

## Design and construction of a Darrieus vertical axis turbine and analyzed by reverse engineering

### Diseño y construcción de una turbina de eje vertical Darrieus y analizada por ingeniería inversa

GARCÍA-HERNÁNDEZ, Miguel Alejandro†, CRUZ-GOMEZ, Marco Antonio\*, JUAREZ-ZERÓN, Tomás Aarón and SAAVEDRA-CRUZ, Nubia

*Benemérita Universidad Autónoma de Puebla, Faculty of Engineering, Tribology and Transportation Group, Academic Body 189 (Disaster Prevention and Sustainable Development, Tribology, BUAP), Graduate Building, First Level, Cubicle No. 16, Blvd. Valsequillo esq. Av. San Claudio, Ciudad Universitaria, Col. San Manuel, CP. 72570, Puebla Mexico.*

ID 1<sup>st</sup> Author: Miguel Alejandro, García-Hernández / ORC ID: 0000-0002-3813-8836, Researcher ID Thomson: AAM-3746-2021, CVU CONACIT ID: 1117920

ID 1<sup>st</sup> Co-author: Marco Antonio, Cruz-Gómez / ORC ID: 0000-0003-1091-8133, Researcher ID Thomson: S-3098-2018, CVU CONACYT ID: 349626

ID 2<sup>nd</sup> Co-author: Tomás Aarón, Juárez-Zerón / ORC ID: 0000-0002-9796-0540, Researcher ID Thomson: S-3099-2018, CVU CONACYT ID: 295058

ID 3<sup>rd</sup> Co-author: Nubia, Saavedra-Cruz / ORC ID: 0000-0002-9606-6893, Researcher ID Thomson: AKK-8861-2021, CVU CONACYT ID: 1118065

DOI: 10.35429/JIE.2021.15.5.1.9

Received July 10, 2021; Accepted December 30, 2021

#### Abstract

The implementation of sustainable generation systems satisfying the demands of the electric power line has begun to become a necessity given the climatic consequences adjacent to the means of obtaining energy by burning fossil fuels. Today, Mexico has 30.14% of sustainable means installed as generators to the Electric Power System, but only as secondary feeders given their intermittency. However, wind farms have shown promise, being able to satisfy more than 30% of the Southeast Peninsula line at peak hours. Therefore, this research aimed to design and build a prototype of a reverse-engineered back-fed Darrieus vertical axis wind turbine. With the psychrometric chart data at 2135 masl, the parameters of angular velocity, tip speed ratio and wind rotor power were identified. In addition, the wind flow behavior was analyzed by means of a finite element modeling bounded by the K-Epsilon turbulence system and the boundary conditions pertinent to the State of Puebla. Finally, the results obtained will be discussed and based on these, how the implementation of this turbine in urban areas benefits.

#### Resumen

La implementación de sistemas de generación sustentable satisfaciendo las demandas de la línea de potencia eléctrica ha comenzado a convertirse en una necesidad dadas las consecuencias climáticas adyacentes a los medios de obtención de energía mediante la quema de combustibles fósiles. Hoy en día, México cuenta con un 30.14% de medios sustentables instalados como generadores al Sistema Eléctrico de Potencia, pero únicamente como alimentadores secundarios dadas sus intermitencias. Sin embargo, los parques eólicos han demostrado ser prometedores, siendo capaces de satisfacer más del 30% de la línea de la Península Sureste en horas pico. Por lo tanto, esta investigación tuvo como objetivo diseñar y construir un prototipo de aerogenerador de eje vertical Darrieus retroalimentada con ingeniería inversa. Con los datos de la carta psicométrica a 2135 msnm, fueron identificados los parámetros de velocidad angular, Relación de Velocidad de Punta y potencia del rotor eólico. Además, el comportamiento de flujo de viento fue analizado mediante un modelado de elemento finito delimitado por el sistema de turbulencias K-Épsilon y las condiciones de frontera pertinentes al Estado de Puebla. Para finalizar, se discutirán los resultados obtenidos y en base a estos; como beneficia la implementación de esta turbina en zonas urbanas.

**Darrieus wind turbine, Reverse engineering, CFD**

**Turbina eólica Darrieus, Ingeniería inversa, CFD**

**Citation:** GARCÍA-HERNÁNDEZ, Miguel Alejandro, CRUZ-GOMEZ, Marco Antonio, JUAREZ-ZERÓN, Tomás Aarón and SAAVEDRA-CRUZ, Nubia. Design and construction of a Darrieus vertical axis turbine and analyzed by reverse engineering. Journal Industrial Engineering. 2021. 5-15:1-9.

