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Global variable-structure controller applied to *l* degree of freedmon manipulators robots with rotational flexible joint

Controlador global de estructura variable para un robot manipulador de *l* grados de libertad con articulaciones rotacionales y flexible

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Abstract

In this work is proposed a methodology for a global variable-structure controller (GVSC) applied to nonlinear, time-varying and underactuated systems affected by both matched and unmatched perturbations, the main idea is designed a GVSC with an integral sliding mode control coupled together with a nonlinear $\mathcal{H}_\infty$ control. It is theoretically proven that, using the proposed controller, the trajectories of the states in the feedback loop systems are forced to stay into the sliding mode and reject the coupled perturbations by the integral sliding mode control, and the stability of feedback loop system into the switching mode and the attenuated uncoupled perturbations are done by nonlinear $\mathcal{H}_\infty$ control. This structure is used to solve the trajectory tracking problem in the *l* degrees of freedom (DOF) manipulators robots with flexible and rotational joints. The performance issues of the GVSC are illustrated in simulation studies made for a three-DOF robot manipulator.

Robust control, Nonlinear systems, Manipulator robots

Resumen

Se propone la metodología del diseño de un controlador global de estructura variable (CGEV) compuesto de un control por modo deslizante integral en combinación de un control $\mathcal{H}_\infty$ no lineal para sistemas no lineales, subactuados y variantes en el tiempo afectado por perturbaciones acopladas y no acopladas. La finalidad de la estructura propuesta es que el control por modo deslizante integral mantenga las trayectorias de los estados del sistema en lazo cerrado dentro del modo deslizante y rechace las perturbaciones acopladas y el control $\mathcal{H}_\infty$ no lineal garantice la estabilidad del sistema en lazo cerrado dentro del modo deslizante y atenué las perturbaciones no acopladas. La validación de la estructura de control propuesta se realiza a través de la simulación de un ejemplo numérico que resuelve el problema de regulación de movimiento en un robot manipulador de *l* grados de libertad (GDL), con articulaciones rotacionales que presenta el efecto de elasticidad.

Control robusto, Sistemas no lineales, Robots manipuladores

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Introduction

This paper deals with the analysis and design of a global control of variable structure (CGEV) based on the control by integral sliding mode in combination with the control H_∞ for nonlinear systems varying in time affected by both coupled and uncoupled perturbations . As a case study will solve the problem of regulation of movement in a robot manipulator of degrees of freedom with rotational joints that have the undesired effect of elasticity in each of them, as well as the effect of the mentioned disturbances.

Regarding this line is the work of Utkin, (1992) with the control by sliding mode that has been used successfully when there is uncertainty in the parameters and to reject coupled external disturbances, besides concluding stability in finite time, the stability of the system In closed loop it is only guaranteed when its dynamic enters the sliding mode, however, the drawback of this controller is the effect of chattering in the control signal.

The control by integral sliding mode appears in the decade of the 90's (Utkin, Guldner and Shi, 2009) present the advantages of the control by sliding mode, and solve the problem of the reach to the surface with a function that guarantees that the dynamics of the system in closed loop initiate and stay in it, in addition to reducing the effect of chattering on the control signal due to the integrator.

In Wen and Jian, (2001) an integral sliding mode control is proposed in combination with an optimal quadratic linear regulator for nonlinear systems that vary in time in the presence of uncoupled perturbations and its main contribution was to propose the variant sliding function in the time and show that the uncoupled disturbances remain in the sliding mode for all time.

In Castaños and Fridman, (2009) continue with the previous work, providing new elements such as separating the disturbances into coupled and uncoupled and propose a controller H_∞ locally for the nominal control in order to attenuate the uncoupled disturbances.

In Rubagotti, Castaño, Ferrara and Fridman, (2011) and Fridman, Barbo, Plestan, (2016) and Galvan and Fridman (2015) extend the last two works by proposing a sliding function where the projection matrix varies according to the states of the system and recently is the work of Miranda, Chavez and Aguilar, (2017) in it proposes a variable structure control composed of a sliding mode control and a non-linear H_∞ control for a mechanical system of a rotational articulation with the elasticity effect. Based on the previous work we propose a global variable structure controller that solves the problem of regulation of movement of the manipulator arm of l degrees of freedom and also attenuated the effects of coupled and uncoupled disturbances.

The present article is organized as follows: section II provides the characteristics of the plant and the global control of variable structure analyze, section II presents the design of the CGEV in general form for a non-linear system variant with the time and not autonomous, section III is designed the CGEV for the case of a robot manipulator of degrees of freedom with rotational and flexible articulations that present coupled and uncoupled perturbations and validates the theory with a practical case of a mechanical system of a degree of freedom in MatLab / Simulink, finally the results obtained and the conclusions are presented.

