach that is given to linear algebra (arithmetic, geometry, computational, axiomatic), difficulties in learning remain so they must accept that linear algebra is a complicated matter for many students. Jean Luc and Anna Sierpinska Dorier as leading researchers have identified two sources of difficulties in students: the nature of linear algebra as such (conceptual difficulties), and the kind of thinking required to understand linear algebra (cognitive difficulties), the which are not reported separately. As conceptual difficulties considered different types of languages and registers used in linear algebra, such as the formal language (Dorier, 1987), algebraic, geometric and abstract language Hillel (2000) also records charts, tabular and Pavlopoulou symbolic, (1993), and cognitive difficulties as regards cognitive flexibility-Dias Alves (1998), trans-level object of thought Sierpinska (1999) theoretical and practical thought Sierpinska, (2000).

This paper aims to identify the challenges and strategies that have undergraduate students to travel on different modes of thought into systems of homogeneous linear equations. In equations (1), (2) and (3) equations SELH and their correspondents ranges are shown in Figure 2 and its graphical representation.

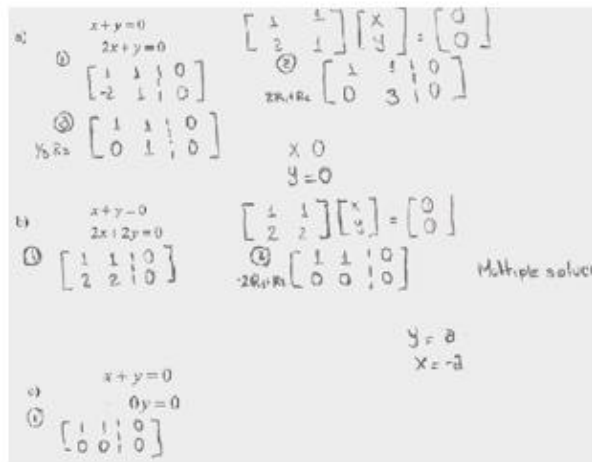

Figure 1 Responses from students

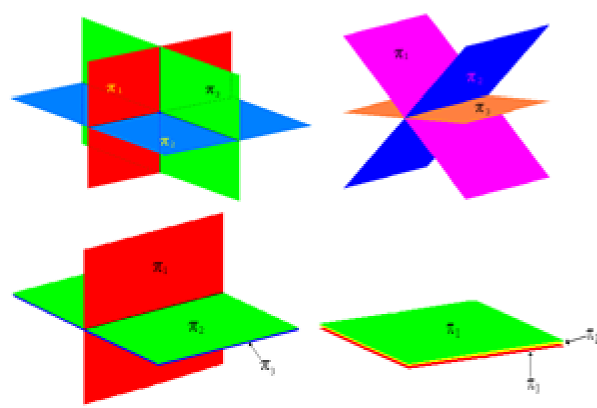

Figure 2 Graphical representation of some SELH

$$
\begin{aligned}
a_{11}x - a_{12}y + a_{13}z &= 0\\
a_{21}x + a_{22}y + a_{23}z &= 0\\
a_{31}x + a_{32}y + a_{33}z &= 0
\end{aligned}
\tag{1}
$$

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Range A is three

$$a_{11}x - a_{12}y + a_{13}z = 0$$
$$a_{21}x + a_{22}y + a_{23}z = 0$$
$$a_{31}x + a_{32}y + a_{33}z = 0$$

(2)

Det A = 0 and the rank of A is two

$$a_{11}x - a_{12}y + a_{13}z = 0$$
$$a_{21}x + a_{22}y + a_{23}z = 0$$
$$a_{31}x + a_{32}y + a_{33}z = 0$$

(3)

Methodology

This section of the research methodology is described, in which one can distinguish two phases, which have been carried out with tools, methodologies of quantitative and qualitative analysis. We begin by describing the objectives. The research was carried out with senior students in Pachuca Instute of Technology, to the race of Civil Engineering and Industrial Engineering.

The overall objective is to analyze the ways synthetic-geometric, analytic-arithmetic and analytic-structural Sierpinska (2000) thinking, difficulties and strategies that have top-level students to solve systems of linear homogeneous equations and concepts related with the structural part of linear algebra.
The analysis of the interviews and questionnaire identifies the difficulties and strategies presented by the students.

