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The MTSK and TSD: didactic planning as a proposal for the learning of rational numbers in fifth grade primary school pupils

El MTSK y la TSD: planificación didáctica como propuesta para el aprendizaje de los números racionales en alumnos de quinto grado de educación primaria

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Abstract

This research shows how, based on an analysis proposal based on the MTSK (Mathematics Teacher Specialized Knowledge) (Carrillo, Montes, Contreras, & Climent, 2017), related to the use of the TSD didactic device (Theory of the didactic situations), it was possible to favor the learning of the content of rational numbers, specifically from the part-whole relationship sub construct and the understanding of the same in students of the 5th grade of a primary school. The results show a greater mastery of the content in question and with it, the students' learning advanced towards different levels of complexity through the established representations and relationships.

Mathematics, Didactics, Specialized

Resumen

La presente investigación muestra cómo, a partir de una propuesta de análisis basada en el MTSK (Conocimiento especializado del profesor de matemáticas) (Carrillo, Montes, Contreras, & Climent, 2017), relacionado con el uso del dispositivo didáctico de la TSD (Teoría de las situaciones didácticas), se logró favorecer el aprendizaje del contenido de los números racionales, específicamente desde el sub constructo relación parte-todo y la comprensión de los mismos en alumnos del 5º de una escuela primaria. Los resultados muestran un mayor dominio del contenido en cuestión y con ello, el aprendizaje de los alumnos avanzó hacia niveles de complejidad distintos a través de las representaciones y relaciones establecidas.

Matemáticas, Didáctica, Especializado

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Introduction

Among the specific objectives that have been proposed in basic education is that students learn to analyze, reflect and solve mathematical problems through conventional algorithms so that they can communicate and interpret different numbers and apply them in everyday life. However, recent results and evaluations (OECD, 2017), show that mathematics present problems in its teaching and learning, particularly, in one of its contents, which are assumed in such evaluations to be of greater complexity: "fractions".

This paper offers an analysis of the way in which teachers perceive fractions and the way in which they work with them, in order to determine to what degree their mastery of the content influences their students' learning.

In this sense, the need arises to understand what the student must learn and for this, the strategies applied for teaching must be considered, so that the student can access the creation of a concept rather than the mastery of a procedure. From this research, it is intended to generate or complement strategies that imply greater results in which school distances, referred to inequalities among students, are less and less, allowing to reach favorable comprehension levels that avoid failure or school lag.

Likewise, it allows building a broader notion about what fractions imply both in the concept and in the procedures for the resolution of problematic situations, in such a way that they lead students to evolve in their conceptualizations in order to advance in increasingly complex levels.

Theoretical framework

We start from the fact that fractions have multiple definitions and representations, but what determines their teaching is the way in which students "grant meaning and establish relationships" (Chamorro, 2003), so that students must adapt new representations and cognitive demands that will be used in different situations.

At this point, we consider one of the delimitations that will be addressed in this research, which is established as a measure and which the author Ma. Del Carmen Chamorro (2003) defines as "relationship between a part and a whole (whether continuous or discrete). The situations that configure this interpretation of the rational number imply situations of measurement and therefore consider a whole divided into parts. The rational number indicates the relationship between the part and the whole" (p. 192).

In Primary Education, it is essential to establish relationships between the first three types of knowledge, that is, "how the representations and language used help to give meaning to the symbols and their manipulation" (Chamorro, 2003, p. 200), in such a way that the students' construction process regarding the topic of fractions can be understood, with the intention that they develop or build representations.

In this same line, the author Juan D. Godino (2004) expresses that the teaching and learning of fractions in students represents a great change in their way of thinking, which brings with it different difficulties in the way they are accustomed to using numbers. Therefore, he mentions that students should understand the notion of fractions progressively in order to build and develop the different meanings derived from them.