1. Problem Statement

Consider the following nonlinear and variant time system of the form:

$$\dot{x} = f(x,t) + g_2(x,t)(u + w_m) + g_2^{\perp}(x,t)w_u \quad (1)$$

where $t \in \mathbb{R}^+$ represents the time, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w_m(t) \in \mathbb{R}^m$, $w_u(t) \in \mathbb{R}^{nm}$ are coupled and uncoupled perturbations respectively, uncoupled perturbations are assumed to belong to the space $L_2^e(0, T)$ where (Khalil, 2015)

$$\mathcal{L}_2^e(0,T) = \int_0^T \|w_u(t)\|^2 dt < \infty$$

and considering that both disturbances are bounded by:

$$\|w_m(t)\|_\infty \leq W_m^+,\ \|w_u(t)\|_\infty \leq W_u^+ \quad (2)$$

W_m^+ and W_u^+ they are constants that are known a priori. The functions $f\ (x,\ t)$ and g_2 $(x,\ t)$ are assumed to be continuous to sections in t for all x and continuously differentiable in x for all t. Sea la función matricial $g_2^{\perp}(x,t) \in \mathbb{R}^{n\times(n-m)}$ of full column range that is the orthogonal complement of g_2 (x, t) such that $g_2^T(x,t)g_2^{\perp}(x,t) = 0$ for all x and all t (Fridman, 2014). The system (1) satisfies the following assumption.

Assumption 1. Be the rank$(g_2(x,t)) = m$ for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}^+$.

If the above assumption is fulfilled then the law of global control of variable structure for the system (1) of the form is proposed:

$$u(t) = u_s(t) + u_1(t) \tag{3}$$

where u_s (t) is the control in the sliding mode responsible for the trajectories converging to the origin while the uncoupled perturbations w_u (t) are attenuated and u_1 (t) is the control by integral sliding mode which incorporates an integrator in the control discontinuous and its function is to reject the coupled perturbations w_m (t) and to prevent the system trajectories from leaving the sliding mode.

2. Design of the CGEV

The design of the CGEV is developed in two stages, the first consists of the design of the control by integral sliding mode and the second of the controller H_∞ non-linear.

2.1. Control design by integral sliding mode

Consider the following sliding surface for the disturbed system (1) of the form

$$s(x,t) = D\left((x(t) - x(t_0) - \int_{t_0}^{t}(f(x_s,t) + g_2(x_s,t)u_s)dt\right) \tag{4}$$

where $D \in \mathbb{R}^{m\times n}$ it is a constant matrix and $x_s(t) \in \mathbb{R}^n$ it is the vector of states in the sliding mode. The sliding function s (x, t) represents the difference between the trajectories of the disturbed system (1) and of the plant in the sliding mode weighted by the matrix D. It is notable to note that the sliding mode starts with the initial condition, that is, $t = t_0, s(x,t) = 0$. It is assumed that the system (1) satisfies the following assumption.

Assumption 2. Let Dg_2 (x, t) be uniformly invertible for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}^+$.

If assumption 2 is satisfied, then the control by integral sliding mode is of the form

$$u_1(t) = -\Gamma\, Sign(s(x,t)) \tag{5}$$

where $Sign(s(x,t)) = [sign(s_1), sign(s_2), \cdots, sign(s_m)\,]^T$, and the sign function is defined in the way:

$$sign(s) = \begin{cases} 1 & yes \quad s > 0 \\ 0 & yes \quad s = 0 \\ -1 & yes \quad s < 0 \end{cases}$$

Regarding the control gain by integral sliding mode, it is subject to:

$$\Gamma > W_m^+ + \frac{\|g_2^{\perp}(x,t)\|}{\|g_2(x,t)\|} W_u^+ \tag{6}$$

where $\|\cdot\|$ refers to the Euclidean norm. The gain (6) is to force the trajectories of the system (1) not to leave the sliding surface $s(x,t) = 0$.

