Interface based on computational geometry to characterize the spatial structure of point patterns for industry tools

Interfaz basada en geometria computacional para caracterizar distribución de puntos espaciales en herramientas de uso industrial

BAUTISTA-ELIVAR, Nazario*†, AVILES-COYOLI, Katia and MARTÍNEZ-SOLÍS, Luis

Tecnológico Nacional de México/Instituto Tecnológico de Pachuca

ID 1st Author: *Nazario, Bautista-Eivar*/**ORC ID**: 0000-0002-9556-2748

ID 1st Coauthor: *Katia, Aviles-Coyoli*/**ORC ID**: 0000-0003-1619-4359 **CVU CONACYT ID:** 568518

ID 2nd Coauthor: *Luis, Martínez-Solís/***ORC ID**: 0000-0002-0917-714X, **CVU CONACYT ID**: 929575

Received July 18, 2018; Accepted November 14, 2018

Abstract

Given a spatial point pattern, we propose a graphical interface is developed to analyze the spatial and angular characteristics based on Voronoi polygons and Delanuay triangles to extract information of the spatial point distribution of industrial tools, using Voronoi polygon and Delanuay triangulation (IVDI). This interface generate TXT files frequencies on Voronoi polygons, it generate graph which connects neighbors points in each Voronoi polygon selected by the user (Ulam tree modified) and it calculate the distances to each neighbor point and the internal angles. The interface obtain measures internal angles of Delanuay triangles, obtain measures neighbors nearest distance throughout the tesselation, obtain measures distance between selected points by the user, calculates the average of mean distances between points, evaluate polygonality and polygonality index, average angular differences, variation index of angle differences, mean-square deviation of angles, and circle radius circumscribing each Delanuay triangle.

Voronoi polygon, Delanuay trinangulation, Industries tools

Resumen

Dado un patrón de puntos espaciales, proponemos una interfaz gráfica para analizar las características espaciales y angulares basadas en polígonos de Voronoi y triángulos de Delanuay (IVDI) para extraer información de la distribución de puntos espaciales en herramientas de uso industrial, empleando polígonos de Voronoi y triangulación Delanuay. Esta interfaz genera archivos TXT de frecuencias de polígonos Voronoi, genera gráficos que conecta puntos vecinos en cada polígono Voronoi seleccionado por el usuario (árbol Ulam modificado) y calcula las distancias a cada punto vecino y grafico de ángulos internos. Mide los ángulos internos de los triángulos de Delanuay, mide la distancia más cercana a lo largo de la distribución de puntos, mide la distancia entre los puntos seleccionados por el usuario, calcula el promedio de las distancias medias, evalúa el índice de poligonalidad, media de diferencias angulares, índice de variación de las diferencias angulares, desviación cuadrática media de ángulos y radio del círculo que circunscribe cada triángulo de Delanuay.

Polígonos de Voronoi, Triangulación Delaunay, Diseño de herramienta industrial

Citation: BAUTISTA-ELIVAR, Nazario, AVILES-COYOLI, Katia and MARTÍNEZ-SOLÍ.S, Luis. Interface based on computational geometry to characterize the spatial structure of point patterns for industry tools. ECORFAN Journal-Democratic Republic of Congo 2018, 4-7: 1-7

* Correspondence to Author (email: nazariobel@itpachuca.edu.mx)
† Researcher contributing first author.

Introdution

Many applications in science, engineering, statistics, and mathematics require structures Voronoi diagrams, and Delaunay tessellations (J. Sudbø, 2000). These functions enable to geometrically analyze data sets in any dimension. Voronoi diagrams are a closest-point plotting technique related to Delaunay triangulation , which is useful by itself to create a triangular grid for scattered data points. Voronoi diagrams define a “natural mesh” for scattered data without explicit point connectivity (Luciano da Fontoura Costa, 2006, NoppadonKhiripe, 2012). Polygon mesh models are typically defined in terms of (a) geometry-the coordinate values of vertices of the meshes making up the model, (b) connectivity-the relationship among the vertices which defines the polygonal faces of the mesh (Iosif Pinelis, 2007). An interface with technology based in Voronoi polygons and Delaunay triangles is presented here to improve spaces setting industrial machines tools (IVDI). Given points grouped as a cloud, represent the tools, this interface is able to develop several graphics of proximity respect to the group of points already shown, this interface allows to obtain display and store from the optimization diagrams through the breaking down from the map in sections of tools.

