Reineke based guide to manage density in *Pinus montezumae* stands in Puebla, Mexico

Guía basada en Reineke para manejar la densidad en rodales de *Pinus montezumae* en Puebla, México

TAMARIT-URIAS, Juan Carlos†*, RODRÍGUEZ-ACOSTA, Melchor and ORDÓÑEZ-PRADO, Casimiro

Instituto Nacional de Investigaciones Forestales Agrícolas y Pecuarias, México.

ID 1st Author: *Juan Carlos, Tamarit-Urias* / **ORC ID**: 0000-0003-3597-4510, **CVU CONACYT ID**: 86510

ID 1st Coauthor: *Melchor, Rodríguez-Acosta* / **ORC ID**: 0000-0001-9771-2685, **CVU CONACYT ID**: 99844

ID 2nd Coauthor: *Casimiro, Ordóñez-Prado* / **ORC ID**: 0000-0001-8520-8406, **CVU CONACYT ID**: 344396

Abstract

Objective: Generate a density management guide (DMG) based on the Reineke density index for even-aged stands of *Pinus montezumae* in Puebla, Mexico. Methodology: Dasometric information from 90 sampling sites of the Forest Management Unit 2103 of the state of Puebla was processed. Each site was 1,000 m², circularly, coming from natural stands in high density and competition; all growth conditions and age range were covered. The variables density per hectare (N) and quadratic diameter (Dq) were used to adjust the Reineke density-size model. The technique of linear ordinary least squares (OLS) was compared against that of stochastic frontier regression (SFR); the quality of the adjustments was assessed using statistical and graphic criteria. The self-thinning line was determined and a DMG was constructed defining four zones of Langsaeter growth. Contribution: The SFR technique in its semi-normal mode better defined the limit of the maximum boundary of the observed data. DMG is useful for prescribing thinnings as intermediate silvicultural treatments for stands of the species. The maximum density index for a reference Dq of 25 cm was 886 trees ha⁻¹.

Resumen

Objetivo: Construir una guía para manejar la densidad (GMD) basado en el índice de densidad de Reineke para rodales coetáneos de *Pinus montezumae* de Puebla, México. Metodología: Se procesó información dasométrica de 90 sitios de muestreo levantados en la Unidad de Manejo Forestal 2103 del estado de Puebla. Cada sitio fue de 1,000 m², de forma circular, se levantó en rodales naturales en alta densidad y competencia; se cubrieron todas las condiciones de crecimiento y rango de edad. Las variables densidad por hectárea (N) y diámetro cuadrático (Dq) fueron usadas para ajustar el modelo densidad-tamaño de Reineke. Se comparó la técnica de mínimos cuadrados ordinarios lineales (MCO) contra la de regresión frontera estocástica (RFE); la calidad de los ajustes se evaluó mediante criterios estadísticos y gráficos. Se determinó la línea de autoaclareo y se construyó una GMD definiéndose cuatro zonas de crecimiento de Langsaeter. Contribución: La técnica de RFE en su modalidad semi-normal definió mejor el límite de la frontera máxima de los datos observados. La GMD es útil para prescribir aclareos como tratamientos silvícolas intermedios para rodales de la especie. El índice de densidad máximo para un Dq de referencia de 25 cm fue de 886 árboles ha⁻¹.

Thinning, Stand density index, Even-aged stands

Aclareos, Índice de densidad del rodal, Rodales coetáneos

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* Correspondence to Autor (E-mail: tamarit.juan@inifap.gob.mx)

† Researcher contributing as first author.

Introduction

The guides or diagrams to manage the density (DMG) of stands have the fundamental purpose of regulating the density by means of the application of intermediate cuts, also commonly called thinnings, to help to produce high volume of quality wood in the shortest possible time (Burkhart and Tomé, 2012; Cabrera-Pérez *et al.*, 2019). They are tools to apply quantitative forestry, they are built based on the determination of the self-thinning line (line of maximum density or imminent mortality).

