# State estimation of discrete event systems using fuzzy timed petri nets

## Estimación del estado de sistemas de eventos discretos utilizando redes de Petri temporizadas difusas

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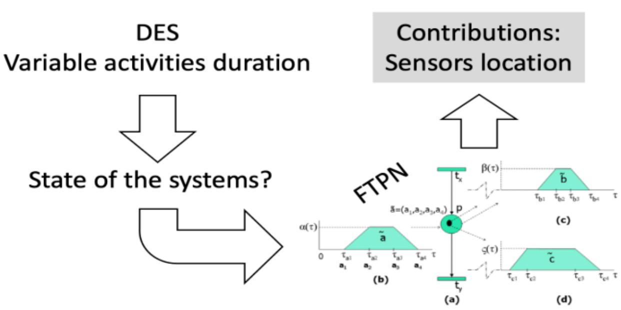
### Abstract

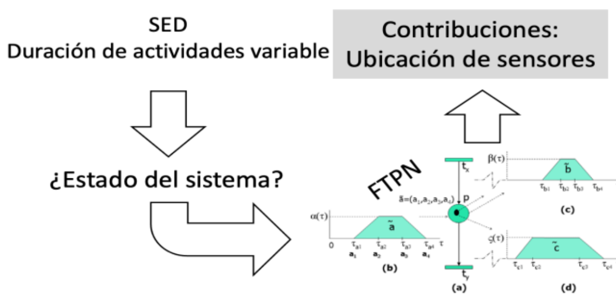
In this work, an extension is made to the inference of the state of discrete event systems using knowledge about the duration of activities. The mentioned systems are characterized by the absence of sensors and where the activities have a variable duration. This causes the uncertainty about the state of the system to increase. The problem is addressed by calculating the fuzzy marking of the *Fuzzy timed Petri Net* (FTPN). In this formalism, fuzzy sets are associated with the places, which have information about the variation of the end of the activities. To keep this uncertainty limited, the location of a minimum set of sensors is studied and rules are established for their placement in order to keep limited the uncertainty in the approximation of the marking in Marked Graph structures and State Machines. In this situation, the estimation device obtains and updates a belief that approximates the condition of the system.




**State estimation, Timed fuzzy Petri nets, Fuzzy marking equation**

### Resumen

En este trabajo se realiza una extensión a la inferencia del estado de sistemas de eventos discretos utilizando conocimiento sobre la duración de las actividades. Los sistemas mencionados se caracterizan por la ausencia de sensores y donde las actividades tienen una duración variable. Esto provoca que la incertidumbre sobre el estado del sistema aumente. El problema se aborda calculando el marcaje difuso de la *Red de Petri difusa temporalizada* (FTPN). En este formalismo, se asocian conjuntos difusos a los lugares, los cuales tienen información sobre la variación del final de las actividades. Para mantener esta incertidumbre limitada, se estudia la ubicación de un conjunto mínimo de sensores y se establecen reglas para su colocación con el fin de mantener limitada la incertidumbre en la aproximación del marcaje en estructuras de Grafos Marcados y Máquinas de Estados. En esta situación, el dispositivo de estimación obtiene y actualiza una creencia que se aproxima a la condición del sistema.




**Estimación de estado, Redes de Petri difusas temporizadas, Ecuación de marcado difuso**

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## I. Introduction

State estimation of a dynamic system is useful when only a subset of the state variables can be directly measured. Observers are the entities providing the system state from the knowledge of the internal structure of the system and its partially measured behavior.

The state estimation problem of *discrete event systems* (DES) has been addressed using a sensor-based approach (Köhler & Zhang, 2022), (Ramírez Treviño, Rivera Rangel, & Lómez Mellado, 2003), (Anguiano-Gijón, Chávez, Cid-Gaona, & Vázquez, 2024) in which the marking of a *Petri net* (PN) model describing a DES is progressively computed from the evolution of its inputs and outputs, (H., Y., K, H, & S, 2024). Also, in (Aguayo-Lara, Gómez-Gutiérrez, Ramírez-Treviño, & Ruiz-León, 2015), (Valencia, Muñoz, & Enríquez, 2020) the optimal sensor placement is studied. In these works, the actual marking is computed after a finite number of event occurrences.