Salvador Llinares (1997), states that the meaning of fractions has two different meanings: "on the one hand, it is presented to us as <<the division of a whole into its parts>>, or <<the parts of a whole>>. On the other hand, within the meanings of Arithmetic there are meanings such as <<broken number>>, <<expression that indicates a division that cannot be carried out>>" (Llinares & Sánchez, 1997, p. 18).

With this he mentions that regardless of the meaning that we give and that prevails in most people, since we are children we are in constant contact with this concept, since we use expressions in which these are immersed.

The understanding of the part-whole relationship opens the way and gives rise to different interpretations of fractions, which, with its use, generates the language and symbols that will form the basis for working with rational numbers. For this reason, it is expressed that the teacher must be careful and pay special attention when working with these contents.

In order to carry out the aforementioned activities, a device is required to help us carry them out in a progressive manner, in which an analysis of the student's knowledge can be performed. For this reason, the Theory of Didactic Situations formulated by (Brousseau, 2007) is implemented.

This model is a proposal that is seen as a production process in which relationships are established, transformed and reorganized.

The environment plays a very important role, since it includes the problems faced by students, with the intention of modifying relationships as they interact and produce knowledge, through the implementation of validation of procedures in problem solving.

Based on this, two types of situations are mentioned:

- Didactic situation: it is intentionally constructed so that the student acquires knowledge, but for this, there must be interactions between subject/medium and student/teacher.

- Adidactic situation: the student faces the problem without the intervention of the teacher. This leads the student to incorporate new ideas, which "give rise to the conscious rejection of erroneous decisions" (Sadovsky, 2005, p. 6).

In turn, Panizza (2003), describes the basic concepts of the Theory of Didactic Situations, which arises from the French School of Didactics of Mathematics. It mentions that:

> (...) The theory of situations appears then as a privileged means, not only to understand what teachers and students do, but also to produce problems or exercises adapted to the knowledge and to the students and finally to produce a means of communication between researchers and teachers. (Brosseau, 1999, cited in Panizza, 2003, p. 3).

Within these concepts, it is important to clarify that the intention that children learn certain mathematical knowledge does not disappear in the adidactic situation, but refers to the fact that "the student must relate to the problem, responding to it based on his knowledge, motivated by the problem and not to satisfy a desire of the teacher, and without his direct intervention in helping him to find the solution" (Panizza, 2003, p. 5).

However, not only the student's knowledge should be considered when developing activities, but also, it is important to take into account the teacher's knowledge "what he/she knows, how, what makes it possible, what he/she needs, which will allow designing training proposals (initial and continuous) consistent with such needs" (Climent, et al., 2014, p. 42).

From this, professional knowledge is seen as support for the teacher's own development, so an analysis of the practice in which it is contemplated must be carried out.

Taking the above aspects as a reference, the (MTSK; Carrillo, Montes, Contreras, & Climent, 2017) Mathematics Teacher Specialized Knowledge, for its acronym in English , which is observed its emergence from Shulman's contributions and its distinction of two components:

- Content knowledge of the subject to be taught (MK).

- Didactic knowledge of the content to be taught (PCK).

From which the MKT is configured, which considers "the teacher's knowledge for teaching mathematics closely linked to the mathematical content and is evidenced from teaching situations (hence its name)" (Climent, et al., 2014, p. 44).

The purpose of MTSK is "to understand what knowledge the teacher has/needs" (Climent, et al., 2014, p. 47) with the intention of transforming it in practice by making sense of teaching-learning situations:

Mathematics Teachers' Specialised Knowledge (MTSK) has a duality in that it is a theoretical proposal that models the core knowledge of the mathematics teacher's professional knowledge and is, in turn, a methodological tool that allows the analysis of different practices of the mathematics teacher through its categories (Climent, et al., 2014).

It is recognized then, that it arises as a response to the difficulties of the MKT, although it retains the two domains proposed by Shulman, and confers three subdomains to each one:

- Mathematical knowledge, which refers to the knowledge of the discipline taught, so it should be considered, to know what and how mathematics knows/should know. Within this, the following subdomains are established:

  – Knowledge of mathematical topics (KoT): the teacher must know the contents and meanings of the contents he/she teaches in a grounded manner.