2.1.1. Equivalent control

The analysis of the equivalent control u_{1eq} (Utkin, 1992) is obtained by making $s(x,t) = \dot{s}(x,t) = 0$, that is

$$\dot{s} = Dg_2(x,t)(u_1 + w_m) + Dg_2^{\perp}(x,t)w_u = 0$$

Clear $u_1(t)$ and make $u_1 \to u_{1eq}$ in the previous equation you have to

$$u_{1eq} = -w_m - \left(Dg_2(x,t)\right)^{-1} Dg_2^{\perp}(x,t)w_u.$$

Substituting the equation (7) in (1) the following system is obtained in the sliding mode:

$$\dot{x}_s = f(x_s,t) + g_2(x_s,t)u_s + \Big(I - g_2(x_s,t)\left(Dg_2(x_s,t)\right)^{-1}D\Big)\, g_2^{\perp}(x_s,t)w_u, \tag{8}$$

where $\dot{x}_s \in \mathcal{S} = \{x \in \mathbb{R}^n : s(x,t) = 0\}$. Note that the uncoupled disturbances remain in the plant in the sliding mode which motivates the use of the non-linear H_∞ control to attenuate its effects.

2.2. Control design H∞ non-linear

Once the trajectories are in the domain of $\mathcal{S}$, then the system (8) takes the form

$$\dot{x}_s = f(x_s,t) + g_2(x_s,t)u_s + \underbrace{\left(I - g_2(x_s,t)\left(Dg_2(x_s,t)\right)^{-1}D\right)g_2^{\perp}(x_s,t)}_{g_3(x_s,t)}w_u(t) \qquad (9)$$

$$z = h_1(x_s,t) + k_{12}(x_s,t)u_s$$

$$y = h_2(x_s,t)$$

where $z(t) \in \mathbb{R}^s$ is the vector of the unknown output to be controlled, $y(t) \in \mathbb{R}^n$ it is the output vector available for system measurement. It is assumed that the system (9) satisfies the following assumptions:

Assumption 3. Functions f, g_2, g_3, h_1, h_2 and k_{12} are assumed to be continuous at t, continuously differential at x_s and of appropriate dimensions.

Assumption 4. Let $f(0,t) = 0, h_1(0,t) = 0$, y $h_2(0,t) = 0$ for all ≥ 0 .

Assumption 5. Let $h_1^T k_{12} = 0$, $k_{12}^T k_{12} = I$ se must satisfy for all $(x_s,t) \in \mathbb{R}^n \times \mathbb{R}^+$.

Assumption 3 ensures that the dynamics of the system are well positioned, while the system is excited with external inputs. Assumption 4 ensures that the origin is the only equilibrium point in the absence of inputs u_s (t) = 0 and disturbances w_u (t) = 0 for the dynamic system (9). Assumption 5 is related to numerical advantages considered in the H∞ standard control problem (Orlov, 2014).

The law of control

$$u_s(t) = \mathrm{K}(x_s,t) \qquad (10)$$

It is a globally admissible driver by feedback of states if the closed loop system (9) and (10) is asymptotically stable globally as long as $w_u = 0$. The gain $\mathcal{L}_2$ of the system (9) is less than γ if the response of z (t), result of w_u (t) for a vector of initial states x_s (0) = 0 satisfies the following inequality

$$\int_{t_0}^{t_1} \|z(t)\|^2 dt < \gamma^2 \int_{t_0}^{t_1} \|w_u(t)\|^2 dt \qquad (11)$$

For all $t_1 > t_0$ and every function continues in sections $w_u(t)$. A permissible local controller (10) constitutes a local solution to the control problem H∞ if there is a region U around the equilibrium point such that the inequality (11) is satisfied for all $t_1 > t_0$ and a continuous function with stretches w_u (t) for which the trajectories of the states of the closed loop system start at the point given by the vector x_s (0) = 0 and remain in U for all $t \in [t_0, t_1]$ (Isidori y Astol, 1992).

Next, the hypothesis under which the solution to the problem of control is given is presented $\mathcal{H}_\infty$.

Hypothesis 1 (Orlov, 2014). There is a positive definite function F (x_s, t) and a soft definite positive function V (x_s, t) such that the inequality of Hamilton-Jacobi-Isaacs

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_s} f + \gamma^2 \alpha_1^T \alpha_1 - \alpha_2^T \alpha_2 + h_1^T h_1 + F \leq 0 \qquad (12)$$

it is fulfilled with

$$\alpha_1(x_s,t) = \frac{1}{2\gamma^2} g_3^T(x_s,t) \left(\frac{\partial V(x_s,t)}{\partial x_s}\right)^T$$
$$\alpha_2(x_s,t) = -\frac{1}{2} g_2^T(x_s,t) \left(\frac{\partial V(x_s,t)}{\partial x_s}\right)^T.$$

Given hypothesis 1 the following theorem is postulated:

Theorem 1 (Orlov, 2014). Assume that hypothesis 1 is valid. Then a solution to the control problem H∞ is given for the closed loop system (9) through the law of control by feedback of states

$$u_s(t) = \alpha_2(x_s,t) \qquad (13)$$

which stabilizes asymptotically the system free of disturbances (9) and makes the gain L_2 of the system in the sliding mode (9) is smaller than γ.

