Capítulo 25

On the Utility of the Hurst exponent in predicting coming crises

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M.Ramos, F.Miranda (eds.) *Optimización-Estocástica-Recursiva-Coherente-Sistémica y sus variantes (probabilidad, econometría y estadística aplicada),* Temas Selectos de Optimización-©ECORFAN-Santiago de Compostela, España, 2012.

Abstract

The aim of this article is to ascertain if and to what extent the Hurst exponent can be used to forecast coming crises. The first and second chapter focuses on the Hurst exponent, giving theoretical insights and a synthesis of its uses in finance. The analysis of a dataset of 35 indices and stocks representing various different geografical areas and economical sectos is presented in chapter 3 while in the last chapter the conclsion that the Hurst exponent has eventually no connection with coming crises is drawn.

25 Introduction

25.1 Uses of the Hurst Exponent

The Hurst exponent is a measure of the autocorrelation of the data being part of a certain time series. The concept of autocorrelation, as the word itself suggests, is connected to the influence that a datum of position say x in a time series has on a successive datum of position say x+1. The effects of such a property can be effectively explained in terms of comparison with the mean: if a value rather superior to the mean is usually followed by another high one (or in other words "forces" the following to be high too), then we say data are correlated in a positive way. In the opposite case, the one of high value being followed by low ones, negative correlation occurs while random data should have no correlation.

As the mathematical process to evaluate the Hurst exponent H is going to be discussed in chapter 2, it is for now sufficient to know what follows:

$H<0.5$: data are negatively correlated
$H=0.5$: data have no correlation
$H>0.5$: data are positively correlated

The name Hurst exponent derives from Harold Edwin Hurst (1880-1978), who first used it studying river Nyle's cycles of heavy rains and droughts. This hydrologic issue was intented to approach the practical problem of optimizing the size of dams while from then on the Hurst exponent got increasingly more used in many scientific fields, including phisics, DNA researches and economics. As far as economics is concerned, the Hurst exponent has been mainly used in finance and for the connection of used approaches with many physics experimental techniques this branch of studies was classified in the econophysics area.

Examples of the use of the Hurst exponent for financial issues are its applications on the high frequency trading and market size studies areas. In cited article [7], the author notices how the Hurst exponent seems to assume different values in developed and emerging markets and stresses the importance of such a result for portfolio management evaluations.

While the Hurst exponent calculated for indices such as NASDAQ (USA), NIKKEI (Japan) and CAC (France) does not cross the 0.5 value, on the contrary it seems to be sistematically superior to this value for IBEX (Spain) and Hang Seng (Hong Kong).

Much more markets are nevertheless quite vaguely sited with small fluctuations in the 0.5 belt: these ones are FTSE (United Kingdom), DAX (Germany), AEX (Netherlands).Hurst exponent analysis proved itself to be of significant help also in high frequency trading market investigations. [1] finds that Hurst exponent values for different but small time horizons differ sensibly from 0.5 which would contradict Efficient Market Hypothesis, stating that they should cluster near 0.5 as prices should not be predictable but follow a random walk distribution.

This last result could have been caused by high frequency trading itself as [18] suggests. The author first set a pre and post high frequency trading era and then analyzes Hurst exponent values for these different periods. The date chosen in the article is June 2005, that is the date of approval of Reg NMS whose Rule 611 obliges the automatic execution of trades at the best quote possible: this automatization of the market is considered to be the decisive factor that made it possible to develop high frequency trading massively. Once that different Hurst exponent values were found for these two periods the author suggests possible causes for this result, that are the breaking of big orders in smaller and reiterated ones and the feedback driven method of many high frequency trading techniques.

25.2 Forecasting the crisis

The most suggestive use of the Hurst exponent is anyway the possibility of anticipating the future. The concept of correlation is indeed the one of connection between past and future data, that can therefore be somewhat precisely forecast.

This idea has been proposed by many scholars in regard to the chance of predicting coming crises and abrupt market movements. Though fascinating, this hypothesis has been the subject of works concentrating only on few indices or markets and even these limited investigations frequently ended up in different and contrasting results. This work aims therefore at broadening the range of investigation and testing by now obtained results on a wider data set.

A very clear work about the ability of predicting cryses of the Hurst exponent is [5]. In this paper the authors evaluate the Hurst exponent for the Polish market index and find evidences that just before a crisis period the Hurst exponent diminishes noteworthy.

A simple method to verify if the crash in the Hurst exponent is preluding a crisis is in addition provided. Considering the simple mean of all Hurst exponent preceding of x days the crash in the market equal to H-x, if the period considered really prelude a crash in the market, then:

- The Hurst exponent should be in decreasing trend
- $H\text{-}21 \leq 0.5$
- $H\text{-}5 \leq 0.45$
- The minimum value of the Hurst exponent in the period just before the crisis should be inferior to 0.4 (as the Hurst exponent trend is decreasing the minimum should stay the nearest to the moment of the market crash)

Even though this method is very satisfying, it should be noted that practically it could become quite difficult to identify exactly the 21 or 5 trading days before the crisis while still being in the period before the crisis.

In [13] the author analyzes the values of the Hurst exponent for 126 societies listed at the Warsaw stock exchang.

This work suggests that the fall of the Hurst exponent under the 0.4 threshold preludes a crash in the prices of that stock. The noteworthy quantity of data considered makes this work a cornerstone in the Hurst exponent analysis while on the other hand the same author in the conclusion of his paper calls for other works considering markets different from the Polish one.Another work finding correspondences between the fall of Hurst exponent and its correlated index is [9].

This work considers uniquely Dow Jones index and the cryses of 1929 and 1987-88 finding that the Hurst exponent forecasts more effectively coming crashes in the case the quotations of the index are in a clear increasing trend.

The same observation is remarked in [12], where the crashes of the Prague stock exchange of the years 2000, 2005, 2006 and 2007 are studied. In cases of 2000, 2005 and 2007 crises the crash is preceded by strong increases (+38.66% in the four months before 2000 crisis, +46% in three months for the crisis of 2005, +30% in respect to pre 2006 crisis values in 2007) and the Hurst exponent starts decreasing noteworthy in the 1-3 months period before the crash.

The case of 2006 crisis is on the other hand not detected as there is no such clear pattern before the crash.

Then a random set of data that was generated from the shuffled logarithmic returns of the index which were cumulated to form the new time series was tested with the Hurst exponent. As in the 2006 case the Hurst exponent does not forecast crashes as no clear preceding trends are identified.

Some years later Kristoufeck published another article, [11], were NASDAQ, Dow Jones and S&P indices are analyzed.

As all of the three indices are extremely similar, also the results obtained with the computation of the Hurst exponent can be synthesized in one single result which is the detection of a fall in the Hurst exponent about a year before 2007 crisis.

This result is actually quite unexpected as by now presented papers show that the Hurst exponent anticipates cryses with no more than a three months' notice, that more often shortens to one single month's notice.

An even more puzzling result is the one in [16]. In this work the authors finds evidences that companies that were going to be bailed out by USA authorities show a long run increasing in the values of the Hurst exponent.

As bailed out companies were reasonably the most hit by the crisis, we would have that an increase in the Hurst exponent would be the signal of coming crashes of the prices of a certain stock, that is the opposite of what was suggested by all previously cited papers.

The authors moreover detect a decreasing trend in the Hurst exponent in sectors that were less strongly hit by the crisis, as the one of Basic Materials.

Now that the current state of Hurst exponent academic investigation has been drawn, next step is to explain the data processing used to obtain Hurst exponent values, that is the topic of chapter 25.2, while the concrete results which were obtained are going to be exhibited in section 25.3. In the end the conclusions of the authors are presented.

25.3 The Hurst exponent and Detrended Fluctuation Analysis

25.3.1 Theoretical introduction

The Hurst exponent is a coefficient that arises naturally in the study of self-similar stochastic processes. The following definition is taken from [8].

Definition A stochastic process $\{X(t) : t \geq 0\}$ is said to be self-similar if for any $a > 0$, there exists $b > 0$ such that $\{X(at)\} \triangleq \{bX(t)\}$

With the symbol $\triangleq$ we denote the equality of all joint distributions for stochastic processes. It is possible to prove ([8]) that for stochastic processes that are nontrivial, stochastically continuous at $t = 0$ and self-similar there exists a unique $H \geq 0$ such that $b = a^H$. In this case, H is called the Hurst exponent of the stochastic process.A variety of self-similar stochastic processes that admit a Hurst exponent have been studied.