Numerous researchers in various fields concern themselves with characterizing spatial distributions of objects. In these research fields, the point process theory undoubtedly helps dealing with these questions. Exploratory statistics of point patterns widely rely on Ripley’s seminal work (Ripley 1976, 1977), namely the K function. A recent review of similar methods is given by Marcon and Puech (2014) who called them distance-based measures of spatial concentration. We will refer to them here as spatial structures since both dispersion and concentration points can be characterized.

The basic purpose of our interface (IVDI) is to test the observed point pattern against complete spatial randomness (CSR), i.e., a homogeneous Poisson process, to detect dependence between point locations (the null hypothesis supposes independent points) assuming homogeneity (i.e., the probability to find a point is the same everywhere).

Similar Ripley-like functions, available in the proposed R (R Core Team 2015) *dbmss* package (Marcon, Lang, Traissac, and Puech 2015), can be classified in three families:

- Topographic measures such as K take space as their reference (Fortin and Dale 2005).

- Relative measuressuchas M (Marcon and Puech 2010) compare the structure of a point type to that of another type (they can be considered as cases and controls).

- Absolute functions such as *Kd* (Duranton and Overman 2005) have no reference at all but their value can be compared to the appropriate null hypothesis to test it.

Methodology

Statistical background

We consider a map of points which often represents sections of tools. We want to apply to this point pattern a variety of exploratory statistics which are functions of distance between points and to test the null hypothesis of independence between point locations. These functions are either topographic, absolute or relative.

They can be interpreted as the ratio between the observed number of neighbors and the expected number of neighbors if points were located independently from each other. If reference and neighbor points are of the same type, the functions are univariate and allow to study concentration or dispersion.

Relative functions

Choosing reference and neighbor point types allows defining univariate or bivariate functions, counting neighbors up to or at a distance defines cumulative, using neighbors points of polygon Voronoi and Delanuay triangulation. Reference points are denoted x_i, neighbor points are x_j. For density functions, neighbors of x_i, are counted at a chosen distance r:

$$n(x_i, r) = \sum_{j,i\neq j} k\left(\left\|x_i - x_j\right\|, r\right) c(i,j) \qquad (1)$$

$k(\|x_i - x_j\|, r)$, is a kernel estimator, necessary to evaluate the number of neighbors at distance r, $c(i,j)$ is an edge-effect correction (points located close to boundaries have less neighbors because of the lack of knowledge outside the observation window). In cumulative functions such as M, neighbors are counted up to r:

$$n(x_i, r) = \sum_{x_j \in N, i \neq} 1(\|x_i - x_j\| \leq r) w(x_j) \quad (2)$$

The number of neighbors is then averaged n is the number of reference points:

$$\bar{n}(r) = \frac{1}{n} \sum_{i=1}^{n} n(x_i, r) \quad (3)$$

Software description

Using $\bar{n}(r)$, we suggest development of an integrated platform based on computational geometry to study concentration or dispersion of points on tools, which in turn use both Voronoi tesselation and Delaunay triangulation, for the purpose to measure the distance, internal angles, radius of circumscribed circle, amid nearby points mean distances average, angular polygonality, polygonality index, mean-square deviation of angles, and variation index angle of differences. The platform holds two options, either being performed by a user or operating with an automatic formulations. This software allows to create Voronoi polygons and Delaunay triangles from a set of XY coordinates, or generated by selecting in an imported image. It locates XY coordinates, using an auxiliary window S, Fig**.** 1.

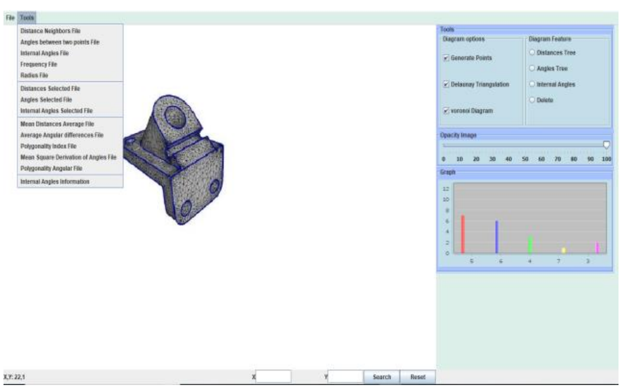

Figure 1 Platform based on computational geometry using Voronoi polygons and Delaunay triangles (IVDI).
Source: self made

Voronoi Frequency

This function displays graphics of frequency respecting to the number of sides in Voronoi mosaic with a data reading window, wider enough to avoid loss of data of polygons compared to other platforms (Image J).