In the determination of the self-thinning line for unspecific contemporary stands, predefined theoretical values are commonly used, such is the case of the Reineke density-size functional relationship, where the value of the parameter that corresponds to the slope was established at -1.605 (Reineke, 1933). In this regard, studies such as that of Pretzsch and Biber (2005), Zhang *et al.*, (2005), Quiñonez-Barraza et al. (2018) and Tamarit-Urias *et al.*, (2019), refer that to determine this line and therefore to construct the respective DMG, the particular allometry of each species must be considered. In this sense, VanderSchaaf and Burkhart (2007) and Comeau *et al.*, (2010), add that the value of this parameter differs significantly between species, so it must be determined for each species and region. To determine this, different adjustment techniques have been proposed to improve the definition of the self-thinning line, among which the linear ordinary least squares and the stochastic frontier regression stand out (Santiago-García *et al.*, 2013; Burkhart and Tomé, 2012; Zhang *et al.*, 2015; Lopes *et al.*, 2016).

Using the function parameters derived from the best estimation technique guarantees that the DMG are powerful tools that allow proper and optimal management of the density of the stands. Density is the main factor that the forester can manipulate to influence the establishment and development of the species of interest, to improve the quality of the wood, the growth rate in diameter and influence the production of the timber volume by redistributing the potential. of growth among the remaining individuals (Daniel *et al.*, 1979; Pretzsch, 2009).

The stand density index (SDI) derived from the Reineke function is used to assess stand density. This index as a measure of relative density and site occupancy indicates the number of trees per hectare for a reference mean cuadratic diaméter, which was originally set at a constant theoretical value for any species at 25 cm, the Index expresses the relationship between tree size and stand density.

P. montezumae is an abundant conifer and widely distributed in the transverse Neovolcanic Axis of Mexico, it grows forming pure contemporary stands at an average altitude of 2.500 m and 800 mm per year of precipitation, the trees of this taxon reach heights of 25 to 30 m, it is highly important for commercial timber harvesting (Perry, 1991; CONAFOR, 2012). Given its economic relevance, it is necessary to carry out studies aimed at generating quantitative silvicultural tools that contribute to improving the technical management of natural stands with this species.

The objective was to build a guide to manage the density based on the Reineke density index for contemporary stands of *P. montezumae* from Puebla, Mexico, by evaluating the quality of fit of the size-density function by the minimum techniques linear ordinary squares and stochastic frontier regression.

Methodology

The study was carried out in forest plots of the Forest Management Unit (UMAFOR) 2103 "Teziutlán" region, located in the northeastern part of the state of Puebla, Mexico. The average altitude is 2.220 m, the average annual temperature fluctuates from 12 to 22 °C and it has Luvisol-type soils. Dasometric information from 90 sampling sites raised in natural stands of *P. montezumae* in a state of high density and competition was processed, which guaranteed to meet the requirement of extreme competition for limited resources (space, water, nutrients and sunlight); all growth conditions and age range were covered. Each site was 1,000 m^2, circular in shape. The dasometric variables by site were the density expressed in terms of the number of trees and the average square diameter, later the density was scaled to the hectare level. With both variables the Reineke density-size functional relationship was adjusted.

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The basic statistics of the processed variables are presented in Table 1.

Variable	Min	Max	Average	E. D.	C.V.
N	190	3390	910	740.4	81.4
Dq	11.5	45.9	25.3	10.2	40.4
Min: Minimum, Max: Maximum, D.E.: Standard deviation, C.V.: Coefficient of variation.					