Figure 1 shows the block diagram of a feedback system based on the CGEV composed of the integral sliding mode control and the non-linear H∞ control applied to a non-linear and non-autonomous plant with coupled and uncoupled perturbations, where $x_d(t) \in \mathbb{R}^n$ is the vector of desired states.

2.2.1. System stability analysis

The stability of the system in closed loop (1), (3) is demonstrated in the theorem of Miranda, Chavez and Aguilar, (2017), which raises the following: be the assumptions 3-5 and hypothesis 1 satisfied and be the system non-linear (1) with the control (5) that satisfies (9) next to the control H_∞ (13). Then the equilibrium point of the closed loop system (1), (5) and (13) is asymptotically stable and the gain L_2 of the system in the sliding mode is less than γ in the presence of disturbances that satisfy (2).

I. Case study: Track tracking problem for a robot with flexible joints

The elasticity effect is very common in manipulator robots, this phenomenon occurs when the movement of the actuator is transmitted to the articulation by means of toothed belts, chains, cables, use of gears, cyclo reducers, Harmonic Drive reducers, etc. all these elements introduce a variation in displacement with respect to time, that is why it is necessary, according to (Rivin, 1985), to incorporate the elasticity effect in the dynamic model of the robot, coupled with this phenomenon we have the uncoupled perturbations that are present as external disturbances, which affects the performance of the system in closed loop.

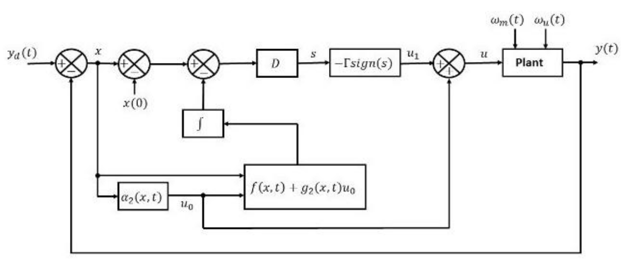

Figure 1 Global control of variable structure
Source: Self Made

The system under study, governed by the following differential equations (Spong and Vidyasagar, 1989):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + K(q-\theta) + w_u = 0 \quad (14)$$

$$J\ddot{\theta} - K(q-\theta) = \tau + w_m$$

Where $q(t) \in \mathbb{R}^l$ is the joint position vector, $\dot{q}(t) \in \mathbb{R}^l$ is the joint velocity vector, $\theta(t) \in \mathbb{R}^l$ is the position vector of the actuator, and due to the elasticity effect $q(t) \neq \theta(t)$ while the mechanism is in motion, $M(q) \in \mathbb{R}^{l\times l}$ is a positive definite symmetric matrix that represents the inertial matrix, $C(q,\dot{q}) \in \mathbb{R}^{l\times l}$ is the matrix of centrifugal and Coriolis forces, $g(q) \in \mathbb{R}^l$ is the vector of gravitational torque, $K \in \mathbb{R}^{l\times l}$ is a positive definite symmetric matrix that represents the torsional constants of flexible joints $(K = diag\{k_1, k_2, \cdots, k_l\})$, $J \in \mathbb{R}^{l\times l}$ is a positive definite symmetric matrix that contains in its main diagonal the product of the moments of inertia of the engine (j_i) by the square of the ratio of turns of the transmission (r_i), that is to say $J = diag\{j_1 r_1^2, j_2 r_2^2, \cdots, j_l r_l^2\}$, $\tau(t) \in \mathbb{R}^l$ is the vector of forces and pairs applied to the joints and $w_u(x,t), w_m(t) \in \mathbb{R}^l$ They denote uncoupled and coupled disturbances respectively due to the uncertainty of the model or external disturbances. Equation (14) represents a mechanical system with flexible joints subject to coupled and uncoupled disturbances.

1.1. Control objective

The trajectory tracking problem of the manipulator robot with flexible joints (14) is established in the following way: given a limited set of functions $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^l$ referred to as desired joint positions, velocities and accelerations, respectively; The objective of control is to ensure the global asymptotic stability of the equilibrium point of the closed loop system, ie

$$\lim_{t\to\infty} \|q(t) - q_d(t)\| = 0 \quad (15)$$

for any arbitrary initial condition $q(0) \in \mathbb{R}^l$, and for the disturbance-free system (14), finally, the gain L_2 of the disturbed system in the sliding mode is satisfied is less than γ with respect to the output $z(t)$.