The state of a system can also be inferred from the knowledge of the duration of activities (X, C. N , & Z, 2024). However, this task becomes complex when the duration of operations is variable in addition to the absence of sensors.

In this situation the observer obtains and revises a belief that approximates the current system state. Consequently, this approach of state monitoring is useful for non-critical applications in which an approximate computation of the state is sufficient.

The uncertainty of activities duration in DES can be handled using or Stochastic Petri Nets (Yang, Duan, Lin, & Chen, 2024) or *fuzzy PN* (FPN), (Kuchárik & Balogh, 2019), (Madhloom, Noori, Ebis, Hassen, & Darwish, 2023), (Xu X.-G. , Shi, Xu, & Liu, 2019), (Deabes, Bouazza, & Algthami, 2023); this PN extension has been applied to knowledge modeling, (Xu X.-G. , Shi, Xu, & Liu, 2019), (Hua & Hu-Chen, 2023), (Hu-Chen, Jian-Xin , Zhiwu, & Guangdong , 2017), planning, (Mahulea, González, Montijano, & Silva, 2021), (X. G. & H. S. , 2009), reasoning, (Yu, Gong, Liu, & Mou, 2022), (Lee, Liu, & Chiang, 2003) and controller design (L. Saleh, J. Mohammed, Sabri Kadhim, M. Raadthy, & J. Mohammed, 2018),(Deabes, Bouazza, & Algthami, 2023).

In these works, the proposed techniques include the computation of imprecise markings.

This paper addresses the problem of state estimating of cyclic DES that exhibit variations on the duration of activities, by approximating the marking of a FTPN model.

A definition of FTPN is presented in which the ending time uncertainty of activities is expressed with fuzzy sets associated with places. Previous results presented in (González-Castolo & López-Mellado, 2006), (González-Castolo & López-Mellado, 2007), (González-Castolo & López-Mellado, 2011) are reviewed and extended with a characterization of the marking estimation degradation and a technique for obtaining discrete marking from approximate marking.

The remainder of this paper is structured as follows. In the next section, theories of fuzzy sets and PN are reviewed. In section III the definition of FTPN is introduced. Section IV analyses the location of measurable locations in the FTPN. Finally, the conclusion is shown.

## II. Background

### *A. Possibility Theory*

In possibility theory, a fuzzy set $\tilde{a}$ is used to delimit ill- known values or to represent values characterized by linguistic variable expressions.

The fuzzy set $\tilde{a}$ in $\tau$ is characterized by a membership function $\alpha_{\tilde{a}}(\tau)$ ,(Jianfeng & Genserik , 2024) which associates to each point $\tau$ in $\boldsymbol{\tau}$ a real number in the interval $[0,1]$; the value $\alpha_{\tilde{a}}(\tau)$ represents the "grade of membership" of $\tau$ in $\tilde{a}$, (Zadeh & Goguen, 1965) .

Usually, a fuzzy set is defined in a trapezoidal form; thus, a fuzzy set $\tilde{a}$ can be characterized as $\tilde{a} = (a_1, a_2, a_3, a_4)$ such that $a_1, a_2, a_3, a_4 \in \mathbb{R}$, where $(a_2, a_3)$ and $(a_1, a_4)$ are the core and support of $\tilde{a}$, respectively. $\tilde{a}^{\uparrow} = (a_1, a_2)$ denotes a subset of $\tilde{a}$ where the values $\alpha_{\tilde{a}}(\tau)$ grow towards 1; similarly, $\tilde{a}^{\downarrow} = (a_3, a_4) \subseteq \tilde{a}$ denotes the decreasing values of $\alpha_{\tilde{a}}(\tau)$. Figure 1(a) illustrates these notions.

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*Definition 1*: Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy sets. The fuzzy sets addition operation is $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$,(Klir & Yuan, 1995).