Circumscribed Circle

These metrics/algorithms are able to find the magnitude of the circumradius, the coordinates of the center of the circumcircle, and the coordinates of the vertices formed in each Delaunay triangle. R is the radius of the circle circumscribing a Delaunay triangle, Fig. 2.

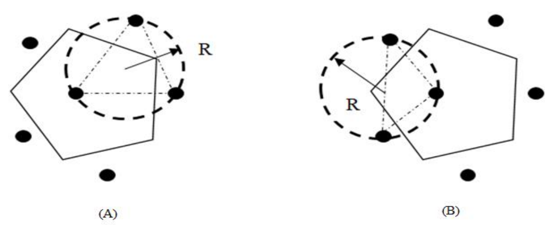

Figure 2 Circuncircle and circumradius for each Delanuay triangle in a Voronoi polygon
Source: self made

Distances Selected

These metrics/algorithms are able to get the distance (d_k) between each pair of points selected by the user. First select the icon distances, located in Diagram Feature screen, after that choose a point of the polygon, by the icon select points, then select again the icon select point for another interest point, finally activate the icon selected file distances from Tools menu to get the data file with its coordinates and the distance that separates them. You might select different pairs of points to find out their distances in a single file, activating the icon selected points, Fig. 3.

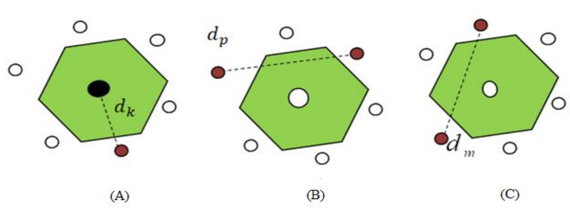

Figure 3 Distances to be chosen by the user in a Voronoi polygon
Source: self made

Article **ECORFAN Journal-Democratic Republic of Congo**
December 2018 Vol.4 No.7 1-7

Angles Selected

These metrics/algorithms generate a file formed from each pair of points selected by the user. The angle υ is relative to the horizontal axis and it works as the rangefinder. First, activating angles tree icon (Fig.1), and then you can select different points for the same file using the select point button, to generate the Angles Selected file, Fig. 4.

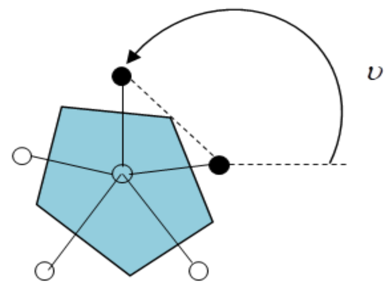

Figure 4 Selection of angles between neighbors to be chosen by the user in a Voronoi polygon
Source: self made

Internal Angles Selected

This metric algorithm generates a file for the angles γ_i of each Delaunay triangle selected by the user. First, activate radio button internal angles (Fig.5), then select points radio button to form the Delaunay triangle, selecting 3 points. The select order of each point defines the angle to be measured. First, if the black point is selected, then you can choose the white point, these two points form a line from which we start measuring the angle and ends at the line formed between the third point, striped circle, forming an angle which is measured from counterclockwise to clockwise, Fig. 5.

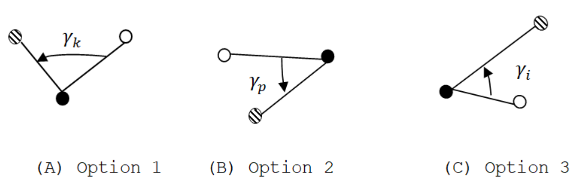

Figure 5 Select nearby points to get an angle between them in a Voronoi mosaic
Source: self made

Description of metrics/algorithm

Internal distance

The internal distance d_i in a particular Voronoi cell, is the distance measured from internal mathematical node (black dot, see Fig. 6) of the Voronoi polygon to each neighboring point (blue dot, see Fig.6) which forms the polygon. This is an Ulam tree modified to measure distances, Fig.6.