Among them, fractional Brownian motion, fractional Gaussian noise and fractional ARIMA (also called ARFIMA) also have an autocorrelation function that depends on the value of H.

The autocorrelation function too allows for a probabilistic treatment of long-rage dependence. In general, values of H strictly higher than $\frac{1}{2}$ indicate a long-term positive autocorrelation, whereas values of H strictly lower than $\frac{1}{2}$ indicate a long-term negative autocorrelation.

It should be noted however ([2]) that not every self-similar process with $H \neq \frac{1}{2}$ exhibits long time autocorrelations, as is sometimes erroneously asserted in literature, so analysis of long-range dependence should not be based on the Hurst exponent alone.

There is evidence ([3]) that the behavior of prices of financial assets can at least be approximated by one of the aforementioned stochastic processes, specifically the versatile ARFIMA model, that even allows for non-stationarities.

Various techniques for estimating the Hurst exponent of the underlying stochastic process, given a discrete time series, have been proposed in literature.

In particular, Detrended Fluctuation Analysis (often abbreviated DFA), first proposed by [17], designed specifically for nonstationary processes, provides an estimator of the Hurst exponent H that characterizes the underlying stochastic process.

A theoretical justification for the use of DFA in the case of fractional Gaussian noise or fractional ARIMA processes can be found in [19].

The initial step of the most basilar version of DFA consists in breaking up the time series in blocks of size s. Then, for each block, the partial sums of the series, $\{Y_i\}$, are calculated.

A straight line is fitted to $\{Y_i\}$ with the method of least squares and the sample variance of the residuals is computed.

The process is repeated for all the blocks and the average of all the variances for all the blocks of the same size is computed.

This number, for large enough s, is asymptotically proportional to s^H, as was proved in the appendix of [19].

25.4 Estimation of the local Hurst exponent

Our data consisted in financial time series representing the daily closure price of 21 stocks and 14 stock indices for thousands of trading days.

The time series that were believed to be generated by some process akin to fractional Gaussian noise or fractional ARIMA were the logarithmic returns, or log returns, defined as $l_i = \ln p_i - \ln p_{i-1}$, where p_i represents the closure price of the asset in the i^{th} trading day.

In order to gain an insight of the market dynamics, the local Hurst exponent was calculated.

The local Hurst exponent is defined ([11]), for each point j of a time series where it is applicable, as the DFA estimation of the Hurst exponent for the sample comprising points $j - L$+1 to j of the original time series, where L is the sliding window length.

The algorithm we employed included several steps and is here described in detail. We denote by $\{p_i\}_{1\leq i\leq N}$ the sequence of prices of an asset for N trading days.

We start with $j = L$. Then the series $\{Y_i\}_{1\leq i\leq L}$, representing the partial sums of the log returns, is constructed as $Y_i = \ln p_{i+j-L}$.

The series $\{Y_i\}_{1\leq i\leq L}$ is divided in $\lfloor L/s \rfloor$ consecutive non-overlapping blocks of size L starting from the beginning and in additional $\lfloor L/s \rfloor$ starting from the end. Therefore no data is neglected even if L is not a multiple of s.

For each block k we denote its subseries of length s by $\{y_i^k\}_{1<i<s}$.

A linear least square fit is performed for the data in $\{y_i^k\}_{1<i<s}$, obtaining a straight line in the form $f_k(x) = m_k x + b_k$.

Then, the (squared) detrended fluctuation F_k^2 is calculated for each block as:

$$F_k^2 = \frac{1}{s}\sum_{i=1}^{s}\left(y_i^k - f_k(i)\right)^2$$

The squared detrended fluctuations for all the blocks are averaged obtaining a number that is a function of s, the length of the blocks, and that we denote by $\langle F^2\rangle(s)$

Steps 2-4 are repeated for all the values of s between some minimum s_{min} and some maximum s_{max}

$\sqrt{\langle F^2\rangle(s)}$ is plotted on a log-log graph for all the considered values of s. The slope of the linear fit to the data is taken as H_j the estimate of the Hurst exponent for the current value of j

The procedure in steps 1-6 is repeated for $j = (L+1)$, then for $j = (L+2)$, and so on, until $j = N$

At the end we obtain a time series of estimated Hurst exponents $\{H_i\}_{L\leq i\leq N}$ that we may compare with $\{p_i\}_{1\leq i\leq N}$.

For $L \leq i \leq N$, H_i represents the Hurst exponent estimated with DFA on the "sliding window of length L" encompassing the prices from p_{i+1-L} to p_i.

For our analysis we selected the same parameters as [11], therefore we chose a sliding window of $L = 500$ trading days (corresponding roughly to two years) and we considered values of s between $s_{min} = 10$ and $s_{max} = 50$.

25.5 Data processing

25.5.1 The data

All the data that are going to be used were taken from historical time series available from the website of Yahoo! Finance (http://finance.yahoo.com/).

The procedure described in chapter 25.2 was applied to the 35 indices and stocks listed in Appendix A that were chosen in a way to have a wide and varied specimen representing both different geographical areas and economical sectors.

In the appendix are also reported the country to which each index is referred to and some Companies names' abbreviations that from now on are going to be used. For the above analyses only adjusted close values have been used.

The results of the calculation of the Hurst exponent are reported in Appendix B. For each index/stock adjusted two graphs are present.

Each graph shows in the x axis the date each datum is referred to; the upper graph shows in the y axis the price of its object in a log scale while the other one shows the values of the Hurst exponent. It should be noted that each datum corresponds to a trading day and that for the first 500 data the Hurst exponent was not evalued because the procedure requires a time window of 500 data. As 500 data represnts approximately a time period of lenghth of two years (500 trading days equals two years), each Hurst exponent's graph has therefore no values for this first period.

The Hurst exponent values obtained are then going to be studied to verify wether or not they can be an useful indicator to forecast 2007 crisis on indices and stocks analyzed.

Once this has been done, the procedure in chapter 25.1 section 25.2 is going to be applied to the data set to find out if it provides consistent outcomes. The results obtained are then discussed.

25.6 Crisis detection with Hurst exponent

To ascertain if the Hurst exponent could forecast a crisis it was first of all needed to identify the period of the crisis. For each stock and index the date when the value of the price reaches its relative maximum in the period ranging from of the 1st of January 2007 and the 31st of December 2009 have been therefore found (from now on we refer to this date as to max date).

This date has been considered to separate the crisis period from the pre crisis one; if an Hurst exponent decrease really anticipates crises, a crash in its values should be present in the days before max date.

In table 1, column "Befire max date", the values of the minimum of the Hurst exponent in the 21 days preceding max date are reported in subcolumn Min. }

The two following subcolumns, H21 and H5, contain the average value of the Hurst exponent in the 21 and 5 days periods preceding this same date.

These values have been reported for every period mentioned in the very first row of the table. These same calculations have been repeated for different time spans, whose initial date is reported in the following three columns.

These different time spans consist in two randomly chosen ones (the 12th of May 2006 and the 15th of March 2012) and finally a day identified looking for the one that minimizes the mean value of H21 of all 35 indices/stocks (the 20th of November 2008).

In the last row of the table, for each column, the number of data superior to their "Before max date" peer are counted. Not available data have been market with a --- symbol in their corresponding boxes.