\* Author Correspondence (Email: miguel.garciaher@alumno.buap.mx).

† Researcher contributing as first author.

## Introduction

The generation of electrical energy through the excitation of the armature winding of a synchronous machine is a fundamental fact in any power system that seeks to satisfy a high load demand; however, 69.86% of the means to turn the rotors of the generators of the grid in Mexico are by burning fossil fuels, combined cycles, and nuclear power plants, which produce large amounts of carbon emissions and substances that degrade the integrity of the ozone layer. It is therefore advisable to explore the other remaining 30.14% of the excitation means for our generators: by sustainable means. *Centro Nacional de Control de Energía [CENACE]. (2021) y Programa de Desarrollo del Sistema Eléctrico Nacional 2017-2031 [PRODECEN]. (2020).*

Among the five means of sustainable energy generation used in Mexico (photovoltaic, wind, biomass, hydroelectric and geothermoelectric), this research focused on wind power, specifically on the Darrieus vertical axis designs, which consist of capturing the winds through the blades of this turbomachinery, which rotates perpendicularly to the direction of the air flows. However, the implementation of vertical wind turbines in Mexico are hardly implemented in comparison with horizontal axis, this is because they have lower aerodynamic efficiency and a higher drag coefficient, therefore, their area of study has also been reduced, despite everything, vertical axis designs have two great advantages: the first is that they do not have to be oriented in the direction of the wind and the second is that they can work in urban areas where air currents are turbulent. *Centro Nacional de Control de Energía [CENACE]. (2020) y el Instituto Tecnológico y de Estudios Superiores de Monterrey. (2020).* In this project, the design and construction of a Darrieus vertical axis turbine retrofitted through reverse engineering and the psychrometric chart at 2135 meters above sea level, the calculation sheets of the parameters of angular velocity, Tip Speed Ratio, and power capacity that the rotor can generate with an average wind speed of 8.03 m/s radially were developed. In addition to corroborating each of these coefficients calculated through a CFD analysis, using the mathematical model K-Epsilon and the Navier-Stokes equations. *Benemérita Universidad Autónoma de Puebla [BUAP]. (2021).*

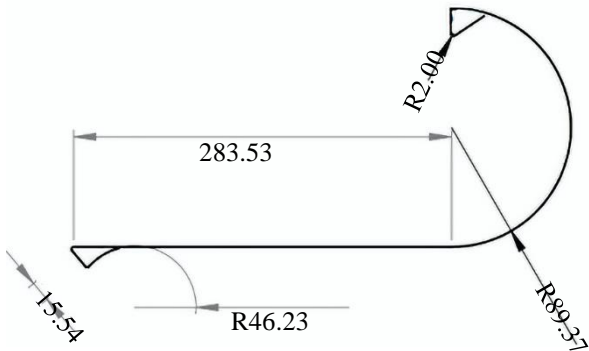
## Methodology

This research has a mixed approach, applying both quantitative and qualitative technologies, using systematic processes as well as recorded and estimated data. The objective of this project is to design and build a prototype of a Darrieus H-type vertical axis turbine, from a selection of recycled and low-cost materials, capable of operating under the climatic conditions of the State of Puebla, Mexico, and to obtain the wind rotor operation data using a reverse engineering process. To this end, the application of the quantitative method was relevant to obtain the parameters of angular velocity, TSR, and project power through the figures provided by an anemometer, the data from the psychrometric chart at 2135 masl, and the geometric characteristics of the turbine, such as its outer radius (0.495 m) and the contact area (0.07081 m<sup>2</sup>). The application of the qualitative method allowed the possibility of obtaining results from estimating the linear wind speed variable as a constant of the average air flow speed (8.03 m/s), in addition to a unit estimation of the drag coefficient; the operating data resulting from these estimates were an average angular velocity of 16.21632 rad/s and a projected power with the rotor assembled directly to the shaft of 22.45677 W. *Saavedra, A. et. al. (2019).* Finally, by the mixed method, a CFD analysis was performed to the CAD model of the turbine, through the ANSYS 2020 R2 Fluent simulator, where a control volume was implemented, in which a constant wind flow governed by the K-Epsilon model was made to observe how the fluid behaved with the wind rotor in operation. From the results obtained, a discussion of results was generated about the constructed design and its implications in the field of sustainable energy generation and sustainability.