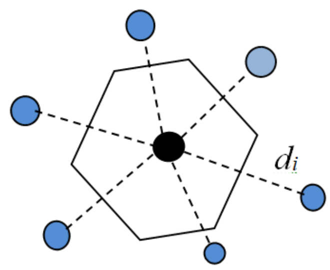

Figure 6 Distance graph (Ulam tree modified) between neighbors in a 6-tides Voronoi polygon
Source: self made

Angular graph

The numbers are the angles between the horizontal line which contains the internal mathematical node and the line which joins each neighboring point to the internal node, it is measured anticlockwise (Ulam tree modified to measure angles), Fig. 7.

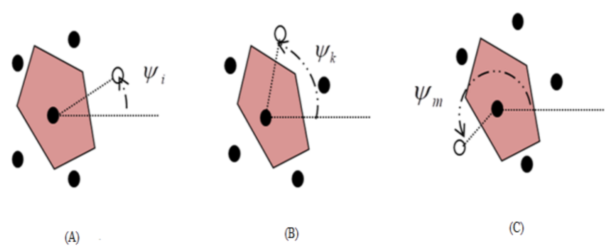

Figure 7 Measure angles between nearby points from a horizontal axis
Source: self made

Internal angles of Delaunay triangles

These are the internal angles of any triangle in the Delaunay triangulation ω_i ; the angles are measured in the positive direction (counterclockwise), Fig. 8.

ISSN-On line: 2414-4924

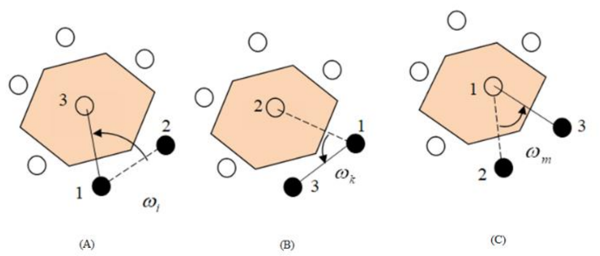

Figure 8 Measure angles between 3 neighbors that form a Voronoi polygon and choose one angle
Source: self made

The sequence to measure the angle ω_i is in the following order: if you select first point 1 (vertex 1) then select point 2 (vertex 2), these points will form a starting line where the angle measurement starts, the point 3 (vertex 3) is where the angle measured ends. The selection and sequence of generating points will indicate the final angle obtained.

Mean distances average

This metric/algorithm determine the mean of average distances from the inner point in every Voronoi cell to its n neighbors and it is calculated by $\sum_{i=1}^{n} \frac{d_i}{n^2}$, where d_i is the internal distance defined above. It also represents a way to measure the cell size and it could be interpreted as a coefficient of expansion or contraction, Fig. 9.

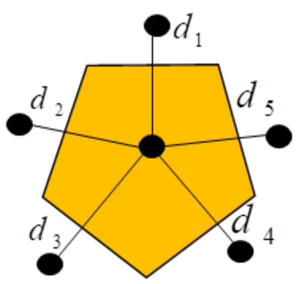

Figure 9 Modified Ulam tree graph for distances in each Voronoi cell
Source: self made

Polygonality Index

This metric/algorithm generates the measure of polygonality index, Eq. (4).

$$\Xi = \frac{1}{\sum_{i=1}^{n} |\chi_i - \beta| + 1}, \qquad (4)$$

Where χ_i is the formed angle between consecutive neighbors for Delaunay triangle (irregular polygon, dotted arrow), β is the angle between consecutive vertex for a regular polygon (solid arrow), $\beta = 360^0/n$ and n is the number of neighbors from Voronoi cell, Fig. 10. Due to the way that χ_i is measured, its measurement is invariant under any rotation movement. The measurement is performed counterclockwise.

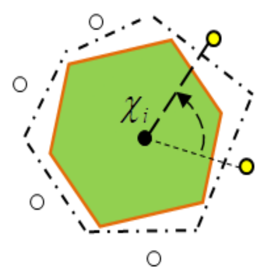

Figure 10 The solid line is a regular polygon, the irregular polygon is represented by the dotted line, and the neighbors are the black dots. The internal dot is a mathematical node
Source: self made

Mean-Square deviation of angles

This metrics/algorithms evaluate the mean-square deviation of angles, Eq. (5),

$$\varepsilon = \sqrt{\sum_{i=1}^{n} (\chi_i - \beta)^2}, \qquad (5)$$

The root square of mean deviation from the angles χ_i, with respect to angle $\beta = 360^0/n$ where n is the number of neighbors of the Voronoi cell for each Voronoi polygon. The magnitudes χ_i , β , and n are defined above. Again, this measurement is invariant under any rotation movement.