  – Knowledge of mathematical structure (KSM): This refers to the knowledge of the relationships that the teacher makes between different contents, either of the course he/she is teaching or with contents of other courses or educational levels.

  – Knowledge of mathematical practice (KPM): Emphasizes the importance of the teacher not only knowing the established mathematical results, but also the ways of proceeding to reach them and the characteristics of mathematical work.

- Didactic knowledge of the content (PCK): It characterizes as a particular knowledge of the teacher, the teaching work itself. The following subdomains are distinguished:

  – Knowledge of learning characteristics (KFLM): encompasses knowledge about the learning characteristics inherent to the mathematical content.

  – Knowledge of mathematical teaching (KMT): knowledge of resources, materials, ways of presenting the content and the potential it may have for instruction, as well as knowledge of appropriate examples for each given content, intention or context.

  – Knowledge of mathematics learning standards (KMLS): knowledge that a teacher has about what a student is expected to learn and the conceptual level at which he or she is expected to learn it at a given time in school.

  – In general, these elements allow the teacher to focus and use them as a tool to have the knowledge that is useful to perform his or her work, so they are used for the analysis of this research and the following scheme is considered:

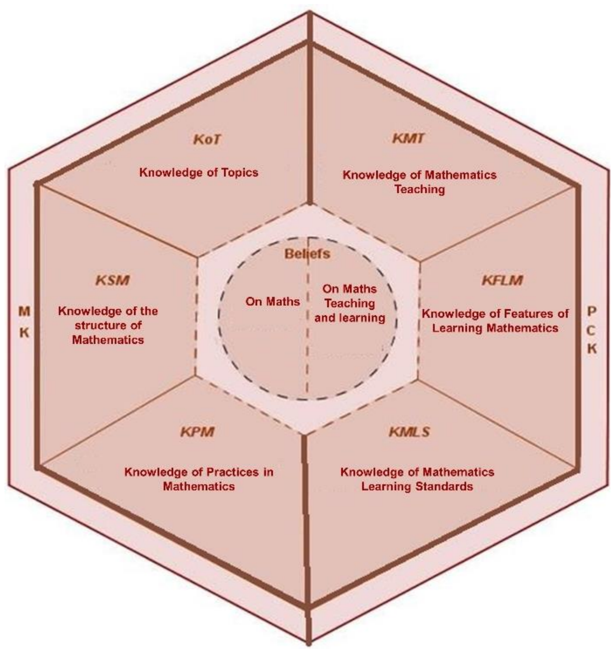

Figure 1 Diagram of the MTSK

Methodology

A qualitative approach was used since it allows the use of data collection involving an interpretative approach to the object of study.

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A didactic device is shown as a proposal that includes the theoretical proposals of the TSD and the MTSK, in addition to contemplating aspects of content mastery with respect to fractions.

The purpose is to visualize planning as an initial guideline for programming, analysis and reflection on professional practice, in this sense, there is an opportunity from the two theories addressed to deepen the knowledge that the teacher has about the content, and its didactification, in addition to translate it into a powerful teaching device for mathematics.

Results

The proposal that concentrates The MTSK, The TSD in the teaching of fractions in the mentioned school grade is shown below:

Subject: Mathematics	**Axis: Number sense and algebraic thinking**
Mathematical Content Knowledge (MK)	
Knowledge of Mathematical Topics **(KOT):** For the development of these activities it is important that students recognize the meaning of a fraction, which is understood as follows: Fraction: It expresses a numerical value that, unlike natural numbers, represents quantities of an object that has been divided into equivalent parts. In turn, it consists of two important elements for its understanding: Numerator: It is the number that indicates the parts we have or have taken and is written above the dash, for example: 2 Denominator: It is the number that indicates the parts into which the object has been divided and is written below the dash, for example: 6 It is necessary to consider that there are also different types of fractions: Proper fractions: Those in which the numerator is smaller than the denominator, for example: 1/3 Improper fractions: Fractions in which the numerator is larger than the denominator, for example: 4/3. Mixed fractions: Fractions in which an integer and a fraction are written together, for example: 1 ½.	
*Knowledge of the Structure of Mathematics (**KSM**): Knowledge package shown in APPENDIX 1.* In this scheme, the relationship of the different contents that are worked on in the topic of fractions are shown, which go from the commensuration of the unit with the context starting with the measurement that begins with lengths and later capacities, with the intention that the teacher introduces formally the fractions: ¼, ¾, 2/5, ½, etc. From this, equivalent parts are generated, to understand the part-whole relationship and the concept of unity that integrates the numerator and denominator. It continues with the formalization of fractions that are converted into integers, for example: 6/6 = 1, and mixed and improper fractions are integrated and their representation from the transformation by converting them.	
Knowledge of Mathematical Practice **(KPM):** This aspect considers the teaching and learning of the content, for which, it is important to take into account the curricular considerations that establish the following: - Students should use mental calculation, estimation of results or written operations with natural numbers, as well as addition and subtraction with fractional and decimal numbers to solve additive and multiplicative problems. It is suggested that students build knowledge and skills with meaning and significance, such as solving problems that involve the use of fractional numbers; also, a work environment that gives students, for example, the opportunity to learn to face different types of problems, to formulate arguments, to use different techniques depending on the problem to be solved, and to use mathematical language to communicate or interpret ideas.	
Didactic Content Knowledge (PCK)	
*Knowledge of learning standards **(KMLS):***	
Mathematics Standards: Students know how to communicate and interpret quantities with natural, fractional, or decimal numbers, as well as solve additive and multiplicative problems using conventional algorithms: 1.1.1.1. Reads, writes, and compares natural, fractional, and decimal numbers. 1.2.1. Solves additive problems with fractional or decimal numbers, using conventional algorithms.	
Expected Learning: Knowledge of different representations of a fractional number: with numbers, with surfaces, etc. Analysis of the relationship between the fraction and the whole.	
Mathematical Competence: - Solve problems autonomously - Communicate mathematical information - Validate procedures and results	

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- Manage techniques efficiently

Topic: "Let's learn fractions with different surfaces".

Content: Representation of fractions on different surfaces.

Knowledge of mathematics teaching **(KMT):**

DIDACTIC MATERIAL:
Different surfaces (triangles, circles, squares, rectangles).

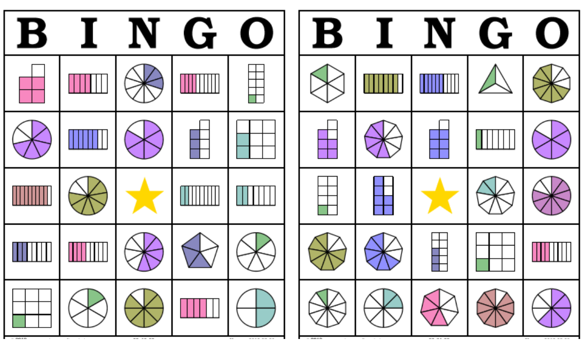
A priori analysis:

The group of 5 "A," has shown in a diagnosis and different exercises that they have difficulties in recognizing the notion of a fraction and the elements of which it is composed, referring in this case to the numerator and denominator, since they have repeatedly confused these terms when writing the representation of a rational number.

Likewise, most of the students have shown that they have complications in making graphical representations because they get confused when establishing relationships between the part-whole notion, mainly in what refers to improper and mixed fractions, because they do not identify the number of elements or figures that must be used to make the representation or conversion to whole numbers.