	Before max date			12/05/06			15/03/12			20/11/08		
	Min	H21	H5	Min	H21	H5	Min	H21	H5	Min	H21	H5
AEX	0.47463	0.49142	0.48589	0.48678	0.50247	0.49619	0.43319	0.44761	0.45315	0.43392	0.46149	0.48582
ATHEX	0.48111	0.49541	0.49985	0.42141	0.47006	0.43486	0.42992	0.45417	0.45920	0.44913	0.46912	0.48761
ATX	0.52144	0.53334	0.52681	0.47495	0.53237	0.48654	0.39812	0.42562	0.45954	0.38202	0.40425	0.42773
DAX	0.40266	0.42426	0.41576	0.41226	0.43405	0.42222	0.41582	0.42578	0.43430	0.39747	0.42669	0.45043
CAC	0.41811	0.43809	0.44426	0.38636	0.40889	0.39516	0.39318	0.40725	0.41371	0.34700	0.37131	0.38838
EXCH	0.48416	0.49253	0.48885	0.47852	0.51136	0.52261	0.41025	0.43114	0.44522	0.39785	0.42301	0.44638
FTSE	0.47517	0.49002	0.49073	0.49000	0.52382	0.51442	0.41127	0.42660	0.43535	0.36059	0.38632	0.41104
IBEX	0.45784	0.47263	0.46233	0.47663	0.49606	0.48582	0.37333	0.37986	0.38049	0.31092	0.32970	0.32863
ISEQ	0.42648	0.45940	0.44228	0.54275	0.59030	0.59495	0.38136	0.40252	0.40584	0.36645	0.37800	0.38236
NASDAQ	0.37403	0.39444	0.39365	0.43680	0.46868	0.47992	0.44470	0.45942	0.47260	0.34544	0.37180	0.38222
NIKKEI	0.46679	0.47567	0.48088	0.43522	0.45244	0.44112	0.44717	0.46426	0.46508	0.36515	0.39474	0.41615
OMXS	0.41642	0.43520	0.43053	---	---	---	0.36317	0.37730	0.38087	0.37099	0.39290	0.41397
S&P/TSX	0.46659	0.48609	0.48908	0.47986	0.50504	0.51833	0.41788	0.43203	0.42983	0.37266	0.40034	0.42895
SMI	0.46359	0.48083	0.49130	0.46501	0.49897	0.49818	0.42444	0.44279	0.45111	0.31375	0.33146	0.33324
AG	0.46944	0.48752	0.47816	0.54736	0.56494	0.56254	0.41191	0.43020	0.42802	0.34957	0.37213	0.38249
Apple	0.48157	0.50198	0.50634	0.43955	0.44958	0.44289	0.38888	0.41253	0.43385	0.51222	0.52522	0.53702
Barclays	0.49486	0.50304	0.50235	0.50382	0.55321	0.57516	0.34770	0.35957	0.36044	0.33789	0.36584	0.34872
Bayer	0.41796	0.43297	0.42282	0.52086	0.52859	0.52711	0.40764	0.42613	0.43921	0.44849	0.45958	0.46545
BCP	0.41273	0.43279	0.41956	0.38466	0.40723	0.40800	---	---	---	0.43275	0.46621	0.44826
Coca Cola	0.40854	0.42068	0.41979	0.55680	0.57021	0.58521	0.42695	0.44156	0.43401	0.30734	0.33861	0.35016
EDF	---	---	---	---	---	---	0.47913	0.48769	0.49404	0.46532	0.48105	0.49809
ENEL	0.40564	0.41755	0.40833	0.53053	0.53958	0.54421	0.42920	0.44557	0.45116	0.35529	0.36964	0.36980
ENI	0.49245	0.52707	0.50013	0.45609	0.47912	0.46837	0.47416	0.48813	0.49081	0.40553	0.44785	0.47891
Exxon	0.52316	0.53402	0.53957	0.48317	0.49463	0.49349	0.46699	0.48616	0.50244	0.31114	0.33574	0.34974
FT	0.45442	0.47181	0.46458	0.37571	0.38831	0.38952	0.39921	0.41586	0.42645	0.32849	0.34194	0.34895
IBM	0.41937	0.43123	0.42409	0.59445	0.60536	0.60802	0.39819	0.40714	0.41094	0.39300	0.42321	0.43093
Microsoft	0.42473	0.45857	0.44293	0.45918	0.49338	0.51825	0.48969	0.51829	0.53524	0.32207	0.33251	0.33840
PSA	0.54434	0.56969	0.55286	0.53596	0.54854	0.55693	0.44618	0.46654	0.47158	0.34442	0.36053	0.36661
RDS	0.35886	0.52145	0.51724	---	---	---	0.48794	0.50106	0.50409	0.41152	0.43972	0.46802
Renault	0.40403	0.41833	0.41223	0.51715	0.53099	0.53323	0.41174	0.43014	0.41957	0.35355	0.39471	0.37211
Santander	0.41708	0.44204	0.42474	0.46370	0.50826	0.47200	0.35561	0.36790	0.36212	0.30670	0.38632	0.36681
SG	0.48148	0.50084	0.50577	0.40161	0.42559	0.44063	0.36069	0.38669	0.37505	0.32274	0.34865	0.34221
Toyota	0.43511	0.45336	0.47083	0.41519	0.44690	0.43100	0.46939	0.48881	0.49323	0.35419	0.38816	0.39142
Vow	0.08343	0.40321	0.25776	0.52732	0.56125	0.57236	0.47078	0.48655	0.48860	0.08549	0.17360	0.12964
Xstrata	0.43532	0.44745	0.44404	0.36115	0.40231	0.37978	0.41765	0.43057	0.43689	0.32978	0.35617	0.40163
Number of data superior than "Before max date" column				18	19	20	9	8	9	5	4	5

Data set's Hurst exponent averages and minima over different periods

A first test on the ability of the Hurst index to foresee crises can be done comparing the values in the three columns composing “Before max date” with the ones of the following two columns, obtained choosing randomly two dates and applying the same procedure.

If the Hurst exponent diminished before 2007 crisis, the values of “Before max date” should be sensibly lower than the other ones and therefore the majority of the data of these last two columns should be superior than the ones of “Before max date”. This prediction is slightly verified in the 12th of May 2006 case, when 19 data coming from “Before max date”out of a total of 34 are minor than their correspondent ones. On the other hand this hypothesis is openly contradicted by the columns 15th of March 2012, when the number of data of “Before max date” minor to their correspondent ones decreases to 9, and 20th of November 2008, where this last number even falls to 5.

The results of identically conducted investigations on the periods of 21 and 5 days after max date and other two randomly chosen date (27th of November 2004 and 1st of September 2009) are moreover presented in Appedix C. Being the results of these analyses very similar to previously shown ones, it could be argued that the Hurst exponent does not seem to suffer any anticipate fall forecasting crises.

The hypothesis presented in [5] has been then tested. This hypothesis stated that the Hurst exponent indicates a coming crisis in the case the following conditions were verified:

- The Hurst exponent is in decreasing trend
- H-21≤0.5
- H-5≤0.45
- The minimum value of the Hurst exponent in the period just before the crisis is inferior to 0.4 (as the Hurst exponent trend is decreasing the minimum should stay the nearest to the moment of the market crash)

To verify the first condition all the periods of 21 successive trading days of all indices/stocks were considered. Once the coefficient indicating the slope of the line coming from a linear regression of the Hurst exponent values of each period were calculated, only the days presenting a negative value of this coefficient were taken into consideration for next steps.

This procedure should have selected only periods verifying the first condition, that is to say decreasing Hurst exponent values. Each period of 21 days was then labeled with the date of the last day, that is the one from which it would have been concretely possible to detect the crisis as in the previous days it would not have been possible to have an idea of how the Hurst exponents value could have evolved afterwards. Next two conditions were then applied (H-21<0.5 and H-5<0.45) and in the end a value of the Hurst exponent of 0.4 have been looked for in each remaining 21 days period. Determining the period where at least one value of the Hurst exponent inferio to 0.4 should have been present was done arbitrarily. It could nevertheless be noted that this period should not be bigger than 21 days, that is the lenght of a period; we then have that decreasing its size should not affect much final results as being the values of the Hurst exponent decreasing (see first condition) the values minor to 0.4 should be close to the end of the period.

A final and additional condition requested was that each crisis detected in this way was at least 30 days far from the subsequent one.

This was done to avoid having many subsequent days all identified as crisis periods because the really interesting value is just the first day from which it was possible to detect each crisis while other immediately subsequent are redundant.

The results of the analysis described in the previous paragraph are reported in Appendix D. First the name of each index/stock is followed by the date when the price hits its relative maximum in the period ranging from the 1st of January 2007 to the 31st of December 2009, that is so far called max date. In the same row, it is also reported the minimum price registered in the same time period, corresponding to the hardest time in the crisis.

The following dates are the ones when the adopted procedure indicates a coming fall in the quotation of the index/stock.

Finally the dates when coming crises are detected follows.