1.2. Design of the CGEV for the manipulator robot

The representation in state variables of the system (14) in terms of errors is given by

$$\dot{e}_1 = e_2 \quad (16)$$

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$$\dot{e}_2 = M(e_1+q_d)^{-1}[M(e_1+q_d)\ddot{q}_d - C(e_1+q_d, e_2+\dot{q}_d)(e_2+\dot{q}_d) - g(e_1+q_d) - K(e_1+q_d-\theta) - w_u]$$
$$\dot{e}_3 = e_4$$
$$\dot{e}_4 = J^{-1}\left[-J\ddot{\theta}_d + K(e_1+q_d-\theta) + \tau + w_m\right]$$

where $e_1(t) = q(t) - q_d(t)$ is the error vector of joint positions, $e_2(t) = \dot{q}(t) - \dot{q}_d(t)$ is the error vector of joint velocities, $e_3(t) = \theta(t) - \theta_d(t)$ is the vector of errors positions of the actuator and finally $e_4(t) = \dot{\theta}(t) - \dot{\theta}_d(t)$ is the actuator speed error vector.

Decoupling the states of the system (16) facilitates the synthesis of the control H_∞, therefore, a virtual control entry is proposed according to the regular form of Utkin, (1992) within the state $\theta(t) = \eta(t) + \theta_d(t)$, and you get the solution based on the virtual control η de $\dot{e} = 0$, as:

$$\eta(t) = K^{-1}[M(q_d)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)] + q_d - \theta_d - K_{p1}e_1 - K_{d1}e_2 \qquad (17)$$

The earnings $K_{p1} = K_{p1}^T > 0$ y $K_{d1} = K_{d1}^T > 0$ they assure that the states $(e_1, e_2)^T \rightarrow 0$ when $t \rightarrow \infty$. Finally, a coordinate change is made with the function $\sigma = e_3 - \eta$ which is derived continuously until the control input τ of the form appears $\ddot{\sigma} = \dot{e}_4 - \ddot{\eta}$ and doing what $\dot{e}_4 - \ddot{\eta} = 0$ is obtained

Proposition 3. Shape stabilizer control:

$$\tau(t) = J\left(\ddot{\eta} + \ddot{\theta}_d\right) - K(x_1 + q_d - \eta - \theta_d) - K_{p2}x_3 - K_{d2}x_4 + u \qquad (18)$$

Where the new states are $x_1 = e_1$, $x_2 = e_2$, $x_3 = \sigma$, $x_4 = \dot{\sigma}$ y $\theta_d(t), \dot{\theta}_d(t), \ddot{\theta}_d(t) \in \mathbb{R}^l$ are the desired angle, speed and acceleration of the motor, respectively; $u(t)$ is the global control defined in (3) y $K_{p2} = K_{p2}^T > 0$ y $K_{d2} = K_{d2}^T > 0$.

The representation of the system in closed loop in terms of the new defined states x (t) that are obtained by substituting (17) and (18) in (16), is given by

$$\dot{x}_1 = x_2 \qquad (19)$$
$$\dot{x}_2 = M(x_1+q_d)^{-1}\left[-h(x,t) - C(x_1 + q_d, x_2 + \dot{q}_d)x_2 - \left(K\, K_{p1}\right)x_1 - K\, K_{d1}x_2 + w_u\right]$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = J^{-1}\left[-K_{p2}x_3 - K_{d2}x_4 + u + w_m\right]$$

where $h(x,t) = [M(x_1+q_d) - M(q_d)]\ddot{q}_d + [C(x_1+q_d, x_2+\dot{q}_d) - C(q_d,\dot{q}_d)]\dot{q}_d + g(x_1+q_d) - g(q_d)$.

The feedback system is decoupled in two sections (one mechanical and one actuator) with a control input τ, and its origin in $x_0 = 0 \in \mathbb{R}^n$ it will be a unique balance point if and only if

$$\lambda_{min}\left\{K_{p1}\right\} > \frac{K_g + K_M\|\ddot{q}_d\| + K_{c2}\|\dot{q}_d\|^2 - \lambda_{min}\{K\}}{\lambda_{min}\{K\}} \qquad (20)$$