*Definition 2:* The intersection and union of fuzzy sets are defined in terms of *min* and *max*. $(\tilde{a} \cap \tilde{b}) = min(\tilde{a}, \tilde{b}) = min\left(\alpha_{\tilde{a}}(\tau), \alpha_{\tilde{b}}(\tau)\right)$ such that $\tau \in$ support of $\tilde{a} \wedge \tilde{b}$ and $(\tilde{a} \cup \tilde{b}) = max(\tilde{a}, \tilde{b}) = max\left(\alpha_{\tilde{a}}(\tau), \alpha_{\tilde{b}}(\tau)\right)$ such that $\tau$ belongs to the support of $\tilde{a} \vee \tilde{b}$, where the *min (max)* operator gets the minimum (maximum) $\tau$ of $\boldsymbol{\tau}$. It uses these intersection and union operators as a *t-norm* and a *s-norm*, respectively. The null element for *min (max))* operation is $1(0)$.

*Definition 3*: Let $\tilde{a}$ and $\tilde{b}$ two fuzzy sets such that $\propto_{\tilde{a}}(\tau) + \propto_{\tilde{b}}(\tau) \leq 1$. The *sum* operation between $\tilde{a}$ and $\tilde{b}$ is computed as:

$$sum(\tilde{a}, \tilde{b}) = \alpha_{\tilde{a}}(\tau) + \alpha_{\tilde{b}}(\tau) \qquad (1)$$

*Definition 4:* Let $\tilde{a}$ a fuzzy set and $w$ a real number. The product between $\tilde{a}$ and $w$ is defined as:

$$prod(\tilde{a}, w) = w \cdot \alpha_{\tilde{a}}(\tau) \qquad (2)$$

such that $w \cdot \alpha_{\tilde{a}}(\tau) \leq 1; \tau \in \boldsymbol{\tau}$.

*Definition 5:* The distribution of possibility before and after $\tilde{a}$ are the fuzzy sets $\tilde{a}^{<} = (-\infty, a_2, a_3, a_4)$ and $\tilde{a}^{>} = (a_1, a_2, a_3, +\infty)$, respectively. They are defined in (Andreu , Pascal , & Valette , 1997) as a function $\alpha_{(-\infty,\tilde{a}]}(\tau) = \sup \alpha(\acute{\tau})$ and $\alpha_{(\tilde{a},+\infty]}(\tau) = \sup_{\acute{\tau} \leq \tau} \alpha(\acute{\tau})$, respectively, (Figure 1(c), (d)).

*B. Petri Nets*

*Definition 6:* An ordinary PN *structure G* is a bipartite digraph represented by the 4-tuple *G (P, T, I, O)* where $P = \{p_1, p_2, \ldots, p_n\}$ and $T = \{t_1, t_2, \ldots, t_m\}$ are finite sets of vertices called respectively places and transitions, $I(O): P \times T \rightarrow \{0,1\}$ is a function that represents the arcs from places to transitions (transitions to places).

Pictorially, places are represented by circles, transitions are represented by rectangles, and arcs are depicted as arrows.

The symbol $\bullet t_j$ ($t_j\bullet$) denotes the set of all places $p_i$ such that $I(p_i, t_j) \neq 0$ $\left(O(p_i, t_j) \neq 0\right)$. Analogously, $\bullet p_i$ ($p_i\bullet$) denotes the set of all transitions $t_j$ such that $O(p_i, t_j) \neq 0$ $\left(I(p_i, t_j) \neq 0\right)$.

The pre-incidence matrix of $G$ is $C^- = [c_{ij}^-] = I(p_i, t_j)$; the post-incidence matrix of G is $C^+ = [c_{ij}^+]$ where $c_{ij}^+ = O(p_i, t_j)$ ; the incidence matrix of $G$ is $C = C^+ + C^-$.

A marking function $M: P \rightarrow \mathbb{Z}^+$ represents the number of tokens (depicted as dots) residing inside each place. The marking of a PN is usually expressed as an n-entry vector.