## Elaboration of the design and construction of the Darrieus vertical axis turbine

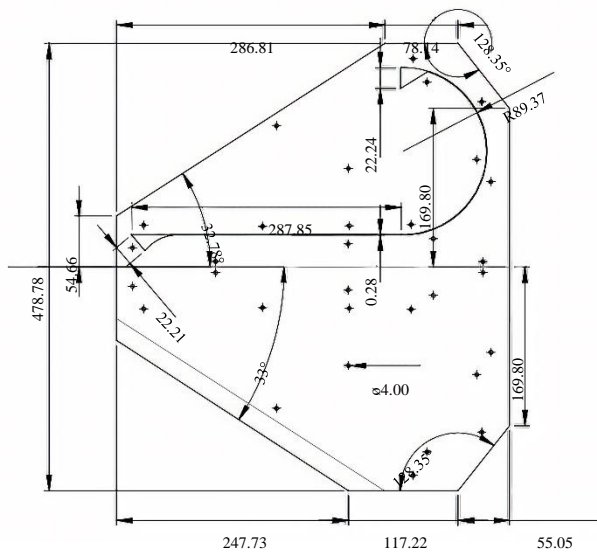
Generally, models of any turbomachinery are designed based on calculations developed concerning the physical conditions to which the system will be subjected. However, in this project; the calculations were approached through the reverse engineering process, so the wind turbine was first modeled based on previous designs.

A model of a Darrieus H-type vertical axis turbine with three blades was proposed, a schematic of the profile and blade dimensions is shown in Figure 1.



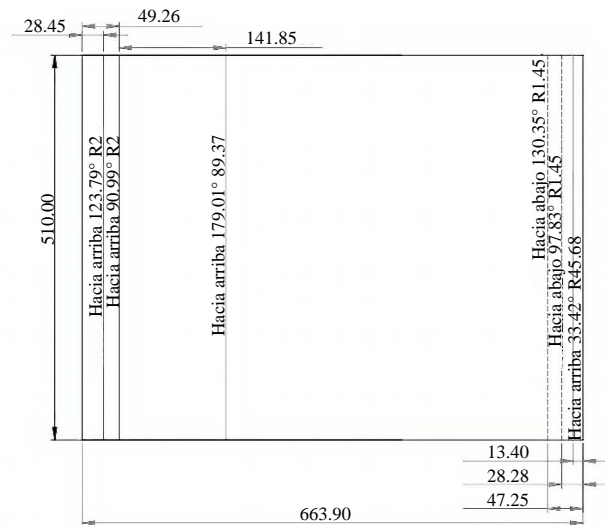
**Figure 1** Diagram of the profile and blade dimensions in millimeters  
 Source: Tribology and Transport Group. Faculty of Engineering. BUAP; Solidworks 2018

From the above schematic, in the freely licensed CAD tool Solidworks 2018, as shown in Figure 2, the base plans for the rotor blades were carried out.



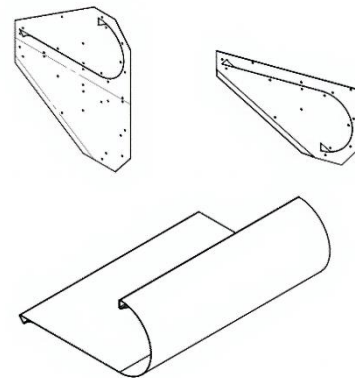
**Figure 2** Base plane in millimeters  
 Source: Tribology and Transport Group. Faculty of Engineering. BUAP; Solidworks 2018

Using a commercial sheet plate of 664 x 510 mm, the angles of curvature of the turbine blades were proposed, as shown in Figure 3.



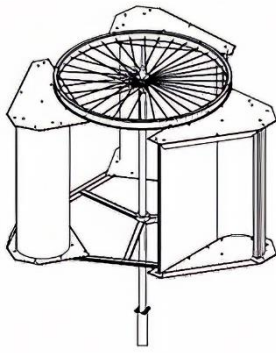
**Figure 3** Drawings for the manufacture of blade curvatures in millimeter  
 Source: Tribology and Transport Group. Faculty of Engineering. BUAP; Solidworks 2018

Following the geometry in the previous drawings, Figure 4 shows the three-dimensional model of the turbine elements, proposing a thickness of 26 mm.



**Figure 4** Three-dimensional models of the base and blade  
 Source: Tribology and Transport Group. Faculty of Engineering. BUAP; Solidworks 2018

As part of the material reuse plan, it was decided to use a bicycle rim with a 260 mm radius to allow the rotation of the turbine, so it was also modeled in the final design, as shown in Figure 5.



