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Article

Pressure drop prediction by a polynomial model for two-phase flow in vertical oil wells

Predicción de la caída de presión por un modelo polinomial para flujo bifásico en pozos petroleros verticales

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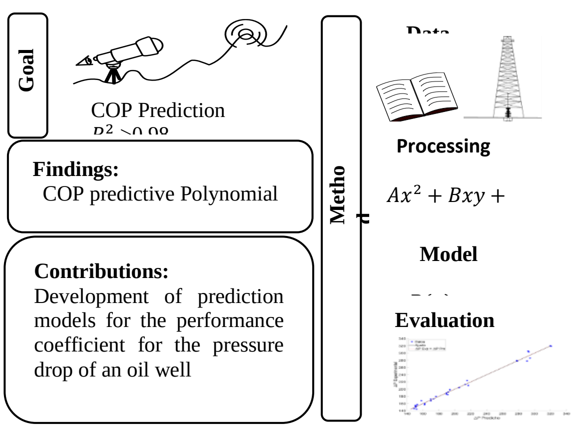
Resumen

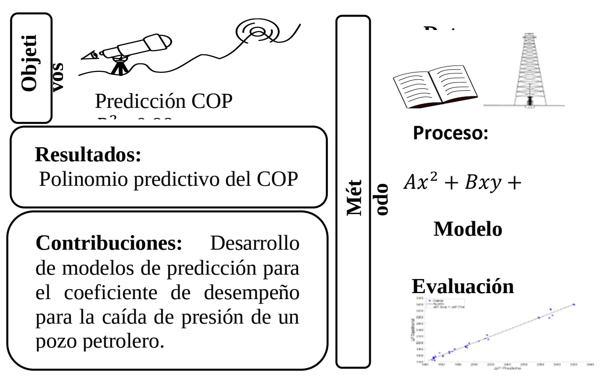
In this paper, a polynomial model is developed to predict the pressure drop in oil wells with two-phase flow. The variables used in the model are gas-oil ratio production, oil production, tubing diameter, solution gas-oil ratio, oil formation volume factor, and oil viscosity. A polynomial model is presented to predict the pressure drop with a coefficient of determination of 0.9901. The residual analysis and level surfaces of the pressure drop against the polynomial model's input variables are presented to validate the model. A regression of the experimental and predicted pressure drops values using the polynomial model is presented. This model contributes to a simpler methodology for the calculation of pressure drops and the consequent application in the modeling of production curves in oil wells for NODAL analysis.



Two-phase flow, residual analysis, validate, regression, experimental, and polynomial model

Resumen

Se desarrolla un modelo polinomial para predecir la caída de presión en pozos petroleros con flujo bifásico. Las variables utilizadas en el modelo son producción de la relación gas-aceite, producción de aceite, diámetro de la tubería de producción, relación de solubilidad, factor volumétrico de formación del aceite y viscosidad del aceite. Un modelo polinomial es presentado con el objetivo de predecir la caída de presión que tiene un coeficiente de determinación de 0.9901. Con el objetivo de validar el modelo son presentados el análisis residual y superficies de nivel de las caídas de presión contra las variables de entrada del modelo polinomial. Se presenta una regresión de los valores de caída de presión experimental y predichos mediante el modelo polinomial. Este modelo contribuye en una metodología más simple para el cálculo de las caídas de presión y la consecuente aplicación en el modelado de las curvas de producción en pozos petroleros para su análisis NODAL.



Flujo bifásico, análisis residual, validación, regresión, experimental y modelo polinomial

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Introduction

During production in hydrocarbon wells, two-phase flow is encountered, which complicates the calculation of various phenomena that occur during the trajectory of the fluid inside the pipe. That is to say, during the period in which the fluid travels through the pipe, it is very common for there to be a change in the distribution of the fluid, and it is also possible to find hanging of the liquid. One of the main variables that is affected for its study is the pressure drop that occurs in this flow path. In order to calculate the pressure drop, it is necessary to use mechanistic models that explain the behaviour of multiphase flow in the pipe according to all the variables included in the processes of this phenomenon. These mechanistic models, which have been developed for decades, have their application problems, due to the limited conditions in which they were developed and therefore they are not accurate in general conditions or far from the original conditions in which they were created.