*Definition 7:* A *Petri Net system* or *Petri Net* (PN) is the pair $N = (G, M_0)$, where $G$ is a PN structure and $M_0$ is an initial token distribution.

In a PN, a transition $t_j$ is enabled at the marking $M_k$ if $\forall p_i \in P, M_k(p_i) \geq I(p_i, t_j)$; an enabled transition $t_j$ can be fired reaching a new marking $M_{k+1}$ which can be computed using the PN state equation:

$$M_{k+1} = M_k + C^+ v_k - C^- v_k \qquad (3)$$

where $v_k(i) = 0, i \neq j, v_k(j) = 1$.

The *reachability set* of a PN is the set of all possible markings reachable from $M_0$, firing only enabled transitions; this set is denoted by $R(G, M_0)$.

A *structural conflict* is a PN substructure in which two or more transitions share one or more input places; such transitions are simultaneously enabled and firing one of them disables the others.

*Definition 8:* A transition $t_k \in T$ is *live*, for a marking $M_0$, if $\forall M_k \in R(G, M_0), \exists M_n \in R(G, M_0)$ such that $t_k$ is enabled $\left(M_n \xrightarrow{t_k}\right)$. A PN is *live* if all its transitions are live.

*Definition 9:* A PN is said to be *1-bounded*, or *safe*, for a marking $M_0$, if $\forall p_i \in P$ and $\forall M_j \in R(G, M_0)$, it holds that $M_j(p_i) \leq 1$.
In this work it deals with *live* and *safe* PN.

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## III. Fuzzy Timed Petri nets

*A. Basic Operators*

First, some useful operators are introduced.
*Definition 10:* Let $\tilde{a}$ and $\tilde{b}$ two fuzzy sets such that $a_2 < b_3$. The extended union operation between $\tilde{a}$ and $\tilde{b}$ is

$$ext\left(\tilde{a},\tilde{b}\right) = \min\left(\tilde{a}^{>},\tilde{b}^{<}\right) \quad (4)$$

this operation is illustrated in Figure 1(e).

*Definition 11:* The latest (earliest) operation selects the latest (earliest) fuzzy set among $n$ fuzzy sets; they are calculated as follows:

$$latest\ (\tilde{a}_1,\dots,\tilde{a}_n) = min\left(max(\tilde{a}_1{}^{<},\dots,\tilde{a}_n{}^{<}), min(\tilde{a}_1{}^{>},\dots,\tilde{a}_n{}^{>})\right) \quad (5)$$

$$earliest(\tilde{a}_1,\dots,\tilde{a}_n) = min\left(min\ (\tilde{a}_1{}^{<},\dots,\tilde{a}_n{}^{<}), max\ (\tilde{a}_1{}^{>},\dots,\tilde{a}_n{}^{>})\right) \quad (6)$$