**Figure 5** Final modeling of the Darrieus vertical axis turbine Source: Tribology and transport group. BUAP; Solidworks 2018

Using the design drawings of the Darrieus vertical axis turbine as a reference, the vertical rotor blades and bases were fabricated from a 26-gauge stainless steel sheet. Figure 6 shows the wind rotor assembled according to the above specifications.



**Figure 6** Darrieus vertical shaft turbine Source: Tribology and Transport Group. Faculty of Engineering. BUAP

### Theoretical analysis of the turbine

To allow the study of the Darrieus turbine, the average wind speed of Puebla (8.03 m/s) was declared to be constant. The rest of the wind rotor parameters were obtained through its geometry, the psychrometric chart data, and the formulas described below. *Benemérita Universidad Autónoma de Puebla [BUAP]. (2020).*

### Identification of TSR rate

The Tip Speed Ratio or TSR was obtained through equation 1, which relates the linear speed of the turbine to the linear speed perpendicular to the wind flows.

$$\lambda = \frac{v}{V} = \frac{wr}{V} \quad (1)$$

Where:

- $\lambda$ : Tip Speed Ratio.
- $V$ : Linear wind speed (m/s).
- $v$ : Turbine linear speed (m/s).
- $w$ : Turbine angular velocity (rad/s).
- $r$ : Turbine radius (m).

In which, the linear velocity of the turbine will be described by equation 2.

$$v = wr = 2\pi fr \quad (2)$$

Where:

- $f$ : Frequency (Hz)

Substituting the data in equation 1.

$$\lambda = \frac{2\pi(60 \text{ Hz})(0.495 \text{ m})}{8.03 \text{ m/s}}$$

Therefore.

$$\lambda = 23.23917$$

However, given the design of the Darrieus vertical axis turbine generated and that it has three blades equidistant to its axis and the same contact area, the TSR coefficient is divided by three, as shown in equation 3.

$$\lambda_R = \frac{\lambda}{3} \quad (3)$$

Where:

- $\lambda_R$ : Real Tip Speed Ratio

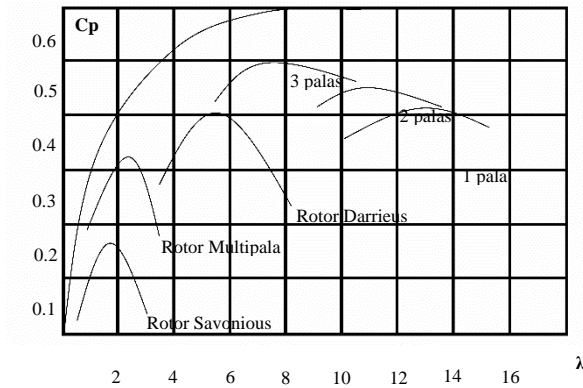
Substituting the data in equation 3.

$$\lambda_R = \frac{23.23917}{3} = 7.74639$$

### Power coefficient

The TSR allows us to measure the behavior of a body that rotates on its axis and is governed by aerodynamic drag and given the shape of the blades for wind capture are perpendicular to the wind flows and have a large contact area; a unit drag coefficient will be estimated.

Graphic 1 shows the power curves for the TSR of each of the designs. Therefore, we can estimate an approximate power coefficient of 0.35.



**Graphic 1** Tip Speed Ratio Curves concerning power coefficients of different wind turbine designs

Source: *Análisis matemático. Potencia Eólica. Hidrolate*; Recovered from <https://hidrolate.wordpress.com/tema/>. (2021)

### Force and power of the wind flow captured by the turbine

Based on the geometric characteristics of the wind rotor, the psychrometric chart of the State of Puebla (2135 masl) and the delimitation of the wind constant of 8.03 m/s, the equations of the force captured by the turbine and the power to be projected from it without mechanical transmissions were developed.