Variation index angle of differences

This metric algorithm gets the Variation index angle of differences, Eq. (6),

$$\delta = 1 \Big/ \sqrt{\sum_{i=1}^{n} (\chi_i - \beta)^2} + 1, \qquad (6)$$

Where χ_i , β and $\beta = 360^0/n$ are defined as above. If the value of δ is close to 1, then the Voronoi polygon is close to regularity, if δ is close to 0, then Voronoi polygon is irregular.

Results

Using the interface IVDI, we calculate $\bar{n}(r)$ by Eq. (3), for a metal gear, figure 11.

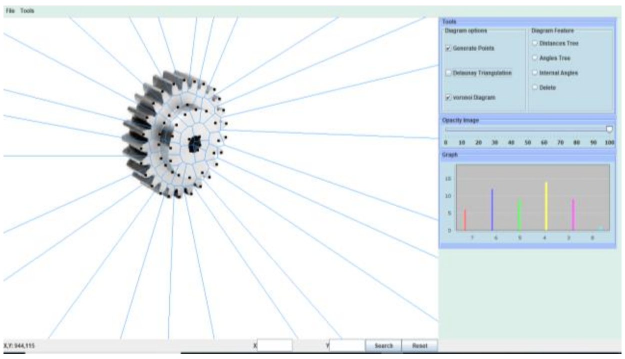

Figure 11 A metal gear
Source: self made

The n statistic is the sum of contribution of $\bar{n}(r)$ for all r values. $\bar{n}(r)$ are made independent by construction a metal gear. We want to evaluate the number of neighbors $\bar{n}(r)$ at distance r, for simulation, Ripley-like function and IVDI, figure 12.

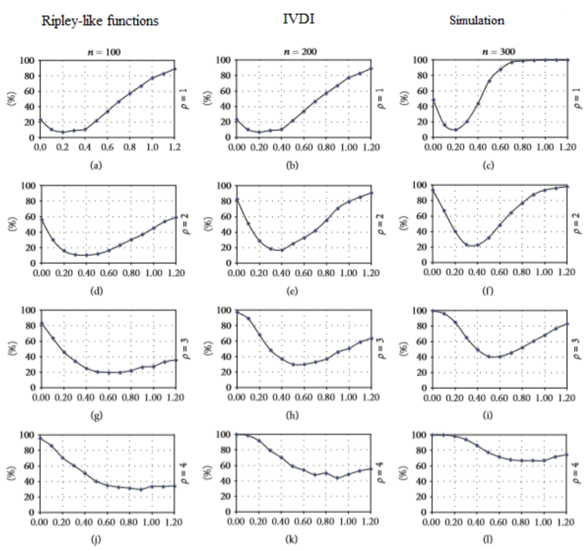

Figure 12 Several iterations to calculate % $\bar{n}(r)$, n is the number of simulated points for a metal gear, ρ aparameter of aggregation, X axis in cm
Source: self made

Taking into account distances above the maximun range of interaction between points limits the power of the test since a fraction of the $\bar{n}(r)$ values are purely stochastic. This is a normal behavior for a goodness-of-fit test. Ripley-like function test applied to a metal gear have a fit equal to 23.5%. In the same conditions, using IVDI, our test returns a fit value equal to 69.3%; it appears to close simulation of $\bar{n}(r)$ introduced in the analysis.

Conclusions

The platform is a tool based on computational geometry for getting structural manifestations and quantification of geometric distribution to characterize the spatial structure of point patterns for tools, based on Voronoi polygon and its dual Delanuay tesselation (IVDI). We built this package to provide an easy-to-use toolbox for users of spatial statistics mainly in tools employed on industries. Our results have goodness-fit that Ripley-like functions. The analysis is limited to testing a point pattern against an appropriate null hypothesis, including the simulation of many point processes as alternate null hypotheses and model fitting beyond exploratory statistics.

Future developments include the use of distance matrices as input of the distance-based functions to allow addressing of the simulation of many point processes. We will also develop subsampling techniques to be able to manage huge datasets (several million points) whose distances cannot all be calculated in a reasonable time.

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