There are numerous models that have been proposed in the literature to calculate the hydrocarbon pressure drop, some of the most important models are those presented in (Beggs & Brill, 1973) which considers the fluid hang-up as well as the flow regime. The model of the authors (Hagedorn & Brown, 1965) considers the calculation of gravity and friction pressure drops, using the fluid hang-up to determine the gravity pressure drops, but without considering the flow regime. The aforementioned models can be complex in terms of calculations, which is why, in this work, a polynomial function is proposed to predict the pressure gradient calculation.

A polynomial function is obtained by means of a multiple linear regression analysis, the objective is to predict the pressure drop knowing the experimental measurements of some variables associated to this process. In fact, polynomial fitting is an attractive technique used to estimate the dependent variable in a system knowing experimental data of the independent variables associated with the system under study, see for example (Escobedo-Trujillo et al., 2014) for more details of the technique. There are different prediction methods such as the machine learning techniques studied in (Dabiri et al., 2024), it is up to the researcher to choose the one that best fits their experimental data.

For this article we used the experimental data of the authors (Chierici et al., 1974) which are given in the first section, then in section 2 we mention the methodology used for the development of the polynomial model and in section 3 we show the results of the polynomial model obtained and its respective residual analysis to verify the goodness of fit or prediction, and finally in section 4 we conclude with the contribution obtained from this work.

Experimental data

The database used in this work was obtained from the research work (Chierici et al., 1974). This database was chosen because it had fluid properties such as solubility ratio and volumetric factor of the oil, unlike the work (Espanol et al., 1969) which only provides flow data and mechanical characteristics of the wells. The Chierici database shows information on 10 variables that affect pressure drops such as: oil specific gravity, gas-oil ratio, water cut, oil volumetric flow, well diameter, temperature, solubility ratio, oil volumetric factor, oil viscosity and gas specific gravity; this information is from 31 oil wells. Table 1 shows the variables with their respective operating ranges.

Box 1
Table 1
Ranges of operations under experimental conditions to obtain pressure drop values

Variable	Rango de operación
Specific gravity of oil (γ_o)	8.3-46
Production of gas-oil ratio (GOR)	25.9 - 404.6
Water cut (W_c)	0-0.5
Oil production (Q_o)	7 – 1848
Diameter of production line (D)	2.8750 - 5
Temperature (T)	27.2-77.2
Solubility ratio (R_s)	22.9 - 404.6
Volumetric oil formation factor (B_o)	1.1398 - 2.4360
Oil viscosity (μ_o)	0.160 - 77.20
Specific gravity of the gas (γ_g)	0.571-1.705
Pressure drop (ΔP)	144.4-320.2

Methodology

The relationship of the pressure drop to the variables that directly affect it, the fluid properties and well characteristics can be approximated by a polynomial model of the form: $\Delta P_{experimental} \approx p(d, l, GOR, Q_{oil}, P_{top}, \gamma_o, \gamma_g, \sigma, \rho_o, \rho_g, \mu_o, R_s, B_o, B_w)$ where p is an unknown polynomial function.

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In general, the relationship between the variables GOR, Q_{oil}, D, R_s, B_o, μ_o and the pressure drop ΔP can be approximated by a polynomial function of the form:

$$\Delta P_{experimental} \approx p(\mathrm{GOR, Qoil, D, Rs, Bo}, \mu_o) + \varepsilon, \quad (1)$$

Where ε is a random error. The reduction of the number of variables in the derivation of the polynomial model to be searched for is due to the fact that after performing an analysis of correlations between the independent variables $d, l, GOR, Q_{oil}, P_{top}, \gamma_o, \gamma_g, \sigma, \rho_o, \rho_g, \mu_o, R_s, B_o$ and the ΔP, it was found that the variables that affect the pressure drop the most areGOR, Qoil, D, Rs, Bo, μ_o.