We can distinguish two phases, exploratory, which consisted of applying a questionnaire to two groups of Engineering Pachuca Institute of Technology.

One of the careers of Industrial Engineering with a total of 13 students and another group of Mechanical Engineering with a total of 8 students who passed the course Linear Algebra and later an interview to a student selected from the same groups.

In the second stage interviews were applied to two successful students of the Engineering who were in differential equations and had approved the matters Vector Calculus and Linear Algebra.

The documentation for the evidence in the analysis was taken from the writings in their worksheets and audio and video recordings made during the course of the interview students. All students who participated in the first phase of the research had previously studied at least one college course were finalizing Vector Calculus and Linear Algebra course. In the second phase the students interviewed were selected based on taste for mathematics and the success they have for their good grades in math, students interviewed had half a year of study vector calculus and linear algebra.

Results

Some of the results are

Evidence that the analytic-arithmetic thinking takes precedence in the respondents because the solution of the systems, obtained by applying a reduction method, when it is questionable whether there would be another way to tell if the system is trivial or no solution (line shown 61 and 63), the respondent says that the only way you taught (line 64), not the characteristics of the system relates to the solution and this is reflected in subsection c) of the same problem where no response, although sometime referred to in subparagraph b) the solution is manifold because the equations are multiples of each other.

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Again he has trouble passing the-arithmetic analytic analytical-structural thought, subsection c) thought, possibly due to having 0y = 0, and therefore does not express his solution, although he was again leads (online 69 and 71), can not express the solution as in paragraph b). As shown in Figure 1.

Conclusions

Product research about teaching and learning of linear algebra are not, in general, statements like, "This is how to teach linear algebra resulting in a better understanding by students of the basic concepts of the theory and a better performance on standardized test questions ", complemented by classroom materials, as: textbooks, software, etc. Such statements and steady supply in the market, textbook linear algebra, tutorials and software, are generally not based on research. In fact, the most reliable research results are, in a sense, "negative" in nature. Explains to some extent why many students fail in linear algebra courses, show why some innovative approaches in teaching linear algebra can fail. These results lead to some "well-founded Recommendations" or "good advice" regarding the practice of teaching, but the council should not be confused with a foolproof recipe; in fact, the recommendations are only conjectures that are still liable to be questioned.

This work tried to pinpoint some reasons why many students find it difficult to learn linear algebra. They are:

1. The axiomatic approach in linear algebra appears in many students. All linear problems within the reach of first-year college students can be solved without using the theory of vector spaces. Therefore, this theory has little chance of being perceived as an intellectual need for these students.

Students can "recognize" its existence, or even "Appreciate" and meaningful information, but the theory is unlikely to become "cognitive supply" in the minds of students. It will be considered superfluous and vain to many students.

2. Linear algebra is an "explosive compound" of languages and systems of representation. There is a geometric language of lines and planes, algebraic language of linear equations, the n-tuples and arrays, the leguaje of "Abstract" of vector spaces and linear transformations. There records "Graphic", "Tabular" and "Symbolic" the language of linear algebra, Figure (2). There is also representations “Cartesian” and “Parametric” subspace. Teachers and texts move between these languages, records and modes of representation without considering the need for conversions and talk about their validity. They seem to assume that these conversions are "Natural" and obvious and need no conceptual work at all. Linguistic and epistemological studies and observations of students demonstrate, however, how these claims can be misleading.

3. Linear algebra is very demanding from the cognitive point of view. At the most general level, an understanding of linear algebra requires a fair amount of "cognitive flexibility" among the different languages so that they can move freely (eg The language of matrix theory and language of the theory of vector spaces), points view (Cartesian and parametric) and semiotic registers. The understanding of linear algebra also requires students in identifiable conceptual structures encapsulate a substantial range that was encapsulated previously as individual objects and actions on these objects in the conceptual structures identifiable. (For example, the functions must be seen as objects themselves, elements of vector space procedures instead of assigning numbers to other numbers).

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In addition, the understanding of linear algebra requires the ability to use the "theoretical thinking." Indeed, it is essential to have a means of control over the ways of thinking, in the same way that the "Insights" and the characteristics of practical thinking mental image. It is however the "practical thinking" necessary to avoid a situation where the linear algebra is nothing more than a strange, secret and formal language that can be written but can be used to think.

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