**Box 1**

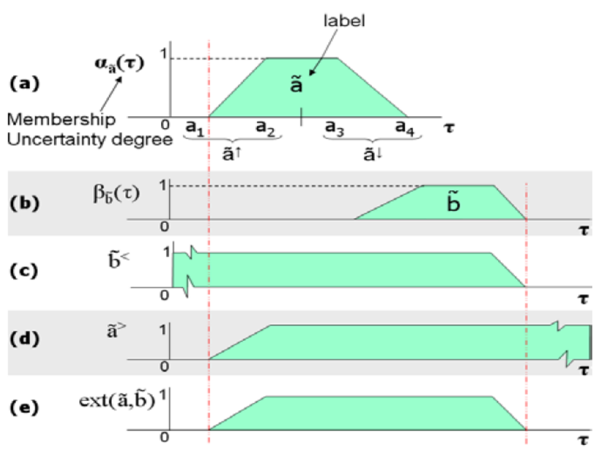


**Figure 1**
Fuzzy sets and operations
*Source: (González-Castolo & López-Mellado, 2011)*

*Definition 12:* The *conjugation operator* is defined as *arg1* $\overset{oper}{\bullet}$ *arg2*, where *arg1*, *arg2* are arguments that can be matrices of fuzzy sets; • is the *fuzzy and* operation and *oper* is any operation referred as +, −, *latest, min,* etc. For two fuzzy sets $\tilde{a}\overset{oper}{\bullet}\tilde{b} = \mathrm{oper}\left(\mathrm{and}(\tilde{a},\tilde{b})\right)$. For matrices $\tilde{A}(m\times r)$ and $\tilde{B}(r\times n)$, $\tilde{A}\overset{oper}{\bullet}\tilde{B} = \left[\underset{k=1,\dots,r}{\mathrm{oper}}\left(and(\tilde{a}_{ik},\tilde{b}_{kj})\right)\right], i = 1,\dots,m$ and $j = 1,\dots,n$, (González-Castolo & López-Mellado, 2011).

*B. Formalism Description*

*Definition 13:* A *fuzzy timed Petri net structure* is a 3- tuple $FTPN = (N,\Gamma,\delta)$; where $N = (G, M_0)$ is a PN, $\Gamma = \{\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_n\}$ is a collection of fuzzy sets, $\delta: P \to \Gamma$ is a function that associates a fuzzy set $\tilde{a}_i \in \Gamma$ to each place $p_i \in P$.

The values $\tilde{\tau}\uparrow$,$\tilde{\tau}\downarrow$, corresponds to ranges $(a_1,a_2),(a_3,a_4)$, respectively. When $\tau \in \tilde{\tau}\uparrow$ $(\tilde{\tau}\downarrow)$, the function $\alpha(\tau)$ goes towards $1(0)$.
A fuzzy set $\tilde{a}$ is referred indistinctly by the function $\alpha(\tau)$ or the characterization $(a_1,a_2,a_3,a_4)$.

***Fuzzy timing of places.*** The fuzzy set $\tilde{a} = (a_1,a_2,a_3,a_4) \in \Gamma$ Figure 2(b) represents the *static* possibility distribution $\alpha(\tau) \in [0,1]$ of the event at which an activity stops, i.e. the instant when a token leaves a place $p \in P$. The activity duration is considered from the instant when *p* is marked. This set does not change during the FTPN execution.

***Fuzzy timing of tokens.*** The fuzzy set $\tilde{b} = (b_1,b_2,b_3,b_4)$ Figure 2(c) represents the *dynamic* possibility distribution $\beta(\tau) \in [0,1]$ associated to a token residing within a $p \in P$; it also represents the instant $\tau$ at which such a token may leave the place.

This instant is computed from the instant when $p$ is marked. $\tilde{b}$ is computed from $\tilde{a}$ every time the place is marked during the marking evolution of the FTPN.

A token begins to be available for enabling output transitions at $\beta(b_1)$. Thus $\tilde{b}^{>}$ represents the possibility distribution of available tokens.

The fuzzy set $\tilde{c} = (c_1,c_2,c_3,c_4)$, known as *fuzzy timestamp*, Figure 2(d) is a dynamic possibility distribution $\varsigma(\tau) \in [0,1]$ that represents the duration of a token within a place $p \in P$. This notion is close to that introduced in (Murata, 1996)

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**Box 2**

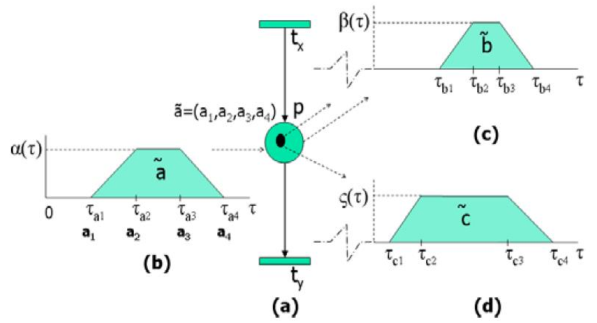


**Figure 2**

a) FTPN, (b) The fuzzy set associated to places. (c) Fuzzy set to place or mark associated. (d) Fuzzy timestamp