Preparation of the medium:

Before starting with the session, an activity consisting of a fraction bingo as follows will be performed:



For this activity we will use different cards like the previous ones, with the intention of recognizing the students' previous knowledge (how they represent a rational number when they hear it, how they identify the numerator and denominator and how they identify proper, improper and mixed fractions).

Group organization: The bingo activity will also bring students together to work in pairs.

CONSIGNA

- I am going to give you a sheet of paper that contains some figures, these are divided and shaded in different parts (the figures will be pointed out). In the two little squares in front of each figure you have to write the fraction that you think is represented.

- Then they will have to take out the figures that they were given as homework (circle, triangle, square rectangle) and represent the fractions that I am going to write on the blackboard, here they must consider that each one will choose the figure they want and the divisions they make must be exactly the same.

- RESTRICTIONS: Students will not be able to ask for help from other pairs, since it is a pair activity.

RETURN OF THE QUESTIONNAIRE: Students will be asked randomly:

What does the activity consist of?

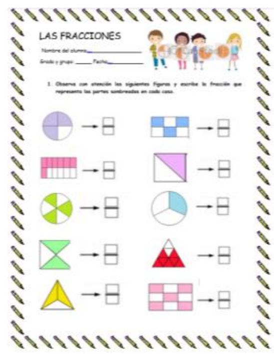
How will we organize ourselves?

DEVELOPMENT OF THE CLASS:

PHASES

ACTION:

-The following worksheet will be given to students to identify the fractions that are represented:



- Next, students will be asked to represent the following fractions, considering that the partitions they make must be exactly EQUAL IN THE FIGURES THEY CHOOSE TO USE FOR THEIR REPRESENTATION:

$$\frac{1}{4} \quad \frac{4}{8} \quad \frac{1}{2} \quad \frac{3}{6}$$

Once this is done, they will be instructed to cut out each of the partitions made in the figure to do the following:

- The children will be instructed to put together different parts of a figure to observe how another fraction is obtained, e.g.:
- Take ¼ plus ¼ of the same figure and realize that they form ½.
- Take ½ plus ½ of the same figure to realize that it forms 1 whole.
- Put together 1/8 plus 3/8 of the same figure to realize that you get 4/8 or else ½.
- Put 1/6 + 1/6 + 1/6 together and realize that you get 3/6 or ½.
- Put 6/6 together and realize that you get 1 whole.
- Put 4/4 together and realize that it forms a whole.

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- Once the activity is finished, the students will be given figures similar to the ones they already had so that they can divide them in the same way as the previous ones, with the intention of forming improper and mixed fractions (here it is that the students manage to represent the indicated fraction and transform it into a mixed fraction), for example:

- First figure that is divided into 8 parts plus the 8 of the second one that has been divided in a similar way gives us a total of 16, of which we take 11 and it is represented as 11/8 or else 1 integer and 3/8.

- From the previous figure, we take 12/8 which can also be represented as 1 integer and ½.

- From the previous figure we take 16/8 which can also represent 2 integers.

- A figure that is divided into 6 parts plus the 6 parts of the second one that has been similarly divided gives us a total of 12, from which we take 10/6 which also represent 1 integer and 4/6.

- From the previous figure we take 8/6 which also represent 1 integer and 2/6.

- A figure that is divided in 2 parts plus the 2 of the second one that has been divided in a similar way gives us a total of 4, from which we take 4/2 that also represents 2 integers.

- A figure that is divided into 4 parts plus the 4 of the second one that has been similarly divided gives us a total of 8, of which we take that, also representing 8/4 which also represents 2 integers.

FORMULATION: Within each team (bina) they will analyze the figures presented in the activity to write the fraction that is represented in each case.

In the same way, they will analyze and reflect on the figure that is easier for them to represent the previous fractions.

VALIDATION: Each group will share with the rest of their classmates the figures they used to represent the fraction, the way they divided each figure and why they did it that way, for which different questions will be asked:

Which fraction did they consider the easiest to represent?