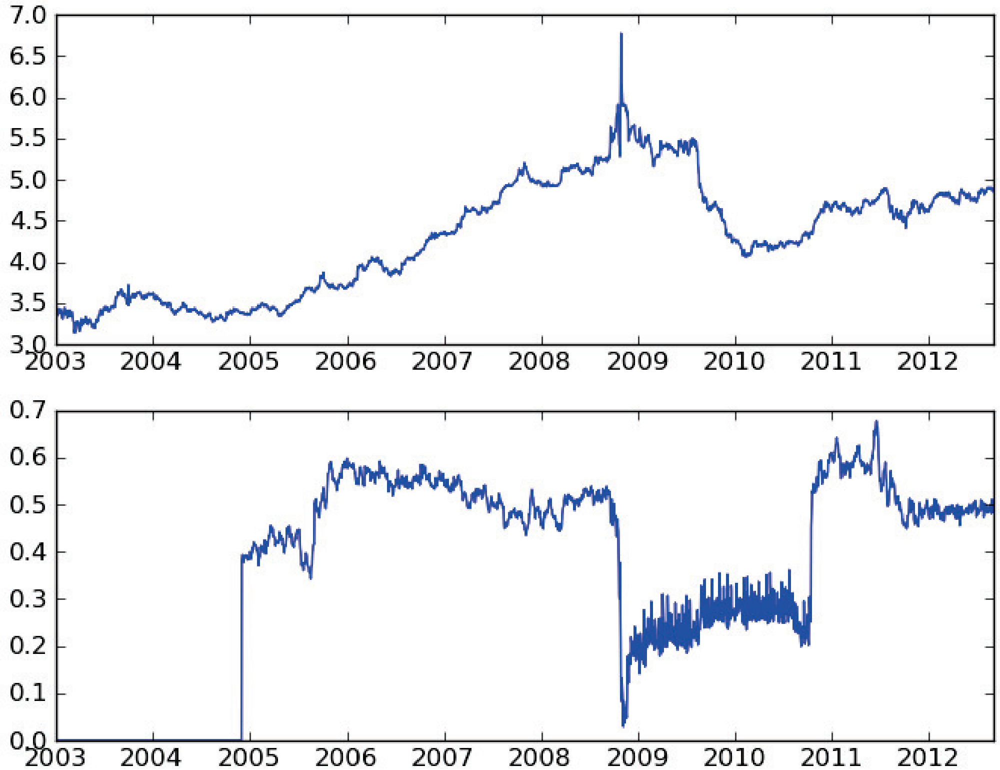
Next step was then to count how many times our procedure detected a crisis in the period between max date and three months before this day, that is to say how many times Hurst exponent analysis effectively forecast 2007 crisis.

Out of 35 indices/stocks only in 3 cases the coming crisis was forecast. It was also noted that this procedure identified many dates that were not followed by any fall in the index/stock considered. This result is evidently in contrast with the hypothesis that the Hurst exponent decreases considerably before a crisis and thus this hypothesis has to be rejected.

This is to say Hurst exponent analysis does not seem apt to forecast crises.

25.7 Discussion

Let’s now consider the quite curious case of Volkswagen. Its Hurst exponent, as shown by the following graph, decreases considerably after an abrupt movement in its price:



Prices and local Hurst exponent values of Volkswagen stock

A brusque movement in the prices as the one of Volkswagen increases their mean value that compared with the data shows first a period of low prices (that are the ones occurring in normal market period) followed by an high one (that is the peak) and then other low prices. As the couple high-low values indicates anticorrelation, this causes the Hurst exponent to crash. The same effect is not necessarily produced by a peak followed by other normal values but could also be caused by one single considerable movement.

Let's consider a sudden decrease in a stock quotation: this movement would cause a fall in the mean value so that quotations before the turning point wuold seem high values followed by low ones. All this is to say that the crisis could be considered as a quick and brusque movement in the prices causing the Hurst exponent to fall. The same effect could be moreover magnified by the increase of volatility usually following crisies, that mimic the big movement effect in a smaller scale.

This hypothesis seems to be confirmed by the time period minimizing the mean of H21 values of the data set. This period is indeed the 20th of November 2008 that is sited in such a way to collect the biggest number of price crashes, including the ones of European companies happened later than the American ones. The procedure previously described to detect crises applied to the period of time of three months centred on the 20th of November 2008 would give positive results 20 times out of 35. Incresing the time window to six months, we have 27 alerting results out of a sample of 35 indices/stocks.

25.7.1 Interpreting obtained results

25.7.1.1 Interpreting found results

But then why have many authors found correspondences between Hurst exponent crashes and coming crises? One first answer is the chance: many papers concentrating only on one single index could have had a sort of bad luck, and this could be the case of [11].

In this article the author only considers NASDAQ, Dow Jones and S&P indices, that being very similar convey to the case of the analysis of one single index. Unluckily NASDAQ is in addition one of the 3 indices out of 35 that were found to react positively to previously described procedure that was used to detect crises.

Another issue could have affected many other works.

Some authors indeed underlines that the Hurst exponent does not work properly if the crisis period is not preceded by a clear and very strong increasing trend in prices. This trend could be considered as a sort of brusque movement similar to the one that caused the Hurst exponent crash in the case of Volkswagen.

The Hurst exponent crash would not therefore detect the coming crisis but just a quick movement in prices. As many economic crashes are preceded by speculative bubbles, this could be a reason to this phenomenon.

25.8 Conclusions

The analysis of the data set reported in Appendix A did not seem to give any positive confirmation to the hypothesis of connection between coming crises and falls in the Hurst exponent. This hypothesis has been therefore abandoned and considered erroneous.

A deeper look at the Hurst exponent suggested that this apparent correlation is a consequence of its property of decreasing in cases of abrupt movements and very volatile market conditions. These conditions could have been caused by speculative bubbles preceding the crashes.

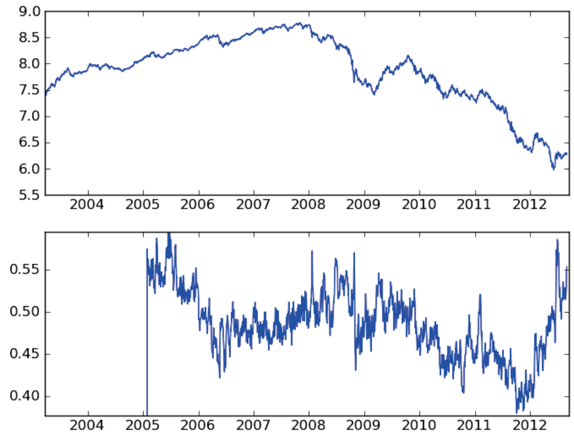
Appendix A:

Stocks/indices considered

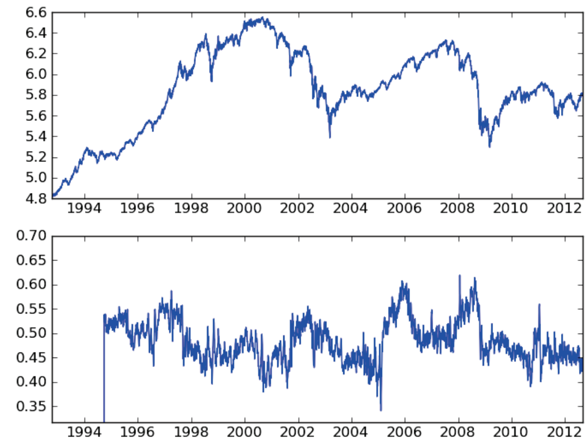
Index/Stock	Abbreviation	Country
AEX	AEX	Netherlands
ATHEX	ATHEX	Greece
ATX	ATX	Austria
CAC	CAC	France
DAX	DAX	Germany
EXCH	EXCH	Norway
FTSE	FTSE	United Kingdom
IBEX	IBEX	Spain
ISEQ	ISEQ	Ireland
NASDAQ	NASDAQ	USA
NIKKEI	NIKKEI	Japan
OMXS	OMXS	Sweden
S&P/TSX	S&P/TSX	Canada
SMI	SMI	Switzerland
Assicurazioni Generali	AG	Italy
Apple	Apple	USA
Barclays	Barclays	United Kingdom
Bayer	Bayer	Germany
Banco Comercial Português	BCP	Portugal
Coca Cola	Coca Cola	USA
Électricité De France	EDF	France
Ente Nazionale per l'energia Elettrica	ENEL	Italy
Ente Nazionale Idrocarburi	ENI	Italy
Exxon Mobil	Exxon	USA
France Télécom	FT	France
IBM	IBM	USA
Microsoft	Microsoft	USA
PSA Peugeot Citroen	PSA	France
Royal Dutch Shell	RDS	Netherlands
Renault	Renault	France
Grupo Santander	Santander	Spain
Société Générale	SG	France
Toyota Motor Corporation	Toyota	Japan
Volkswagen	Vow	Germany
Xstrata	Xstrata	United Kingdom

Appendix B:

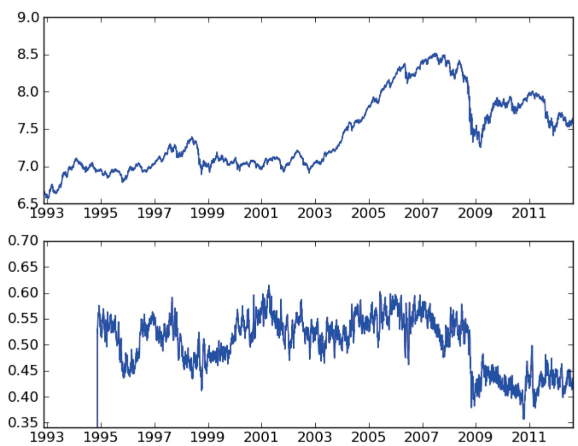
Hurst exponent graphs



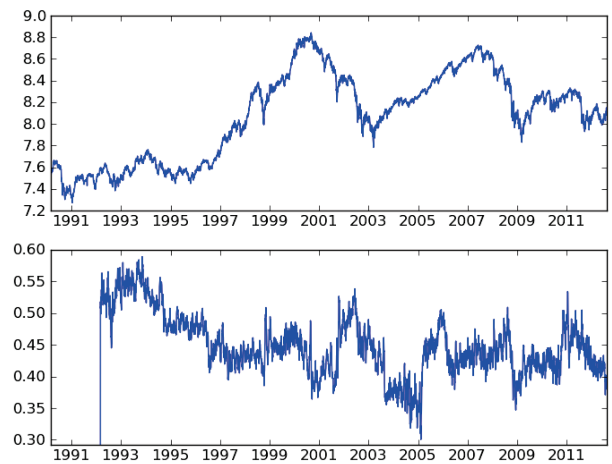
AEX



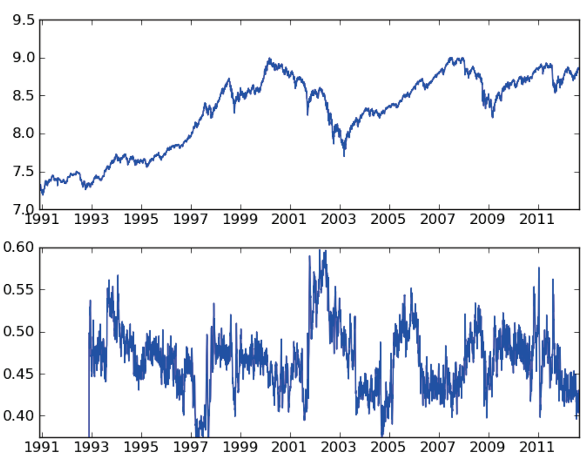
ATHEX



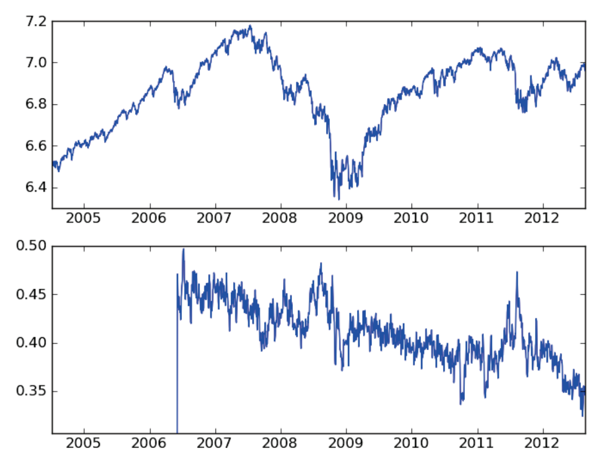
ATX



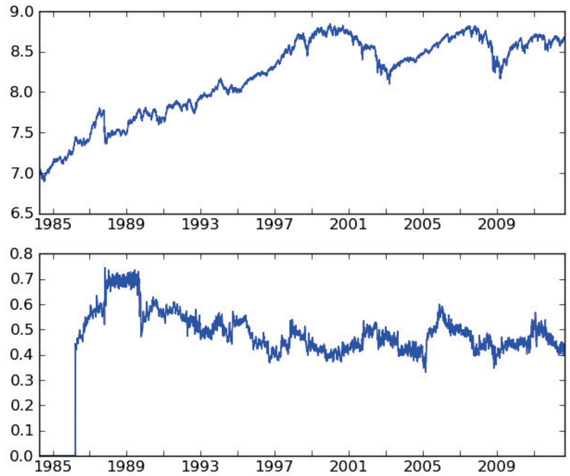
CAC



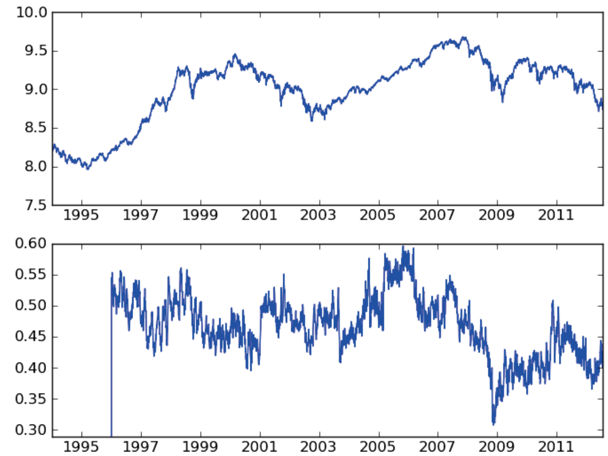
DAX



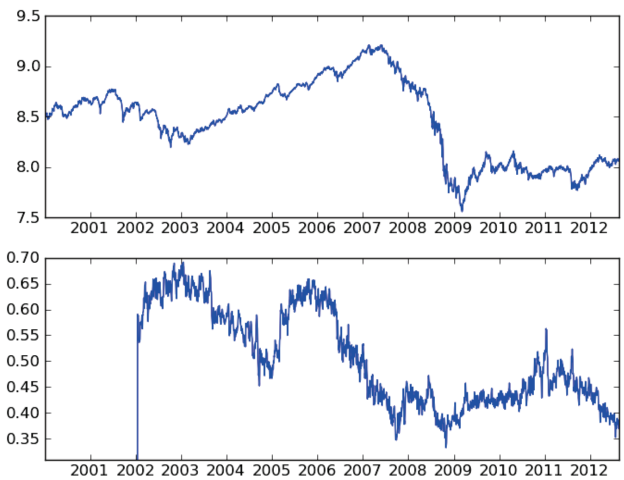
EXCH



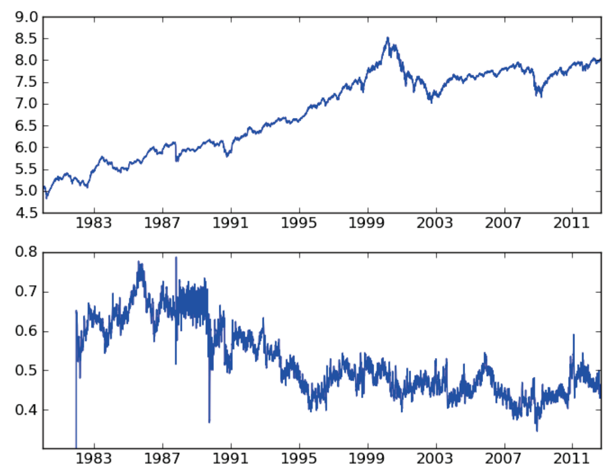
FTSE



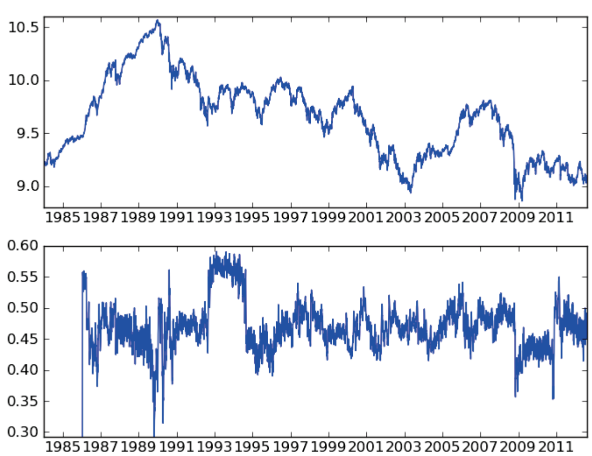
IBEX



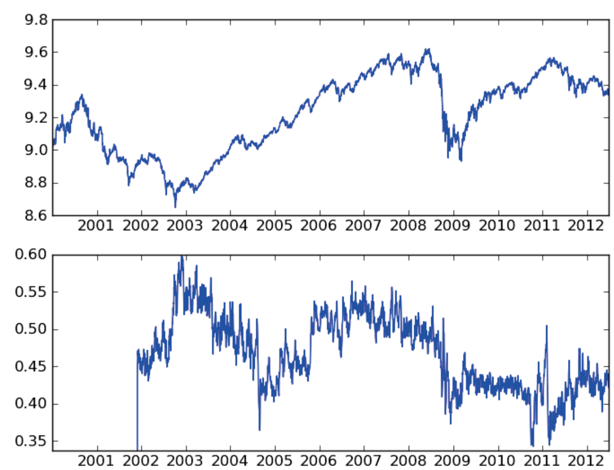
ISEQ



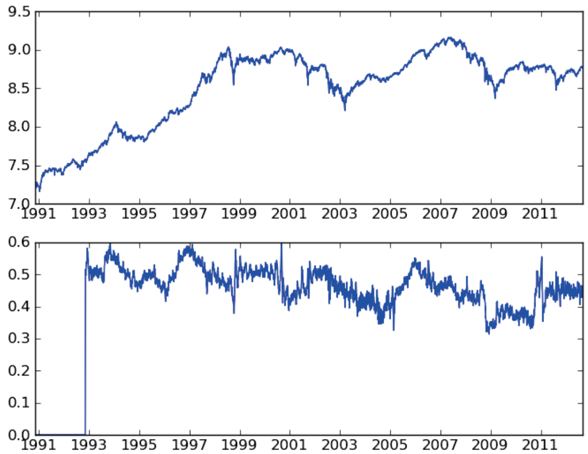
NASDAQ



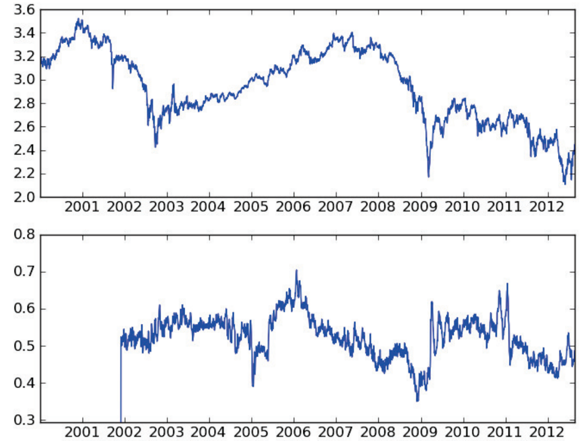
OMXS



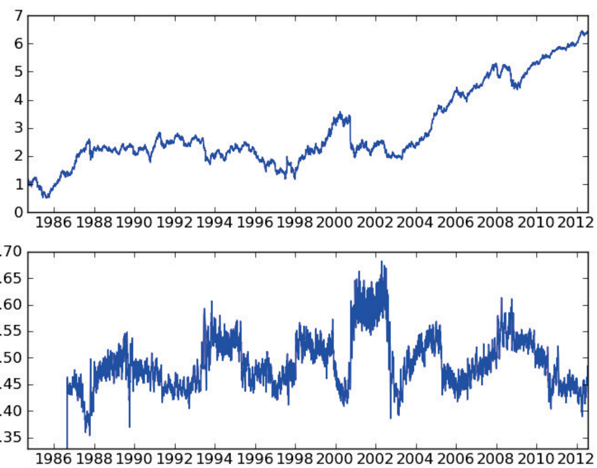
S&P/TSX



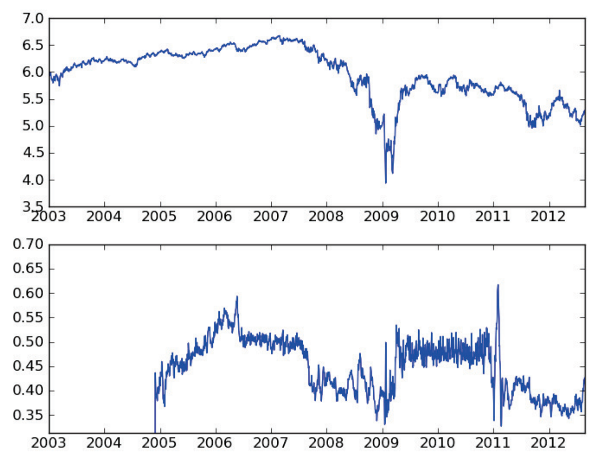
SMI



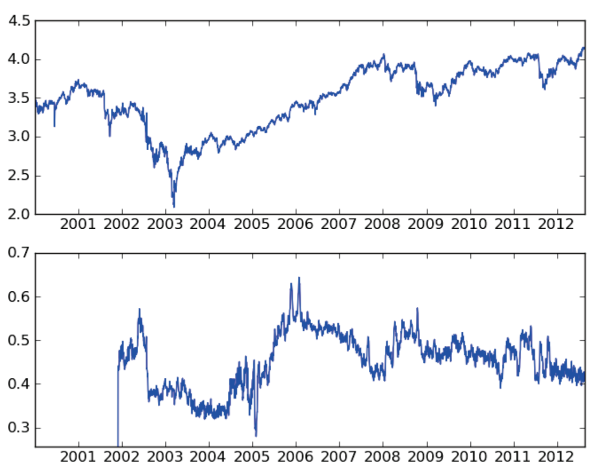
AG

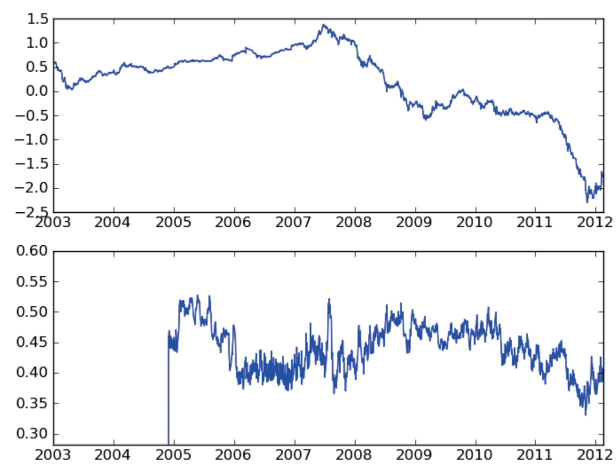


Apple



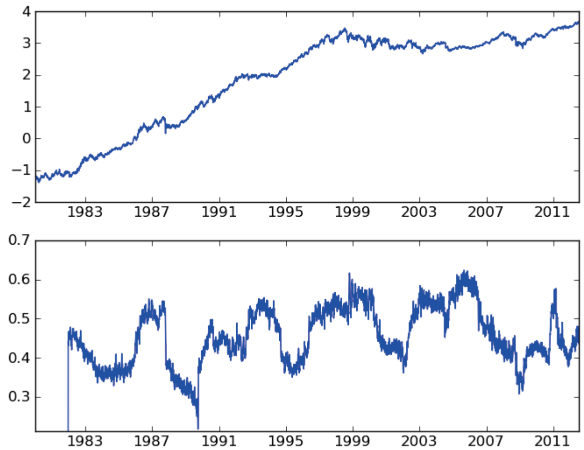
Barclays



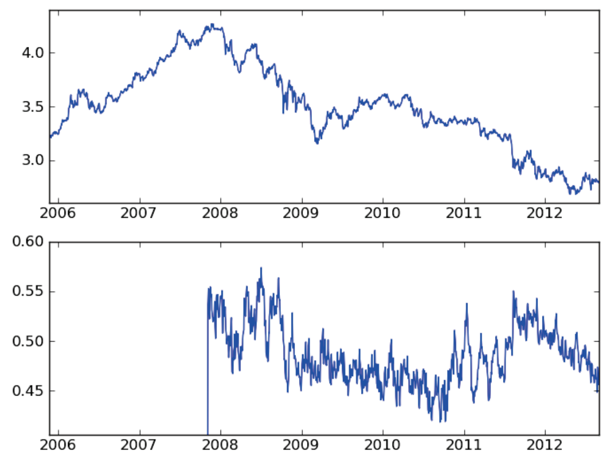


BCP

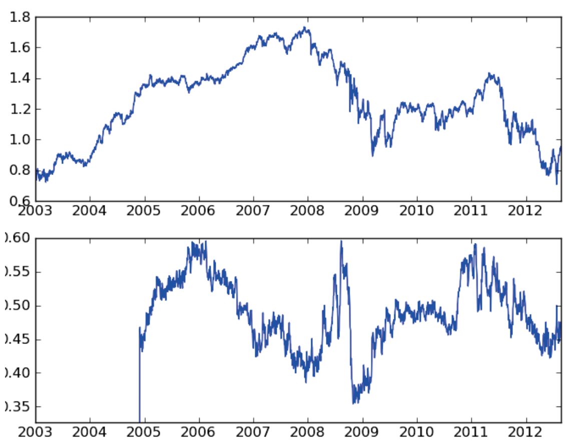
Bayer



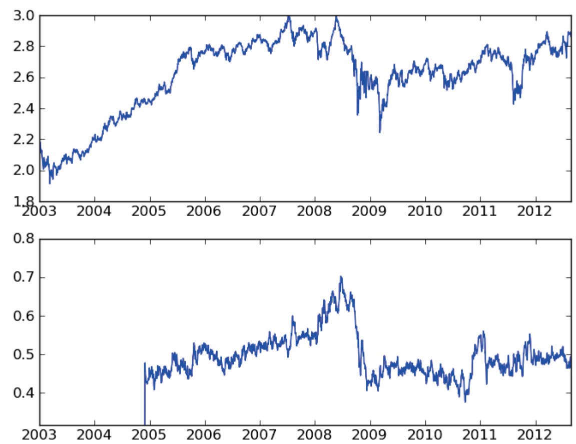
Coca Cola



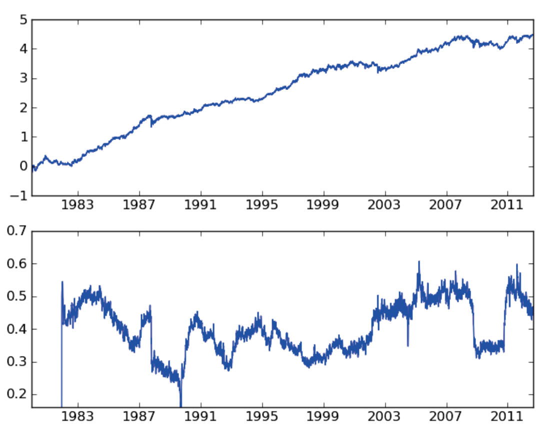
EDF



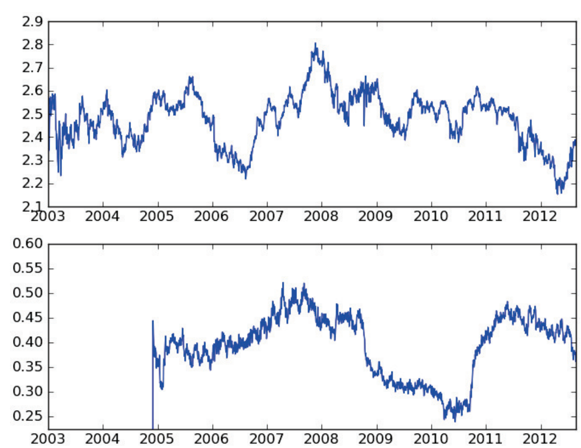
ENEL



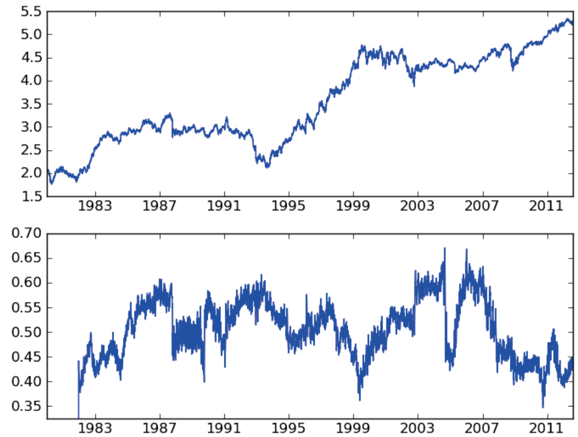
ENI



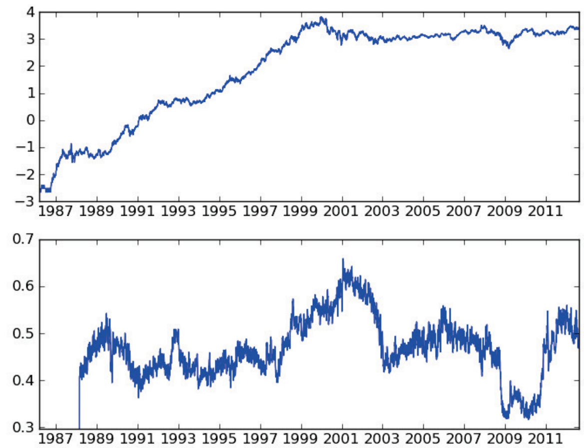
Exxon



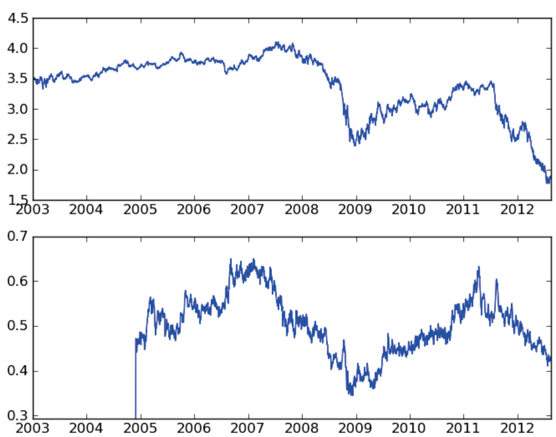
FT



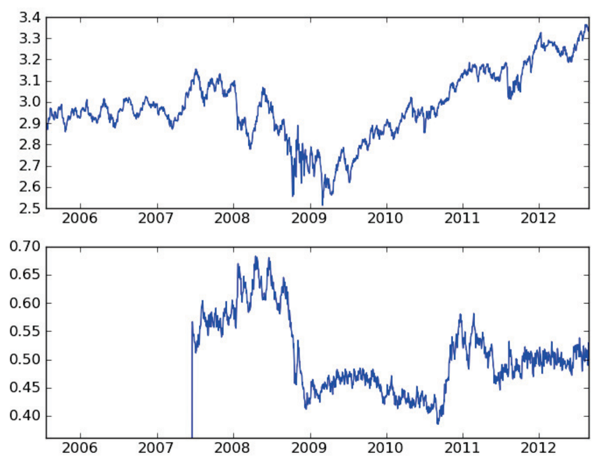
IBM