*Source: (González-Castolo & López-Mellado, 2011)*

***Fuzzy enabling date***

*Definition 14:* The fuzzy enabling date $e_{t_k}(\tau)$ of the transition $t_k$ is a possibility distribution of the instant $\tau$ at which $t_k$ is enabled. It is computed from the outgoing instants $\tilde{b}_{p_i}$ of the tokens within the input places to $t_k$ as (7).

$$e_{t_k}(\tau) = latest\left(\tilde{b}_{p_i}\right) \forall p_i \in \bullet\, t_k \quad (7)$$

***Fuzzy firing date***

Definition 15: The fuzzy firing transition date $o_{t_k}(\tau)$ of a transition $t_k$ expresses the possibility distribution of the instant at which $t_k$ may fire. It is determined with respect to the set of transition $\{t_j\}$ in conflict simultaneously enabled.

$$o_{t_k}(\tau) = min\left(e_{t_k}(\tau), earliest\left(e_{t_j}(\tau)\right)\right) \forall_{t_k} \in p_n \bullet;\ p_n \in \bullet\, t_j \quad (8)$$

***Computation of $\tilde{b}$***

Now, using the above notions, it can state the calculation of the residency of the $\tilde{b}$ tokens. For a given place $p_s$, the possibility distribution $\tilde{b}_{p_s}$ may be computed from $\tilde{a}_{p_s}$ and the firing dates $o_{t_j}$ *(τ)* of a $t_j \in \bullet p_s$ using the following expression:

$$\tilde{b}_{p_s} = ext\left(o_{t_j}(\tau)\right) \oplus \tilde{a}_{p_s} \ \forall t_j \in \bullet\, p_s \quad (9)$$

***Fuzzy timestamp computation***

*Definition 16:* The fuzzy timestamp $\tilde{c}_{p_s}$ is computed from the occurrence dates of both $\bullet p_s$ and $p_s\bullet$.

$$\tilde{c}_{p_s} = ext\left(earliest\left(o_{t_i}(\tau)\right)\right), latest\left(o_{t_j}(\tau)\right) \forall t_i \in \bullet\, p_s, t_j \in p_s \quad (10)$$

Actually, $\tilde{c}_{p_s}$ represents the fuzzy marking in $p_s$ at instant τ.

*D. Uncertainty background*

*Definition 16:* The *ending uncertainty* (*eU*) is the uncertainty of possibility to find a mark in a place $p_j$ or a previous place $p_{j-1}$ in a circuit, (González-Castolo & López-Mellado, 2011).

*Proposition 17:* The *ending uncertainty trace* (*eUt*) is defined with the function $\xi(\tau)$ that represents the marking *eU* in the path, (González-Castolo & López-Mellado, 2011).

The fuzzy state equation of *Fuzzy Time Marked Graphs* (FTMG) was presented in (González-Castolo & López-Mellado, 2007) as (11).

$$\breve{M}(\tau) = \min\left(M(\tau - \Delta\tau) + C^{+}\Delta_{\tilde{0}\uparrow} - C^{-}\Delta_{\tilde{0}\downarrow}, \vec{1}\right) \quad (11)$$

In (González-Castolo & López-Mellado, 2007), the procedure to obtain the fuzzy marking of *Fuzzy Time State Machines* (FTSM) was presented. Remembering that due to the uncertainty that induces the FTSM structure, the process to obtain the tokens is more complex that FTMG.

**IV. Location of measurable places in a FTPN**

In (González-Castolo & López-Mellado, 2011), the minimum number of measurable locations to keep bounded the uncertainty in the FTMG and FTSM is determined. It will now analyze the location of these measurable locations and the quality of the marking approximation.

*A. Location of measurable places in a FTSM*

In proposition 17, the boundedness condition of the *eUt* in a FTSM is given, which establishes that it is necessary to have a measurable place for each circuit. Analyze the following scenarios through a simple example of a FTSM with two t-components (Figure 3).

In the first scenario (Figure 3(a)) there is only one measurable place that belongs to both circuits; with this, the boundedness condition is fulfilled using a single sensor.