Which figure did they use?

Why did they use that figure and not another one?

Which fraction was the most difficult to represent? Why?

Which figure did they use to represent the most difficult fraction?

Which of the figures was the most difficult to divide into equal parts?

How can they form a whole with the shapes they have (students will have to choose one of the shapes to give an example)?

How do you get a half (students will use the different shapes to form a half)?

How can they represent 1 whole 3/8?

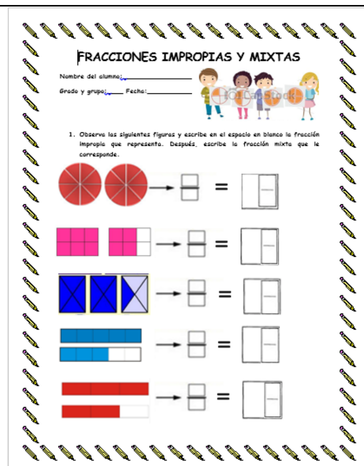
Using one of the figures, how can they obtain 2 integers?

How can they get 1 integer a half (use a figure to explain?

INSTITUTIONALIZATION:

Once the previous activities are finished, the elements of a fraction will be explained, which in this case are the numerator and denominator, and which are important for students to be able to represent a fraction, as well as proper fractions.

We will also explain improper and mixed fractions and how to obtain a mixed fraction from an improper one, taking some examples with the figures formed by the students, as part of this, a worksheet will be integrated:



Knowledge of the characteristics of learning mathematics (KFLM):

- Among the most common difficulties that can be found in students when developing the activities, are the following:

- Students do not identify the difference between numerator and denominator and it is noticeable when writing the representation of fractions.

- Students perform the divisions of a given fraction but these are not equal.

- Students have trouble identifying how a fraction is obtained from joining different parts of a division, i.e., establishing part-whole relationships.

- Students have difficulty understanding improper fractions and it is noticeable because they cannot represent them with the given figures.

Students have trouble transforming an improper fraction to a mixed fraction because they do not establish relationships with the given figures.

Chamorro (2003) emphasizes that the representations of fractions allow the student to give them meaning and establish relationships to understand this notion, thus making it easier for him to find meaning in solving problems where he feels motivated to apply different strategies. Based on this, Godino (2004), Llinares & Sánchez (1997) establish different proposals for teaching:

- Chamorro, embraces a recursive theory in which seven aspects are considered: inventing, structuring, formalizing, observing properties, having images, constructing images and doing.

- Godino, proposes three types of models or representations: areas (in which rectangular or circular figures are used), by means of sets (representation with objects), and linear (numerical line).

- Llinares and Sánchez, develop a sequence in which 6 topics are broken down into different points: unit (identifying the number of units, quantities larger or smaller than the unit), parts of a unit using concrete materials (number of parts of a unit, parts of the same size, dividing a unit into equal parts), oral names by part of the unit (establishing the name of fractions, using fractions to answer "how many", identifying fractions equal to one, identifying fractions equal to one), identifying fractions equal to one, identifying fractions equal to two, identifying fractions equal to one, identifying fractions equal to one, and so on, identifying fractions equal to one), writing fractions to represent parts of the unit -translations between representations- (from oral to written form or vice versa, from concrete to written form or vice versa), representing fractions with drawings (transition from objects to diagrams, repetition of previous steps) and extending the notion of fractions (fractions greater than one, mixed fractions, use of sets, comparison of fractions).

Despite the fact that each one proposes different strategies, it is possible to determine that in all cases learning is carried out in a progressive manner, with which the pupil acquires a structured knowledge that goes from the informal to the formal, in such a way that the children learn to interpret, to construct concepts with which they can solve problems.

Likewise, Godino, Llinares & Sánchez coincide in the use and implementation of geometric figures such as the circle, rectangle and square, because they are easy to divide according to the denominator and they are also the ones with which the students are most familiar, however, within this, there is a difference, since the second authors propose the establishment of other figures such as the rhombus and the pentagon to expand the notion of rational numbers.