The discontinuous control (5) is designed from (19) where

$$f(x,t) = \begin{bmatrix} x_2 \\ x_\psi \\ x_4 \\ J^{-1}\left[-K_{p2}x_3 - K_{d2}x_4\right] \end{bmatrix} \qquad (21)$$

with $x_\psi = M(x_1+q_d)^{-1}\left[-h(x,t) - C(x_1+q_d, x_2+\dot{q}_d)x_2 - \left(K + K_{p1}\right)x_1 - K\, K_{d1}x_2\right]$, and the functions

$$g_2(x,t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ J^{-1} \end{bmatrix}, g_2^\perp(x,t) = \begin{bmatrix} 0 \\ M(x_1+q_d)^{-1} \\ 0 \\ 0 \end{bmatrix} \qquad (22)$$

Based on the system (8) where the expression is defined

$$\left(I - g_2(x_s,t)\left(Dg_2(x_s,t)\right)^{-1}D\right)g_2^\perp(x_s,t)$$

and by substituting D = [0,0,0, J], then the representation on the sliding surface is reduced to the form:

$$\dot{x}_s = f(x_s,t) + g_2(x_s,t)u_s + g_2^\perp(x_s,t)w_u \qquad (23)$$

where the u_s control is designed through the non-linear H_∞ control technique. The target output for the motion regulation problem is proposed

$$z = \begin{bmatrix} u_s \\ \rho\, tanh(x_{s1}) \\ \rho\, x_{s2} \\ \rho\, tanh(x_{s3}) \\ \rho\, x_{s4} \end{bmatrix} \qquad (24)$$

where ρ is a positive constant and $\tanh(x_{si}), i = 1{,}3$, it is the hyperbolic tangent function. The position and speed of the articulation and the actuator are available for feedback.

$$y = x_s \quad (25)$$

while equations (21) - (22), (24) - (25) are represented in the generalized form (23) and the remaining functions of the form (9) are:

$$h_1 = \rho \begin{bmatrix} 0 \\ tanh(x_{s1}) \\ x_{s2} \\ anh(x_{s3}) \\ x_{s4} \end{bmatrix}, h_2 = x_s, K_{12} = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad (26)$$

The vector and matrix functions of the system (23) are of appropriate dimension.

Theorem 2. Assumptions 3-5 and hypothesis 1 satisfied under the following functions

$$V(x_s,t) = \frac{1}{2}x_{s1}^T\left(K + K\,K_{p1}\right)x_{s1} + \frac{1}{2}x_{s2}^T M(x_{s1}+q_d)x_2 + \beta tanh(x_{s1})^T M(x_{s1} + q_d)x_{s2} + \frac{1}{2}x_{s3}^T K_{p2}x_{s3} + \frac{1}{2}x_{s4}^T J x_{s4} + \gamma tanh(x_{s3})^T J x_{s4} \quad (27)$$

$$F(x_s) = \epsilon \tanh(x_{s1})^T \tanh(x_{s1}) + \epsilon\, x_{s2}^T x_{s2} + \epsilon \tanh(x_{s3})^T \tanh(x_{s3}) + \epsilon\, x_{s4}^T x_{s4} \quad (28)$$

with $\beta > 0$, $\gamma > 0$ and then $V(x_s,t)$ it will be defined positive for all $x_s \in \mathbb{R}^n$ and radially desacoted if and only if

$$\lambda_{max}\{K_{p2}\} > \gamma^2\lambda_{max}\{J\}, \quad \lambda_{min}\{K_{p2}\} > \gamma^2\lambda_{min}\{J\}. \quad (29)$$

is fulfilled. In addition, the Hamilton-Jacobi-Isaacs inequality is satisfied if

$$\lambda_{min}\{K_{p1}\} > \frac{(0.5\beta K_{c1}\|\dot{q}_d\| + 0.5K_{h2} + 0.5\beta K_{h1} + 0.5a_1)^2 + K_{h2} + \frac{1}{\beta}(\rho^2+\epsilon)}{a_2 + \beta^2\lambda_{max}\{M(x_1+q_d)\} - \beta^2\sqrt{n}K_{c1} - \beta K_{h1} - \beta(\rho^2+\epsilon)} > \frac{\beta K_{h2} + \rho^2 + \epsilon - \beta\lambda_{min}\{K\}}{\beta\lambda_{min}\{K\}} \quad (30)$$

$$\lambda_{min}\{K_{p1}\} > \frac{\beta\lambda_{max}\{M(x_1+q_d)\} + \beta\sqrt{n}K_{c1} + K_{h1} + \rho^2 + \epsilon}{\lambda_{min}\{K\}} \quad (31)$$

$$\lambda_{min}\{K_{p2}\} > \frac{0.25\gamma\lambda_{max}^2\{K_{d2}\}}{\lambda_{min}\{K_{d2}\} - \gamma\lambda_{max}\{J\} - \rho^2 - \epsilon} + \frac{\rho^2+\epsilon}{\gamma} \quad (32)$$