PaIn order to determine the polynomial model described in (1), polynomials were made by combining the variables as shown in formula (2). This equation describes the combinations that are possible to make to obtain the polynomial.

$$\Delta P = \sum_{i=1}^{n} a_i x_i + \sum_{i,j=1}^{n} b_{i,j} x_i x_j + \sum_{i,j,k=1}^{n} c_{i,j,k} x_i x_j x_k + \ldots, \quad n = 6 \quad (2)$$

The criterion used in this work to select the best polynomial is the coefficient of determination. (R^2) between the proposed polynomial p and the experimental pressure drop $\Delta P_{experimental}$.

Results

In order to implement the methodology given in section 2, the database of the authors' research work (Chierici et al., 1974) was used with the difference that the number of variables mentioned in Table 1 is reduced to just Q_{oil}, GOR, d, R_s, B_o, μ_o y ΔP, This is because the combinations performed showed better results with these variables. Various combinations were made with the previously mentioned variables to find a simple polynomial in algebraic structure. After several algebraic calculations in which a polynomial model was propose d and the coefficient of determination between the pressure drop predicted by the polynomial model and the experimental pressure drop was calculated, the polynomial model selected because of its high coefficient of determination is

$$\begin{aligned}\Delta P \approx 538.8272 &+ 0.2411(\mathrm{GOR}) \\ &+ 0.0788(GOR)(\mu_o) \\ &+ 0.0434(Q_{oil}) \\ &- 6.5879x10^{-6}(Q_{oil}^2) \\ &+ 1.8547x10^{-9}(Q_{oil}^3) \\ &- 0.0109(Q_{oil})(d) \\ &- 2.9451(R_s) + 0.0177(R_s^2) \\ &- 3.2894x10^{-5}(R_s^3) \\ &- 321.1895(B_o^2) \\ &+ 131.8653(B_o^3) \\ &- 1.1807(\mu_o)\end{aligned} \quad (3)$$

The fit of the polynomial model (3) was expressed by the coefficient of determination R^2 which was 0.9901, indicating that 99.01 % of the variability of the pressure drop could be explained by this polynomial model. Figure 1 shows the linear relationship of the experimental pressure drop ($\Delta P_{experimental}$) y the one predicted by the polynomial model (p) with a coefficient of determination value of 0.995.

Box 2

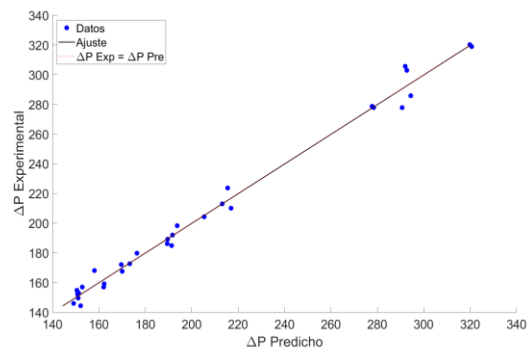

Figure 1

Regression of experimental and predicted pressure drop values using the polynomial model.

Source: Own elaboration.

Graphically in Figure 1 we can see that $\Delta P_{experimental} \approx p(\mathrm{GOR, Qoil, D, Rs, Bo}, \mu_o) + \varepsilon$,

On the other hand, let us recall that the variables involved in the polynomial model are: GOR, Q_o, D, R_s, B_o y μ_o, then to give a geometric idea of the behaviour of the polynomial p, the experimental (ΔP_Experimental) and predicted (ΔP_Pre) pressure drops are plotted, however, it should be noted that the polynomial has six different variables, Therefore, level surfaces are plotted for each variable, in the two-dimensional ones, one of the variables of the polynomial is varied and the others are left fixed (see Figure 2) and in the three-dimensional ones, two variables are varied and the others are left fixed (see Figure 3).

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As can be seen, there is good agreement between the values predicted by the polynomial model and the experimental data of pressure drops in oil wells.