Table 1 Basic descriptive statistics of the analyzed dasometric variables
Source: Own elaboration using sample data

The final database was audited by graphic inspection, corroborating that the variables of interest will present logical graphical behavior in the form of an inverse jota (Figure 1), in this way it was prepared for statistical adjustment by regression.
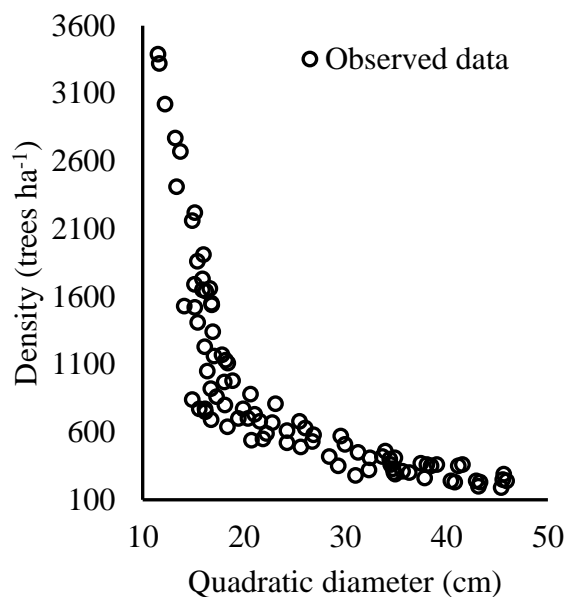

Figure 1 Logical graphical behavior of the variables used in the regression analysis of the Reineke density-size function
Source: own elaboration using the data observed in the experimental sample

The Reineke density-size function expressed linearly when adjusted by the linear ordinary least squares method (OLS-L) had the following linearized mathematical structure:

$$Ln(N) = \alpha - \beta Ln(Dq) + \varepsilon \qquad (1)$$

Where: Ln is the natural logarithm function, ε is the error term that is distributed as $\varepsilon = idd\ N(0, \sigma^2)$.

When the function was adjusted using the stochastic frontier regression (SFR) econometric technique, the model structure took the form:

$$Ln(N) = \alpha - \beta Ln(Dq) - u + v \qquad (2)$$

In the SFR the error component is divided into: 1) an error component that accounts for the technical inefficiency in the data (u_i) and, 2) an error component associated with the measurement of the individual observations (v_i). ui is an asymmetric term that reflects the technical inefficiency of the observations and is assumed to be distributed independently of v_i and regressors. v_i is assumed as a symmetric disturbance distributed independently of u_i, it collects random variations in production due to factors such as random errors, errors in the observation and measurement of data, it is assumed that it is distributed in the form v = idd N ($0, \sigma^2_v$).

Under these assumptions, statistical distributions are selected for u_i that tend to one side, as is the case of the semi-normal and exponential SFR modality. If the value of technical inefficiency u_i is assumed to be zero, which is less likely with increasing values of u_i, then the idd model N + ($0, \sigma^2_u$) refers to the semi-normal model. If the u_i (i = 1 ..., N) are non-negative random variables idd N + ($0, \sigma^2_u$), then the model is known as the normal-truncated (zero) model (Bi, 2004; Zhang *et al*., 2005; Pretzsch and Biber, 2005). For the adjustment with SFR, the modalities of Semi-Normal Model (SNM), Truncated Normal Model (TNM) and Exponential Normal Model (ENM) were tested.

In the OLS adjustment, the Model procedure of the SAS / ETS® statistical package (Statistical Analysis System Institute Inc., 2011) version 9.3 was used. When SFR was used, the function was adjusted using the QLIM procedure of the same statistical package and version; in which the maximum likelihood technique is used to estimate the border and the parameter of the technical inefficiency of the observations (u_i). The adjustment of both techniques (OLS-L vs SFR in its three modalities) was compared.

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The quality of the adjustment by OLS-L was compared with the adjustments obtained when using SFR, the goodness-of-fit statistics used were the likelihood logarithm (logLik), the Akaike information criterion (AIC) and the Schwarz criterion. (SchC), the higher the logLik value and the lower in AIC and SchC the model is more appropriate; as well as the significance of the parameters, the variances of the error components, the ratio of variances of the error components (λ) and the total variance (σ) (Weiskittel *et al*., 2009; Comeau *et al*., 2010; Quiñonez-Barraza *et al*., 2018). To reinforce the selection criteria, a graphic comparison of the self-thinning lines that each of the adjustment techniques and modalities generated was also performed, in which the behavior of the path of the self-thinning lines superimposed on the observed data was inspected.