$$\lambda_{min}\{K_{d2}\} > \gamma\lambda_{max}\{J\} + \rho^2 + \epsilon \quad (33)$$

where $a_1 = \beta\lambda_{max}\{K\}\lambda_{max}\{K_{d1}\}$, $a_2 = \beta\lambda_{min}\{K\}\lambda_{min}\{K_{d1}\}$, $\epsilon > 0$ it is a sufficiently small constant. So based on Theorem 1 the law of control by feedback of states is

$$u_s(t) = \alpha_2(x_s,t) = -\frac{1}{2}[\gamma \tanh(x_{s3}) + x_{s4}] \quad (34)$$

stabilizes the equilibrium point in asymptotic and global form of the disturbance-free system (23) and (34) will ensure that the gain L2 of the closed-loop system is less than γ.

Proof. It is proposed to separate the inequality of Hamilton-Jacobi-Isaacs (12) into two parts, that is to say

$$H(x_s,t) = H_1(x_s,t) + H_2(x_s,t)$$

Where

$$H_1(x_s,t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_s}f + h_1^T h_1 + F$$
$$H_2(x_s,t) = \gamma^2\alpha_1^T\alpha_1 - \alpha_2^T\alpha_2$$

$H_1(x_s,t)$, $H_2(x_s,t)$ are developed and we get their maximum levels

$$H_1(x_s,t) \le \beta\lambda_{max}\{M(x_{s1}+q_d)\}\|x_{s2}\|^2 - \lambda_{min}\{K\}\lambda_{min}\{K_{d1}\}\|x_{s2}\|^2 - \beta\lambda_{min}\{K + K\,K_{p1}\}\|\tanh(x_{s1})\|^2 + \gamma\lambda_{max}\{J\}\|x_{s4}\|^2 - \lambda_{min}\{K_{d2}\}\|x_{s4}\|^2 - \gamma\lambda_{min}\{K_{p2}\}\|\tanh(x_{s3})^T\|^2 + \beta K_{c1}\|x_{s2}\|^2 + \beta K_{c1}\|\dot{q}_d\|\|\tanh(x_{s1})\|\|x_{s2}\|^2 + K_{h1}\|x_{s2}\|^2 + K_{h2}\|\tanh(x_{s1})\|\|x_{s2}\| + \beta K_{h1}\|\tanh(x_{s1})\|\|x_{s2}\| + \beta K_{h2}\|\tanh(x_{s1})\|^2 + \beta\lambda_{max}\{K\}\lambda_{max}\{K_{d1}\}\|\tanh(x_{s1})\|\|x_{s2}\| + \gamma\lambda_{max}\{K_{d2}\}\|\tanh(x_{s3})\|\|x_{s4}\| + (\rho^2 + \epsilon)\|\tanh(x_{s1})\|^2 + (\rho^2+\epsilon)\|x_{s2})\|^2 + (\rho^2+\epsilon)\|\tanh(x_{s3})\|^2 + (\rho^2+\epsilon)\|x_{s4})\|^2$$

The inequalities of vector functions of secant and hyperbolic tangent and their properties were considered (Kelly, Santibañez and Loria, 2005). The previous inequality can be represented as follows

$$H_1(x_s,t) \le -\begin{bmatrix} \|\tanh(x_{s1})\| \\ \|x_{s2}\| \\ \|\tanh(x_{s3})\| \\ \|x_{s4}\| \end{bmatrix}^T Q_1 \begin{bmatrix} \|\tanh(x_{s1})\| \\ \|x_{s2}\| \\ \|\tanh(x_{s3})\| \\ \|x_{s4}\| \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} q_{1\_11} & q_{1\_12} & 0 & 0 \\ q_{1\_12} & q_{122} & 0 & 0 \\ 0 & 0 & q_{1\_33} & q_{1\_34} \\ 0 & 0 & q_{1\_34} & q_{1\_44} \end{bmatrix} \quad (35)$$

where:

$q_{1\_11} = \beta\lambda_{min}\{K + K\, K_{p1}\} - \beta K_{h2} - \rho^2 - \epsilon$

$q_{1\_12} = -0.5\beta K_{c1}\|\dot{q}_d\| - 0.5K_{h2} - 0.5\beta K_{h1} - 0.5\beta\lambda_{max}\{K\}\lambda_{max}\{K_{d1}\}$

$q_{1\_22} = \lambda_{min}\{K\}\lambda_{min}\{K_{d1}\} - \beta\lambda_{max}\{M(x_{s1} + q_d)\} - \beta K_{c1}\sqrt{n} - K_{h1} - \rho^2 - \epsilon$