- Contact area =  $A = 0.07081 \text{ m}^2$
- Drag constant =  $C_A = 1$
- Wind speed =  $v = 8.03 \text{ m/s}$
- Air density =  $\rho = 1.225 \text{ kg/m}^3$
- Turbine radius =  $r = 0.495 \text{ m}$

The force exerted by the air flows exerted on the turbine blades is defined by equation 4.

$$F_w = C_A \frac{1}{2} \rho A v^2 \quad (4)$$

Substituting the data in equation 4.

$$= \left(\frac{1}{2}\right)(1.225 \text{ kg/m}^3)(0.07081 \text{ m}^2)(8.03 \text{ m/s})^2$$

Therefore

$$F_w = 2.79661 \text{ N}$$

The power to be harnessed by the wind speed is described in equation 5.

$$P_w = C_A \frac{1}{2} \rho A v^3 \quad (5)$$

Substituting the data in equation 5.

$$= \left(\frac{1}{2}\right)(1.2 \text{ kg/m}^3)(0.07081 \text{ m}^2)(8.03 \text{ m/s})^3$$

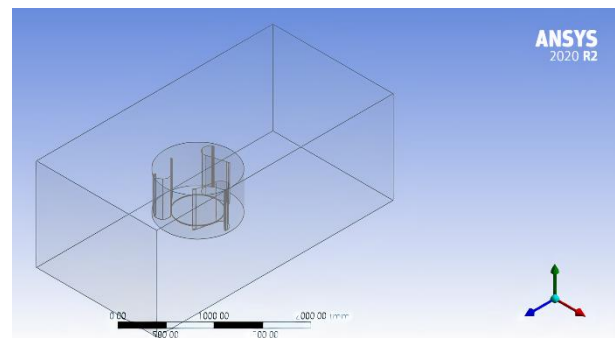
Therefore

$$P_w = 22.45677 \text{ W}$$

### CFD analysis of the Darrieus wind turbine

To observe the behavior of the wind flow while the wind turbine was rotating, the CAD model was imported into the ANSYS 2020 R2 CFD program.

As shown in Figure 7, a control volume was implemented to define the domain through which the wind flow would pass through the wind rotor. Likewise, a rotation domain was also declared to allow the model to rotate.



**Figure 7** Implementation of control and rotation domains  
Source: *Tribology and transport group. Faculty of Engineering. BUAP; ANSYS R2 2020. Student license*

The K-Epsilon viscosity model achievable with a near-wall treatment scalable to the wall was declared. In addition to programming the boundary parameters given in Table 1.

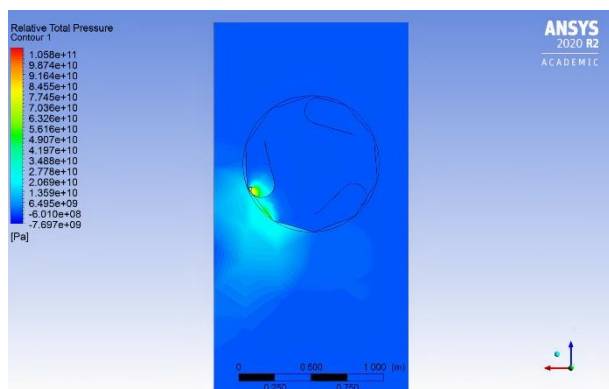
Parameters	Coefficients and Features
Fluid	Air Density = 1.225 kg/m <sup>3</sup> Viscosity = 1.784e-05 kg/m·s
Inlet	8.03 m/s to -Z
Relative pressure in control volume openings (except Inlet)	1.00863 atm
The direction of rotation of the rotation domain	Y = 1
The maximum speed of rotation of the rotation domain	16.21632 rad/s
Number of Prandt TDR	1.2
Number of Prandt TKE	1

**Table 1** Boundary parameters of CFD analysis

Source: Tribology and Transport Group. Faculty of Engineering. BUAP. (2021)

From these data, the simulation was developed in the CFX tool of ANSYS R2 2020, obtaining the results of pressure, the velocity of the air flows in the turbine.

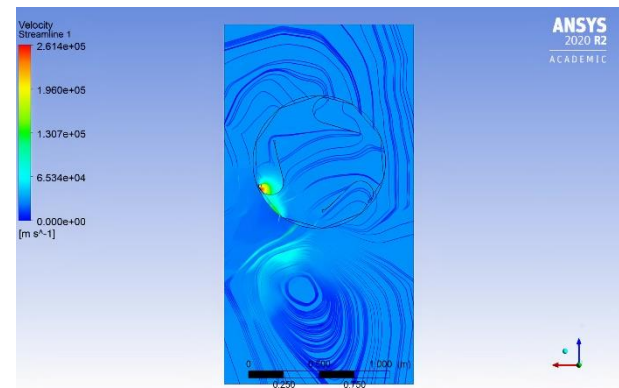
Figure 8 shows how the first air flow intake exerts a relative pressure of 63.2 MPA which starts to push the blade concerning the turbine shaft, making it rotate.