Box 3

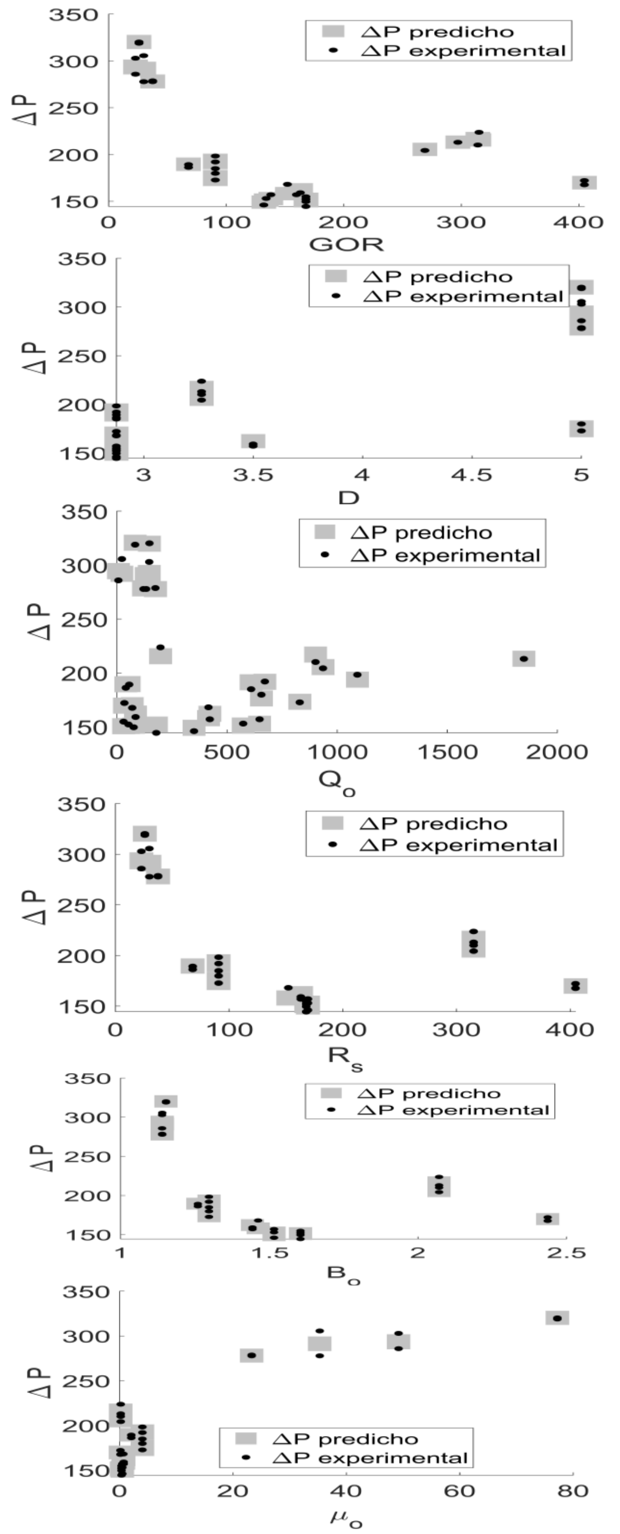

Figure 2

Contour lines. Comparison of experimental pressure drops $(\boldsymbol{\Delta P_{experimental}})$ **and the predicted by calculating the** $\boldsymbol{\Delta P_{predicha} = p(GOR, Qoil, D, Rs, Bo, \mu_o)}$ en function of one variable at a time and leaving the rest of the variables as constants..

Source: Own elaboration.