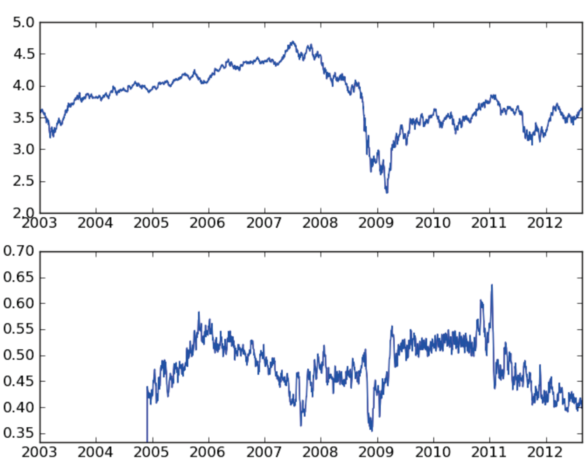
Microsoft



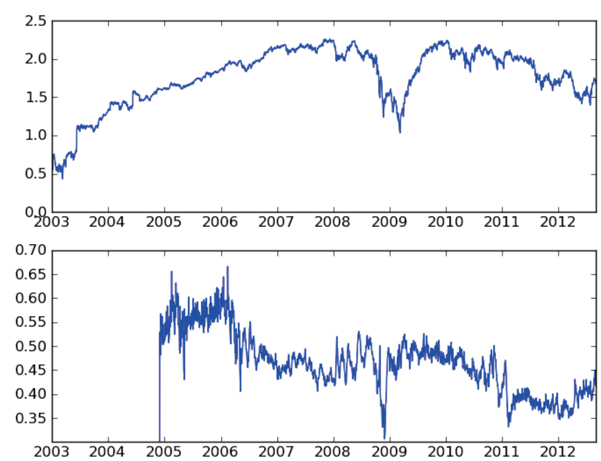
PSA



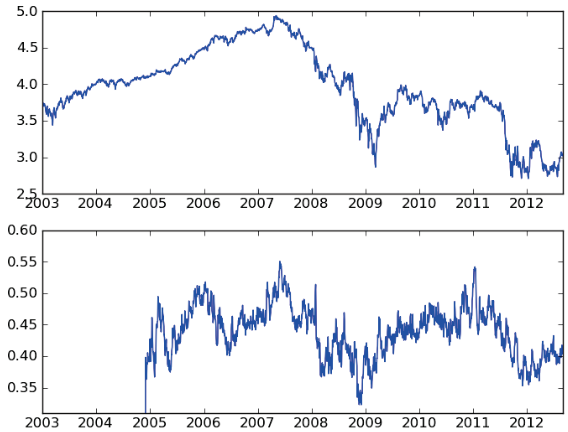
RDS



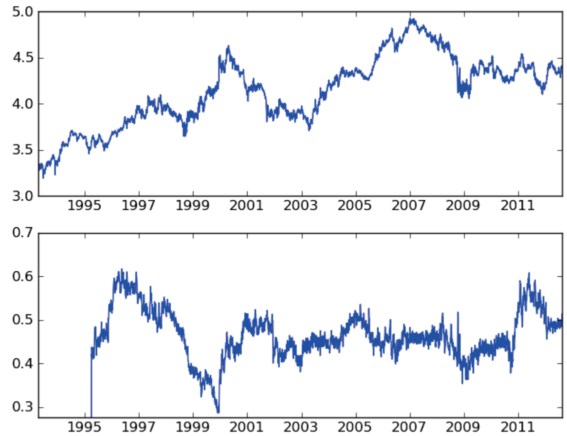
Renault



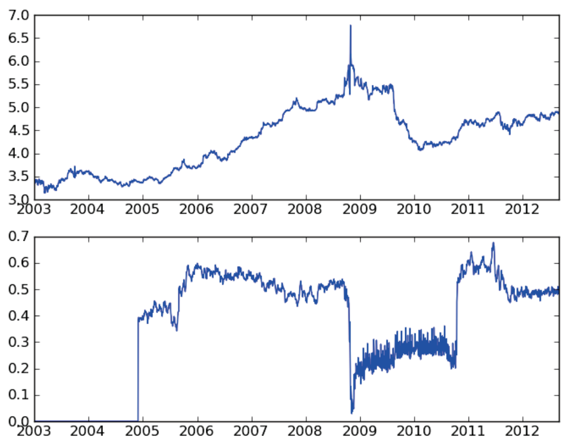
Santander



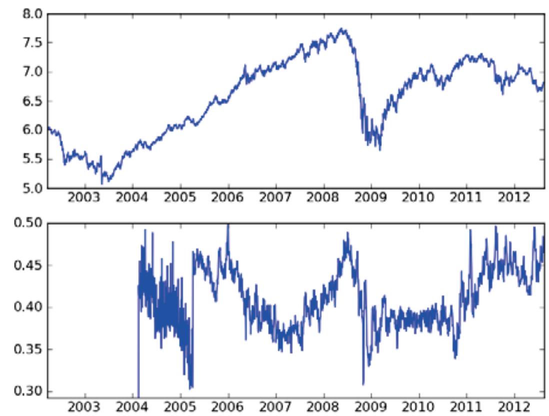
SG



Toyota



Vow



Xstrata

Appendix C:
Other tests on Hurst exponent values

	Before max date			After Max			27-nov-04			01-set-09		
	Min	H21	H5	Min	H21	H5	Min	H21	H5	Min	H21	H5
AEX	0.47463	0.49142	0.48589	0.46926	0.49197	0.47905	0.36976	0.40987	0.44188	0.47140	0.48278	0.49418
ATHEX	0.48111	0.49541	0.49985	0.48676	0.50470	0.50064	---	---	---	0.48621	0.50233	0.51674
ATX	0.52144	0.53334	0.52681	0.51455	0.52907	0.52695	0.52727	0.54277	0.53951	0.42952	0.44117	0.44894
CAC	0.41811	0.43809	0.44426	0.42709	0.44689	0.45036	0.31868	0.34933	0.37183	0.40805	0.42034	0.43049
DAX	0.40266	0.42426	0.41576	0.39807	0.42322	0.40280	0.39657	0.41149	0.41888	0.46905	0.48695	0.50065
EXCH	0.48416	0.49253	0.48885	0.48246	0.50746	0.49587	0.55012	0.56235	0.56054	0.41708	0.43611	0.44315
FTSE	0.47517	0.49002	0.49073	0.47498	0.48525	0.48344	0.37405	0.41413	0.42193	0.43533	0.44616	0.45900
IBEX	0.45784	0.47263	0.46233	0.45479	0.47447	0.46394	0.48174	0.49967	0.49915	0.38485	0.39531	0.40400