**Figure 8** CFD relative pressure results of airflow in the Darrieus vertical axis turbine

Source: Tribology and Transport Group. Faculty of Engineering. BUAP; ANSYS R2 2020. CFX. Student license

In Figure 9, it can be observed how the wind flows take helical shapes when colliding with the internal walls of the blades at a relative velocity, generating those turbulences that are released to the environment at approximately 90° after the wind capture.



**Figure 9** CFD linear velocity results of air flow in the Darrieus vertical axis turbine

Source: Tribology and Transport Group. Faculty of Engineering. BUAP. ANSYS R2 2020. Student license. (2021)

### Rectification of the linear velocity results using the Navier-Stokes equations and the Galerkin approximation

Since the ANSYS R2 2020 CFX software obtains the results using the finite volume discretization method, the linear velocity data will be corroborated through the Navier-Stokes equations without fear of a large lag in the CFD analysis solutions.

For the analytical resolution described in equation 6, the effects of viscosity, density, pressure, and temperature were neglected to achieve simpler governing equations, leaving the steady-state velocity as the unknown. Guanoluisa, E. (2011).

$$\vec{P} = [K]\vec{\phi} \quad (6)$$

Where:

- P: Elementary characteristic vectors.
- [K]: Elemental stiffness matrix.
- $\Phi$ : Node vectors for linear polynomial interpolation.

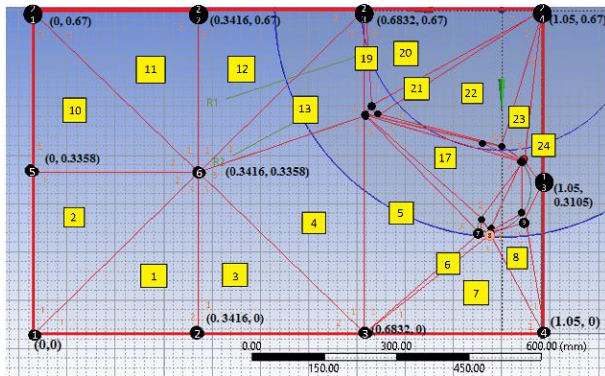
From Equation 6, we rename  $\Phi$  as the velocity of the air currents, as shown in Equation 7.

$$\vec{\phi} = \vec{V} \quad (7)$$

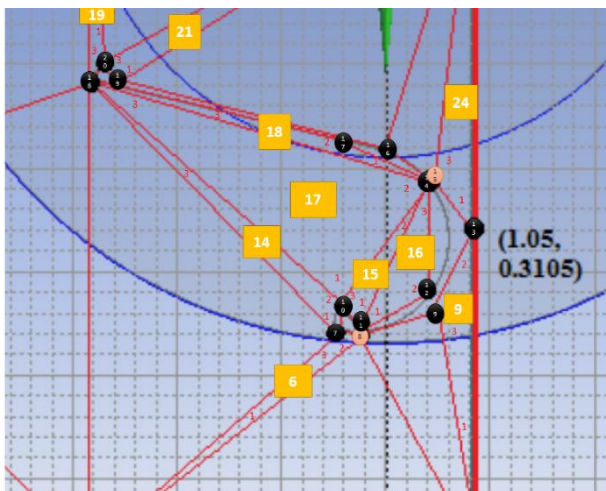
Therefore, by clearing V we obtain eq. 8.

$$\vec{V} = [K] \setminus \vec{P} \quad (8)$$

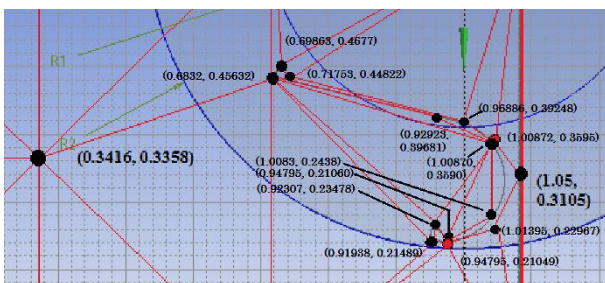
Using the Galerkin approximation, triangular elements were drawn to discretize a section of the top view of the control volume, as shown in Figures 10, 11, and 12. Guanoluisa, E. (2011).