However, although uncertainty is limited when the measurable place is not marked, it approximates only the path it follows, such as that obtained without measurable places.

In a second scenario, there are two measurable places (Figure 3(b)); thus, it is possible to know which path is being executed when a measurable place is marked; However, when the decision is unmarked, after approximating the marking of the place $p_1$, there approximates the path to be followed only until a measurable location is marked again.

Finally, in a third scenario (Figure 3(c)) the measurable places are located just after the decision place $p_1$, so the path being executed is known from the start.

It is evident that the third scenario is the most convenient since the calculation of the branching relationship is avoided and in general, it can affirm that this characteristic is maintained by locating the measurable places after each decision place.

**Box 3**

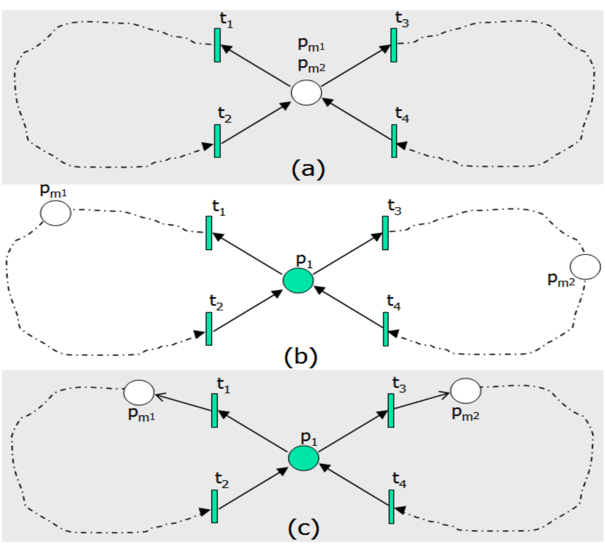


**Figure 3**
Fuzzy marking evolution
*Source: Own elaboration*

*B. Location of measurable places in a FTMG*

In (González-Castolo & López-Mellado, 2011), the boundedness condition of the *eUt* in a FTMG is given, which establishes that it is necessary to have at least one measurable location in the system. It now analyzes the following scenarios (Figure 4).

In the first scenario (Figure 4(a)) the measurable place $p_m$ is located on one of the parallel paths starting from the distribution transition $t_d$ where there are several non-measurable places between $t_d$ and $p_m$. When $p_m$ is marked, the review-update procedure explained above is performed, which is done to reduce uncertainty.

In the second scenario (Figure 4(b)) it has a measurable place $p_m$ immediately after the synchronization transition td. Now that $p_m$ has a mark, the certain is obtained.

Finally, in a third scenario (Figure 4(c)) the measurable place is located just before the synchronization transition $t_d$; the conditions on uncertainty are like those of the previous scenario.

The second and third scenarios are the most convenient for the location of the measurable place since the review-update procedure of the uncertainty in the rest of the parallel branches that start from the distribution transition is avoided.

**Box 4**

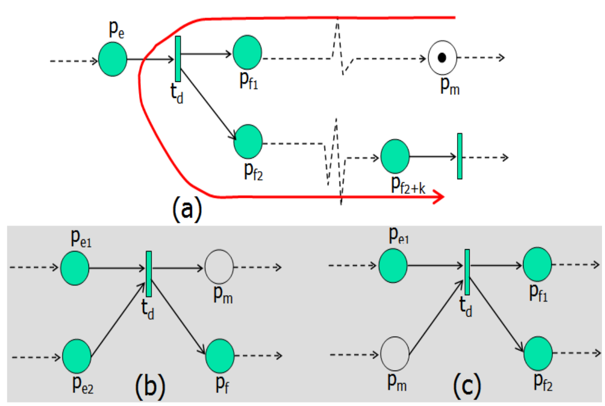


**Figure 4**
Location of places with associated sensor signals
*Source: Own elaboration*

*C. Marking approximation quality*

The bound of the degree of certainty $D(\tau)$ of the marking of the FTPN determines the quality of the estimation of the state. It will analyze here the evaluation of this quality and propose a procedure to adjust the quality by introducing sensors.