With the values of the parameters of the best selected adjustment, the self-thinning line was delimited on a graph on a logarithmic scale, this line corresponds to the maximum number of trees that a hectare without self-thinning can support and is equivalent to 100% of the SDI, which is express how:

$$SDI = N(Dq/Dq_r)^{-\beta} \qquad (3)$$

Where N is the density, Dq is the quadratic diameter, Dqr is the reference quadratic diameter, and β is the parameter that corresponds to the slope of the function.

While the quadratic diameter is expressed:

$$D_q = \sqrt{(40000/\pi)(AB/N)}, \qquad (4)$$

Where AB is the basal area in m^2 ha^{-1}, N is the number of trees ha^{-1} and π is equal to 3.1416.

Based on Smith *et al*. (1997) and Gilmore *et al*. (2005) and based on the maximum density line, assuming a reference Dq of 25 cm, the four different growth areas of Langsaeter were estimated and defined, which make up bands of relative densities.

These zones were determined as a percentage of the maximum SDI; Zone 1, which is underutilized at the site, corresponds to 25%, Zone 2, which is transitional, is defined between 25-35%, Zone 3, which corresponds to maximum growth per hectare, is located between 35-65%, and zone 4, which is that of self-thinning or eminent mortality, is located between 65-100%.

Results

The statistical adjustment of the Reineke function when using linear OLS and SFR in its three modalities, shows that in all cases the function parameters are significant (Table 2). However, OLS-L has the drawback that to define the self-thinning line, the value of the intercept must be modified using some subjective approach, this because the fitted line corresponds to the average of the cloud of observed data and not to the upper limit. absolute, which causes the line thus defined in the upper border of the data to be inefficient (Zhang *et al*., 2005; Santiago-García *et al*., 2013; Quiñonez-Barraza *et al*., 2018).

Parameter	α	β
OLS-L		
Estimator	12.014570	-1.740140
S. E.	0.216000	0.068040
T value	55.62	-25.58
Significance	<0.0001	<0.0001
SFR-SNM		
Estimator	12.081814	-1.645050
S. E.	0.385089	0.139818
T value	31.37	-11.77
Significance	<0.0001	<0.0001
SFR-TNM		
Estimator	12.620250	-1.807118
S. E.	0.003431	0.001081
T value	3678.09	-1671.4
Significance	<0.0001	<0.0001
SFR-ENM		
Estimator	12.546772	-1.835806
S. E.	0.210061	0.057682
T value	59.73	-31.83
Significance	<0.0001	<0.0001
E.E .: Standard error.		

Table 2 Estimated values of the parameters of the Reineke model by OLS-L and by SFR modality
Source: own elaboration using results of the adjustments

For this reason, it was decided to select the best quality of fit among the different proven modalities of the SFR technique, which guarantees that a line of the absolute maximum is obtained directly and immediately and that technically it is the correct limit (Bi, 2004; Comeau *et al*., 2010); In addition, the SFR technique comparatively has the advantage that it estimates extreme values (boundary) of a data set instead of the mean of a function estimated by the OLS method.

The comparative evaluation between the different SFR modalities reveals that the values of the adjustment statistics (logLik, AIC, SchC), the same as the total variance of the error (σ^2) and the variance ratio of the error components (λ) were similar (Table 3).