On the other hand, the Theory of Didactic Situations is presented, approached by Panizza (2003) and Sadovsky, who approach it from different phases (action, formulation, validation and institutionalisation) to understand the way in which the student acts when in contact with the environment (problem situation), this allows to interpret the work that he/she executes, since not only his/her results are considered, but it is contemplated through a process.

However, it is important to recognise that in the teaching-learning process, the student does not act alone, but requires a mediator of the knowledge he/she is expected to acquire, that is to say, he/she requires the support of the teacher (not in the erroneous sense of being a mere transmitter of knowledge), as it is he/she who develops situations in accordance with the needs of the classroom.

This is why we include a methodological proposal by the authors Medrano, Ávila, Montes, Aguilar, & Carrillo (2013), called MTSK (Mathematics Teachers' Specialised Knowledge), which includes the analysis of the content domain that the teacher requires/needs to put into practice, this through two domains and their three corresponding sub-domains, as it is from this that he/she will design the strategies that he/she considers relevant to carry out with the students.

This does not mean that both aspects are seen separately, but on the contrary, it is about establishing a relationship between the student and the teacher through the analysis of practice, in which the perspectives mentioned above are included. This is to determine the extent to which the teacher's conception of mathematical content influences the child's learning.

In this way, the teacher's actions and the process carried out by the pupil in the acquisition of knowledge are reflected upon, allowing the teaching-learning process and the development of mathematical competences to be optimised and favoured.

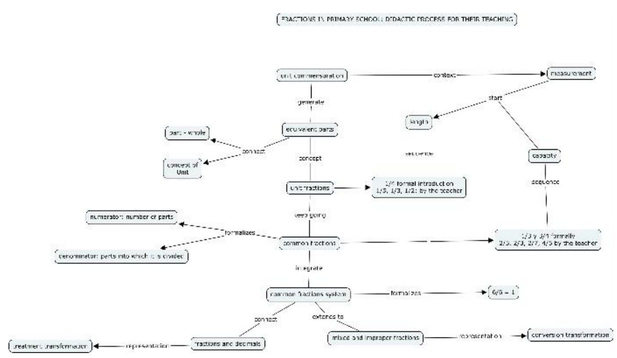
Conclusions

In the process of didactic planning, the theory of didactic situations, considered as a theory that models the processes derived from the interaction between the elements of the didactic triangle: teacher, student, knowledge, takes on special relevance within the KMT (Knowledge of mathematics teaching) as part of the teacher's specialized knowledge, given that it is a specific theory on the teaching of mathematics; although the MTSK model does not take sides with any particular didactic theory, in this paper we highlight the importance that TSD can have as part of the teacher's specialized knowledge and the opportunities it provides when guiding the didactic planning process in a complex content for its teaching: fractions.

As we exemplified in the previous section, the link between the MTSK and the TSD is achieved when both become, in a complementary manner, the articulating guides that make possible the elaboration of assertive didactic planning for the construction of mathematical knowledge in elementary school students.

In particular, it is important to highlight that, when planning, the teacher must have a deep knowledge of the subject (KoT); adequately place the content within the curricular progression (KMLS); know the characteristics of the students' learning of the content (KFLM), which allows him/her to design the tasks he/she will face; recognize prior knowledge and the connections of the content to be planned with other content of lesser and greater complexity (KSM); specify the meaning of doing mathematics and the way it will provoke validation on the part of the children (KPM); and, of course, the knowledge of theories on teaching makes it possible to make decisions regarding the structure of the planning design, but above all the global approach to work with the content (KMT), therefore, in the previous section we can note that priority is given to an action phase (active role of the learner), then the formulation and validation (to argue and validate what was built in the action phase) and closes with institutionalization, that is, giving the status of formal knowledge to mathematical knowledge.

Annexes



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