ISEQ	0.42648	0.45940	0.44228	0.42986	0.45469	0.44662	0.46661	0.48473	0.49242	0.40862	0.42033	0.42782
NASDAQ	0.37403	0.39444	0.39365	0.38705	0.40592	0.39058	0.42936	0.44793	0.44794	0.41527	0.43644	0.44806
NIKKEI	0.46679	0.47567	0.48088	0.46772	0.48229	0.47216	0.44614	0.46308	0.46618	0.43115	0.44502	0.44842
OMXS	0.41642	0.43520	0.43053	0.41977	0.43497	0.43208	---	---	---	0.39832	0.41222	0.42007
S&P/TSX	0.46659	0.48609	0.48908	0.48171	0.49972	0.49020	0.40223	0.41785	0.42409	0.41336	0.42451	0.42876
SMI	0.46359	0.48083	0.49130	0.45750	0.46782	0.47360	0.37081	0.40745	0.42463	0.36338	0.38401	0.39632
AG	0.46944	0.48752	0.47816	0.48192	0.52121	0.49398	0.48925	0.53033	0.54233	0.51068	0.53072	0.54118
Apple	0.48157	0.50198	0.50634	0.48845	0.50253	0.50588	0.50254	0.52137	0.51920	0.51947	0.54124	0.55174
Barclays	0.49486	0.50304	0.50235	0.47168	0.50122	0.49489	---	---	---	0.45102	0.47889	0.49069
Bayer	0.41796	0.43297	0.42282	0.40730	0.45179	0.41442	0.32851	0.36012	0.36373	0.47191	0.49371	0.50803
BCP	0.41273	0.43279	0.41956	0.39259	0.42797	0.42287	---	---	---	0.44851	0.46064	0.46584
Coca Cola	0.40854	0.42068	0.41979	0.38963	0.41759	0.40496	0.54658	0.56174	0.55967	0.40691	0.41870	0.41528
EDF	---	---	---	0.50268	0.52813	0.52383	---	---	---	0.44716	0.46928	0.47167
ENEL	0.40564	0.41755	0.40833	0.38535	0.40626	0.40505	---	---	---	0.47198	0.48316	0.48616