Model	Statistical	Value
OLS-L	σ^2	0.031230
SFR-SNM	logLik	-19.933630
	AIC	47.867270
	SchC	57.866510
	σ^2	0.507300
	λ	1.213980
SFR-TNM	logLik	4.606700
	AIC	-1.213390
	SchC	8.785850
	σ^2	0.332300
	λ	31536946
SFR-ENM	logLik	-2.162250
	AIC	12.324510
	SchC	22.323750
	σ^2	0.272320
	λ	1.599040
logLik: logarithm of plausibility; AIC: Akaike Information Criterion; SchC: Schwarz criterion; σ^2: variance of the error; λ: ratio of variances.		

Table 3 Values of goodness statistics by evaluated adjustment technique.
Source: own elaboration using results of the adjustments of the Reineke function

In all the modalities, significance was detected in the variance of the component of the error relative to the term of technical inefficiency of the observed data.

Although the SFR-SNM model did not present the best values in the adjustment statistics, the comparative graph of the definition of the line of self-thinning (Figure 2) reveals that this modality had the best pattern, since the line fits better. to the upper limit of the border of the observed data and produces the best definition of the maximum border and therefore of the self-thinning line, which is why it was selected to construct the DMD. In this sense, Zhang *et al*. (2005) points out that this adjustment modality is the most appropriate regression method to estimate the coefficients of the Reineke function and the subsequent line of self-thinning, being possible to use all the available N-Dq information from plots or sites that are locate near an upper limit. The SFR approach to define the line of maximum density has been used in different coniferous and broadleaf species by Weiskittel *et al*. (2009), Comeau *et al*. (2010), Reyes-Hernandez *et al*. (2013), Lopes *et al*. (2016) and Camacho-Montoya *et al*. (2018).
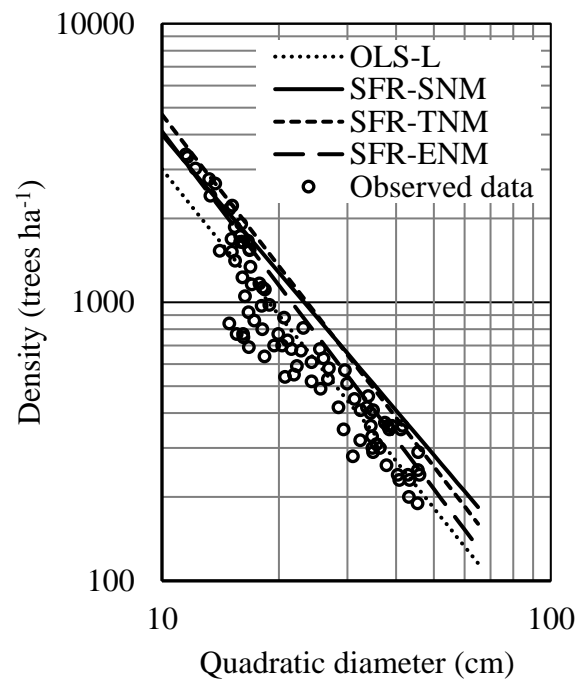

Figure 2 Behavior of the trend of the self-thinning line generated by the evaluated adjustment techniques.
Source: data observed in the experimental sample

The Wald test and the likelihood ratio showed that the value of the slope (β) of the model estimated using SFR-SNM is statistically different (with $\alpha = 0.05$) from the theoretical value proposed by Reineke (1933) of -1.605. This result coincides with that reported by Reyes-Hernandez *et al*. (2013) who used the same adjustment technique and modality in stands of *Picea glauca* in the Alberta region, Canada, whose slope value was 1.96.

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The 95% confidence interval determined for the slope parameter was -1.652 for the lower limit and -1.899 for the upper limit. This interval is similar to that determined by Quiñonez-Barraza *et al*. (2018) from -1.714 to -1.910 who used the same technique and modality for a group of species of the genus *Pinus* of Durango, Mexico. In this context, the argument that the slope (β) is not always close to the theoretical value and that it can differ significantly between species is confirmed (Comeau *et al*., 2010; Santiago-García *et al*., 2013), which is partly due to the fact that different populations present different mortality rates depending on their density, growth habits, factors of site productivity and even the age of the mass (Bi *et al*., 2000; Weiskittel *et al*., 2009; Reyes-Hernandez *et al*., 2013). This evidence justifies the development of a specific allometry for each species and thus avoid errors when estimating and controlling density (Pretzsch and Biber, 2005).