Box 4

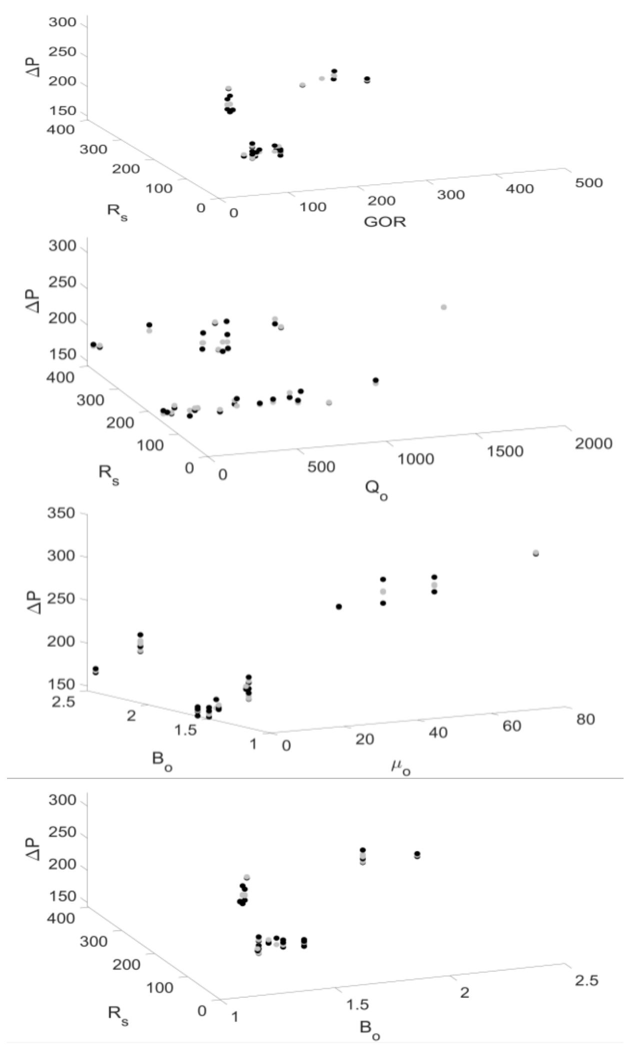

Figure 3

Level surfaces. Comparison of experimental and predicted pressure drops by calculating the ΔP as a function of two variables at a time and leaving all other variables as constants.

Source: Own elaboration

Residual analysis

Recall the existence of a random error ε in (1). To mathematically demonstrate the good polynomial fit of the given model (3), a residual analysis is performed to analyse the behaviour of this random error (Montgomery & Runger, 2003). To this end, note that taking the average value (E) in (1) we get $E(\Delta P_{Experimental}) = E(p(\text{GOR, Qo, D, Rs, Bo}, \mu_o) + \varepsilon)$

Which by the properties of the average value is the same as

$$\Delta P_{Experimental} = p(\text{GOR, Qo, D, Rs, Bo}, \mu_o) + E(\varepsilon) \quad (4)$$

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Note that, from (4), the experimental pressure drop and the polynomial $p($ GOR, Qo, D, Rs, Bo, μ_o $)$ se Therefore, we proceed to verify graphically that E(ε)=0. In order to carry out such a verification, note that from (1) we obtain that the residuals or random errors ε are expressed by the following difference

$$\varepsilon \approx \Delta P_{Experimental} - p(\text{GOR, Qo, D, Rs, Bo}, \mu_o). \quad (5)$$

In Figure 4 the residuals or random errors (ε) have been plotted and as can be seen there are values whose confidence intervals do not cross zero, these values are called outliers, in the graph there are 2 outliers marked in red, i.e. 93.54% of the residuals pass through 0. Finally, it is crucial to know if these outliers affect the proposed polynomial model (3), for this reason the 2 outliers were removed and the regression was redone without these points and a coefficient of determination of R2 = 0.9933 was obtained. The difference is 0.0032, this result infers that the outliers do not represent a significant amount for the polynomial model proposed in (3). On the other hand, the histograms of the standardised random residuals were performed, see Figure 5, you may notice from that figure the mean of the random errors is approximately zero, i.e. E(ε)≈0. The latter allows us to conclude that indeed on average the experimental pressure drop and the pressure drop predicted by the polynomial model (3) coincide because the average of the random errors is zero.