Evaluating the estimation quality considering FTPN models with the minimum number of sensors, the estimation quality is determined by evaluating $D(\tau)$ at the instant when a measurable place is marked in periodic operation. In the case of FTMG the calculation is obvious since there is a t-invariant (Figure 5).

**Box 5**

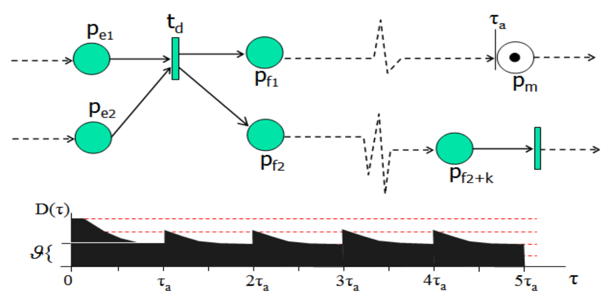


**Figure 5**
Quality of state estimate $\vartheta$
*Source: Own elaboration*

In the case of FTSM it is necessary to make this evaluation $V(\tau)$ in each of the possible paths between two measurable places; the lowest evaluation determines the quality of the entire net. In the case of placing sensors in the immediate locations of the decision places, which may not be a minimum allocation of measurable places (Figure 6), the analysis would be limited to the evaluation of the circuits.

**Box 6**

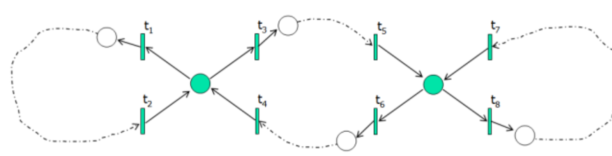

**Figure 6**
FTSM with more than one measurable place by t-component
*Source: Own elaboration*

*D. Tuning to a desired quality*

It is possible to maintain a desired quality value above a specific threshold $\vartheta$ by adding additional sensors to the system.

For a given FTPN model with a certain distribution of measurable locations (which can be the minimum or the so-called best in each of the subclasses), the general idea is to run the model from the initial marking, continuously evaluating $D(\tau)$. When $D(\tau) < \vartheta$ then it is necessary to place a sensor associated with the location on the trajectory where $V^i(\tau)$ is lowest.

By performing this procedure iteratively, it can determine the minimum number of measurable locations that ensure the quality desired $\vartheta$.

## Conclusions

This paper addressed the problem of DES state estimation for which activity durations are poorly known; fuzzy sets represent the uncertainty of the end of activities. Current research addresses the inclusion of sensors in FTPN to reduce the uncertainty about marking the observed locations to zero and to keep the uncertainty of marking bounded for any evolution of the system.

The inclusion of sensory information in the fuzzy model, called semi-fuzzy in this case, allows to keep the uncertainty in the approximation of marking bounded. The analysis presented shows that a reduced number of sensors is enough to give this property to the state estimation device. Furthermore, it is possible to adjust this uncertainty through a simple analysis, adding measurable locations in the model.

This approach provides a closer approximation to the real state of the DES. This method is useful for non-critical applications of state monitoring and decision making to act on the system.

## Conflict of interest

The authors declare no interest conflict. They have no known competing financial interests or personal relationships that could have appeared to influence the article reported in this article.

## Authors' Contribution

*González-Castolo, Juan Carlos*: Contributed to the project idea. He carried out the analysis and systematization of results, as well as writing the article.

*López-Mellado, Ernesto*: Carried out the systematization of the background for the state of the art. He also contributed to the review of the article.

*Ramos-Cabral, Silvia*: She contributed to the writing and revision of the article.




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**Abbreviations**

| | |
|---|---|
| DES | Discrete events systems |
| eU | Ending uncertain |
| eUt | Ending uncertainty trace |
| FTMG | Fuzzy timed marked graph |
| FTPN | Fuzzy timed Petri nets |
| FTSM | Fuzzy timed state machine |
| PN | Petri nets |

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