The maximum SDI estimated with the adjustment of the SFR-SNM when using a 25 cm quadratic reference diameter (Dqr) was 886 trees ha^{-1}, which is less than that determined by Quiñonez-Barraza *et al*. (2018) of 1.107 for the same Dqr value in a group of species of the genus *Pinus* from Durango, Mexico, where he selected the SFR technique in the TNM modality as the best to define the line of self-thinning. In contrast, for the same adjustment modality and the same reference Dq, the maximum SDI determined in this study is similar to that estimated by Martínez (2017) who reports 898 trees ha^{-1} for mixed *Pinus* spp forests. in the Zimatlán region, Oaxaca, Mexico.

Based on the self-thinning line and a Dqr = 25 cm, the DMG was constructed for unispecific contemporary stands of *P. montezumae*. The estimated number of trees, which correspond to the limit values in percentage of the SDI that define each line that delimits the growth zones were 225, 310, 575 and 886 trees ha^{-1} for 25%, 35%, 65% and 100 %, respectively; which in turn correspond to the lines of free growth (1), transition (2), maximum growth per hectare (3) and self-thinning (4) (Figure 3).

With information on the mean square diameter and average density per hectare of stands with this species, obtained from sampling sites for timber inventory or derived from a timber growth and yield system, the level of competition can be evaluated and if necessary It is possible to prescribe thinning programs and application intensities in terms of the number of trees per hectare to be removed and its equivalent in m^2 ha^{-1} of basal area (AB).
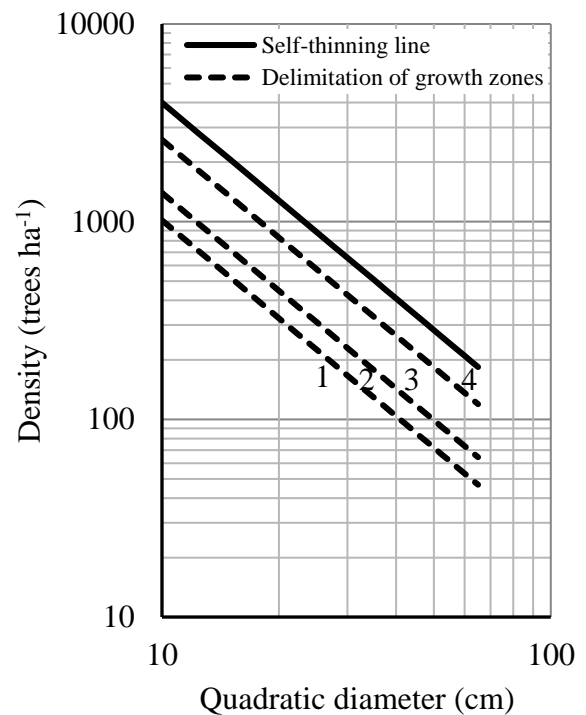

Figure 3 Guide to Density Management based on the Reineke IDR for *P. montezumae*
Source: own elaboration based on the adjustment of the Reineke model in the RFE-SNM modality

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Conclusions

The DMG built based on the Reineke SDI for *P. montezumae* stands at UMAFOR 2103 in Puebla, Mexico, derived from the adjustment by the SFR-SNM technique, is a quantitative silvicultural tool with easy technical and operational application for the forestry professional in charge of the silvicultural management of this species, since it allows you to assess the need to apply thinning. For its reconstruction some program in the form of a spreadsheet such as Microsoft Office® Excel can be used. The DMG will contribute to better applying silvicultural practices through adequate density management, making intermediate cuts as silvicultural treatments and optimizing the commercial shift of the species in the study area.

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