ENI	0.49245	0.52707	0.50013	0.50950	0.53876	0.51281	---	---	---	0.46121	0.47008	0.47226
Exxon	0.52316	0.53402	0.53957	0.49147	0.50558	0.51186	0.47555	0.48861	0.48422	0.33407	0.34321	0.34702
FT	0.45442	0.47181	0.46458	0.43155	0.44535	0.45033	---	---	---	0.29273	0.30601	0.30188
IBM	0.41937	0.43123	0.42409	0.41870	0.43251	0.43250	0.42216	0.44823	0.45279	0.41885	0.43644	0.44378
Microsoft	0.42473	0.45857	0.44293	0.40871	0.44751	0.44927	0.45564	0.48379	0.48776	0.35348	0.36559	0.36835
PSA	0.54434	0.56969	0.55286	0.51759	0.55042	0.54986	---	---	---	0.43750	0.44966	0.46019
RDS	0.35886	0.52145	0.51724	0.51637	0.54910	0.52704	---	---	---	0.45386	0.47008	0.47046
Renault	0.40403	0.41833	0.41223	0.40189	0.42286	0.41543	---	---	---	0.49619	0.51225	0.52799
Santander	0.41708	0.44204	0.42474	0.41650	0.42815	0.41921	---	---	---	0.46574	0.48458	0.49412
SG	0.48148	0.50084	0.50577	0.48681	0.51558	0.49165	---	---	---	0.42524	0.43803	0.44056
Toyota	0.43511	0.45336	0.47083	0.44770	0.46019	0.46160	0.48228	0.50205	0.49732	0.41637	0.43267	0.44105
Vow	0.08343	0.40321	0.25776	0.02920	0.08112	0.09688	---	---	---	0.23876	0.27328	0.27832
Xstrata	0.43532	0.44745	0.44404	0.44704	0.46125	0.44898	0.36359	0.38320	0.39684	0.37173	0.38289	0.38916
Number of data superior than "Before max date" column				14	18	16	11	11	12	12	11	13