**Figure 10** Analogous discretization with the numbering of elements and global and local nodes  
 Source: Tribology and transport group. Faculty of Engineering. BUAP. (2021)

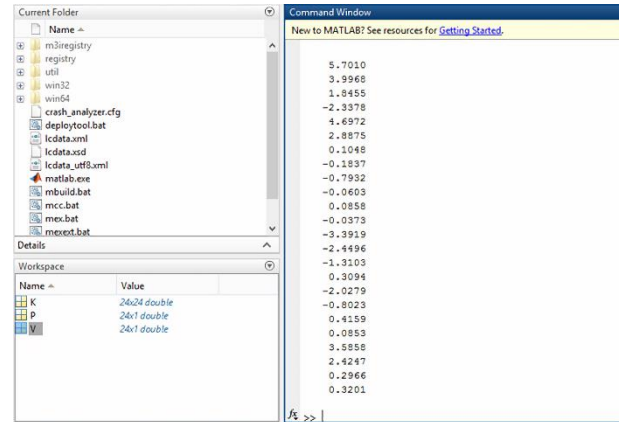


**Figure 11** Analogous discretization of blade details with the numbering of elements and global and local nodes  
 Source: Tribology and transport group. Faculty of Engineering. BUAP. (2021)



**Figure 12** Coordinate system of the analog discretization, warp details  
 Source: Tribology and transport group. Faculty of Engineering. BUAP. (2021)

Using the coordinate system and the numbering of the twenty-four global and local nodes, the matrices and vectors of equation 6 were developed, whose data were displayed in Matlab R2019a software to obtain the velocity vector described by equation 8, the solutions of the clearance are shown in figure 13.



**Figure 13** Vector V solution  
 Source: Tribology and transport group. Faculty of Engineering. BUAP; Matlab R2019a. (2021)

Since ANSYS indicates the direction of the results visually, the directions of the analog solution are defined by the sign corresponding to its result. So comparing; for example, element 16, which corresponds to the inner base of the blade, whose flow velocity is 1.3103 m/s according to the analog solution; is close to the 1.307 m/s of the ANSYS CFD solution, making the obtained results valid.

**Results**

Table 2 shows all the calculated data obtained from the turbine geometry and the corresponding psychrometric data.

Description	Literal	Calculated Value
Ratio Rate Tip Speed	TSR	7.74639
Power Coefficient	Cp	0.35
Calculated power	Pw	22.45677 W

**Table 2** Calculated operating values  
 Source: Tribology and Transport Group. Faculty of Engineering. BUAP. (2021)

Regarding the airflow analysis, it can be interpreted that the wind intercepted by the turbine blades spreads out around the contact area, encapsulating the air eddies and releasing the greatest amount of kinetic energy by turbulence when reaching 90° after the first capture.

When comparing the steady-state velocity solutions obtained by the Navier-Stokes equations and the Galerkin approximation with the results obtained by the CFD analysis performed in ANSYS; it can be observed that there is a relatively pronounced lag, however, given the number and shape of the nodes and elements, this can be attributed to the accuracy of the study.

### Discussion of results

The Darrieus vertical axis turbine design proposed in this research presents a medium-high tip speed ratio with good turbulence behavior of the air flows. Therefore, in terms of sustainability, it would be a great benefit for urban areas with high wind speeds, providing them with an affordable and environmentally friendly energy production alternative.

In the sustainable area, the implementation of this wind turbine design would begin to generate a culture committed to the integrity of its environment. It would also allow Mexican society to actively participate in international compliance with the Paris Agreement on climate change and the reduction of carbon emissions by 2050.

### Conclusions

In this study, a three-bladed Darrieus H-type vertical axis design with a large contact area, capable of operating optimally under the psychrometric conditions of the State of Puebla in Mexico, was proposed for future implementation in different urban areas. However, the results obtained show low aerodynamic performance and pose a great challenge to meet the daily demands of city residences. In addition, safety measures must be considered for the inherent risks of this type of turbine, such as, for example, the use of tensioning cables at the top of the prototype to ensure its structural stability in the event of strong winds or a study to ensure that the building in which the installation will be carried out will be able to withstand the extra load. Even so, the Darrieus rotor represents a great advantage in the production of electrical energy in enclosed cities, where wind flows collide with buildings, forming air eddies; and since these designs do not need to be directed into the air currents; they become the appropriate wind generation devices for these environments.