Box 5

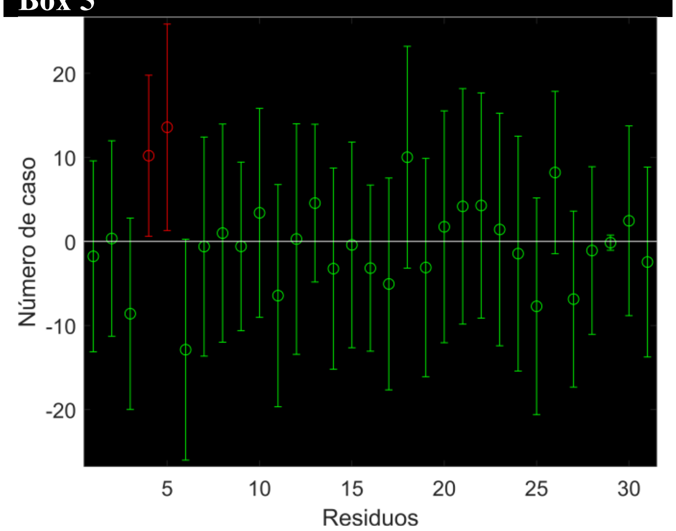

Figure 4

Confidence intervals of the residuals at the 95% confidence level

Source: Own elaboration

Box 6

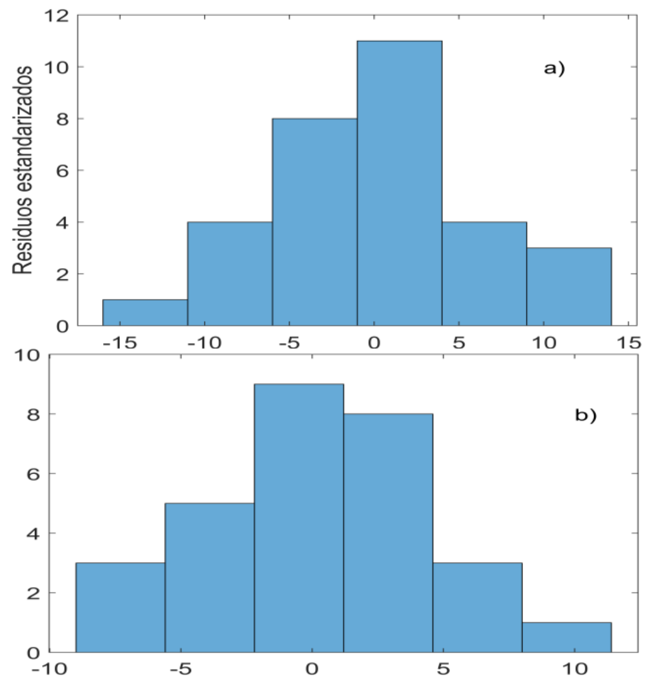

Figure 5

Confidence intervals of the residuals at the 95% confidence level.

Source: Own elaboration

Conclusions

The conclusions obtained through the development of this polynomial model were the following. Through the multiple linear regression analysis, a polynomial function of the variables related to the process of pressure drops in two-phase oil wells was successfully obtained.

The results obtained with the fit showed a high accuracy with the experimental data R2 > 0.99. In addition, a standardised residual analysis was recommended in addition to the coefficient of determination to check that the average random error value was zero to ensure a good fit of the polynomial model (3). As can be seen in the polynomial model, the variables with the greatest effect on the pressure drop are the gas-oil production ratio (GOR), solubility ratio (Rs) and volumetric oil formation factor (Bo) because they are the variables that directly influence the pressure drops by elevation. In two-phase wells, the phase distribution governs the behaviour of the pressure drop, which is why the production gas-oil ratio has a considerable impact on the polynomial model, as do the aforementioned fluid properties such as the solubility ratio (Rs) and volumetric oil formation factor (Bo), which provide information about the ratio of the oil production volume to its volume in the reservoir.

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With the above mentioned, it is concluded that more efforts are required to obtain reliable data on the gas-oil production ratio (GOR), solubility ratio (Rs) and volumetric oil formation factor (Bo) because they are the variables that most affect the pressure drop. This methodology can be used to simplify the complex calculations used to calculate pressure drops.

Declarations

Conflict of interest

The authors declare that they have no conflicts of interest. They have no known competing financial interests or personal relationships that might have appeared to influence the article reported in this paper.

Authors' contribution

The contribution of each researcher in each of the points developed in this research was defined based on:

Hernández-Santos, Abisai: He contributed to the idea of the project, carried out the data analysis and supported the writing of the article.

Escobedo-Trujillo, Beatris Adriana: Contributed to the idea of the project, method and research technique.

Alaffita-Hernández, Francisco Alejandro: Contributed to the idea of the project, developed the algorithm for obtaining all the figures shown in the work and carried out data analysis.

Colorado-Garrido, Dario: Contributed to the idea of the project, systematised the background for the state of the art, and reviewed the writing of the article.

Availability of data and materials

The database used in this work was obtained from Chierici (Chierici et al., 1974).

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Abbreviations

Not applicable

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Background

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Basics

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Support

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