### References

- (2018). Programa de Desarrollo del Sistema Eléctrico Nacional 2018-2032. Recuperado de: <https://www.gob.mx/sener/acciones-y-programas/programa-de-desarrollo-del-sistema-electrico-nacional-33462>
- (2021). Análisis matemático, Potencia Eólica, Hidrolate. Recuperado de: <https://hidrolate.wordpress.com/tema/>.
- (2021). Estadística de la Energía Generada Liquidada Agregada (MWh) Intermitente y Firme por Tipo de Tecnología. Sistema Eléctrico Nacional. Días de Operación del Mes de: febrero 2021. Proceso de Liquidación: Original (L0). <https://www.cenace.gob.mx/SIM/VISTA/REP-ORTES/EnergiaGenLiqAgregada.aspx>
- (2021). Red Automática de Monitoreo Meteorológico 12: DIAU-BUAP. Benemérita Universidad Autónoma de Puebla. <http://urban.diau.buap.mx/estaciones/ramm/ramm.html>
- ANSYS. (2021). <https://www.ansys.com/>
- Barrera, J., Jurado, F., Razo, J., González, R. Aerogenerador de eje vertical para aplicaciones In-situ. Análisis y modelado de un aerogenerador de eje vertical. *Innovación y sustentabilidad Tecnológica. Instituto Tecnológico de Mianatla.*
- Batista, H. (2000). Las turbinas eólicas. Universidad Nacional de la Plata.
- Battaglia, L. (2009). Elementos finitos estabilizados para flujos con superficie libre: Seguimiento y captura de interface. Universidad Nacional de Litoral. 24-26.
- Canalejo, D. (2011). Generador eólico para uso doméstico. Escuela Universitaria Politécnica de Mataró. 5-7.
- Estrella, B. (2008). Prototipo Experimental Turbina Eólica de Eje Vertical. Tecnológico de Monterrey. 8-19.
- Guanoluisa, E. (2011). Aplicación del método de elementos finitos en la dinámica de fluidos. 38-50.
- GARCÍA-HERNÁNDEZ, Miguel Alejandro, CRUZ-GOMEZ, Marco Antonio, JUAREZ-ZERÓN, Tomás Aarón and SAAVEDRA-CRUZ, Nubia. Design and construction of a Darrieus vertical axis turbine and analyzed by reverse engineering. Journal Industrial Engineering. 2021



Gutiérrez, P. (2020). Sistemas de energía eólica y solar para la alimentación de luminarias. 44-47.

Huacuz, J. (2016). Tendencias en el Desarrollo de la Tecnología Eólica. *Premio Nacional de Tecnología e Innovación*.

Lucero, F. (2016). Diseño de un aerogenerador que apoye la demanda energética de un hogar promedio en Bogotá.

Matlab. (2021).  
<https://www.mathworks.com/products/matlab.html>

Saavedra, A., Alejos, R. (2019). Diseño de la geometría de un aerogenerador de eje vertical tipo Savonius. 2-3.

Sanchez, C. (1956). Cálculo aerodinámico de compresores y turbinas. Recuperado en: <http://oa.upm.es/6418/>

Sánchez, C. (2020). Simulación numérica de algunos modelos de turbulencia con aplicaciones a la aerodinámica de fluidos. Universidad Politécnica de Cartagena. 13-34.

Solidworks. (2021).  
<https://www.solidworks.com/es>

Steele, J. (1977). Finite Element Analysis and Program for Stokes Fluid Flow. Instituto Rochester de Tecnología. Recuperado en: [https://scholarworks.rit.edu/theses/?utm\\_source=scholarworks.rit.edu%2Ftheses%2F7384&utm\\_medium=PDF&utm\\_campaign=PDFCoverPages](https://scholarworks.rit.edu/theses/?utm_source=scholarworks.rit.edu%2Ftheses%2F7384&utm_medium=PDF&utm_campaign=PDFCoverPages)

Torres, A. (2015). Simulación y análisis de interacción fluido-estructura en alabe de turbina de motor a reacción mediante método por elemento finito. Instituto Politécnico Nacional.

Tseng, Y., Kuo, C. (2011). Engineering and construction torsional responses of glassfiber/Epoxy composite blade shaft for a small wind turbine. *Procedia Engineering 14*.

Vázquez, M. (2007). Diseño de una turbina eólica de eje vertical con rotor mixto.

Villanueva, J., Álvarez, R. (2007). Modelado del coeficiente de potencia de un aerogenerador por efecto de fricción. Universidad Nacional Autónoma de México.

Weht G., Giovachinni, J., Sacco C., D'Errico. Mario. (2011). Métodos de elementos finitos aplicado a flujo compresible con gas en equilibrio. *Mecánica Computacional*, Volumen 30. 547-562.