$q_{1\_33} = \gamma\lambda_{min}\{K_{p2}\} - \rho^2 - \epsilon$

$q_{1\_34} = -0.5\gamma\lambda_{max}\{K_{d2}\}$

$q_{1\_44} = \lambda_{min}\{K_{d2}\} - \gamma\lambda_{max}\{J\} - \rho^2 - \epsilon$

To determine if Q_1 is defined positive, Sylvester's theorem is applied in (35), and it is determined that, by selecting $K_{p1}, K_{p2}, K_{d1}, K_{d2}$ such that they satisfy inequalities (30) to (33) accordingly $H_1(x_s, t)$ it will be a negative definite function.

In the analysis of $H_2(x_s, t)$ the functions are involved $\alpha_1(x_s, t)$ and $\alpha_2(x_s, t)$ as a result of the proposed Lyapunov function, that is to say

$$\alpha_1(x_s, t) = \frac{1}{2\gamma^2}[\beta \tanh(x_{s1}) + x_{s2}] \quad (36)$$

$$\alpha_2(x_s, t) = \frac{1}{2}[\gamma\ \tanh(x_{s3}) + x_{s4}] \quad (37)$$

Developing $H_2(x_s, t)$ you have to

$$H_2(x_s, t) \le \frac{1}{4\gamma^2}\|x_{s2}\|^2 + \frac{\beta}{2\gamma^2}\|\tanh(x_{s1})\|\|x_{s2}\| + \frac{\beta^2}{4\gamma^2}\|\tanh(x_{s1})\|^2 - \frac{1}{4}\|x_{s4})\|^2 + \frac{\gamma}{2}\|\tanh(x_{s3})\|\|x_{s4}\| - \frac{\gamma^2}{4}\|\tanh(x_{s3})\|^2$$

$$H_2(x_s, t) \le -\frac{1}{4}\begin{bmatrix} \|\tanh(x_{s1})\| \\ \|x_{s2}\| \\ \|\tanh(x_{s3})\| \\ \|x_{s4}\| \end{bmatrix}^T Q_2 \begin{bmatrix} \|\tanh(x_{s1})\| \\ \|x_{s2}\| \\ \|\tanh(x_{s3})\| \\ \|x_{s4}\| \end{bmatrix} \quad (38)$$

$$Q_2 = \begin{bmatrix} \frac{-1}{\gamma^2} & \frac{-\beta}{\gamma^2} & 0 & 0 \\ \frac{-\beta}{\gamma^2} & \frac{-\beta^2}{\gamma^2} & 0 & 0 \\ 0 & 0 & 1 & -\gamma \\ 0 & 0 & -\gamma & \gamma^2 \end{bmatrix}$$

Matrix Q_2 will be undefined for any positive constant γ and β. Finally, the inequality of Hamilton-Jacobi-Isaacs $H(x_s, t)$ it will be a negative definite function if

$$\lambda_{min}\{Q_1\} + \frac{1}{4}\lambda_{min}\{Q_2\} > 0. \quad (39)$$

1.3. Pendulum of a degree of freedom with rotational and flexible articulation

Be the mechanical system with flexible articulation defined as:

$$m\ddot{q} + g\sin(q) + K(q - \theta) = w_u \quad (40)$$
$$J\ddot{\theta} - K(q - \theta) = \tau + w_m$$

where the parameters of the plant and controller are shown in the following tables.

Description	Notation	Value	Unit
Mass of the joint	m	1.0001	Kg
Constant stiffness	K	100	Nm/rad
Moment of Inertia of the engine	J	0.02	Kg m
Moment of Inertia of the motor Gravitational constant	g	9.81	m/s

Table 1 Parameters of the pendulum of a degree of freedom
Source: Self Made

The position, the speed of the actuator and the articulation are available for the measurement at all times.

Parameters	Value
γ	7
ϵ	0.01
ρ	0.1
β	8
K_{h2}	44.7747
K'	10.791
K_g	10.791
K_{p1}	5.1
K_{p2}	3.73
K_{d1}	0.09
K_{p2}	0.18
Γ	5

Table 2 Parameters of the CGEV
Source: Self Made

The desired reference signals are defined as: $q_d(t) = \pi\sin(1.885t)$, $\theta_d(t) = q_d(t) + K^{-1}g\sin(q_d)$ and the initial condition is placed in $x(0) = \left[\frac{1}{2}\pi, 0, \frac{1}{2}\pi, 0\right]^T$, finally the system is disturbed with $w_m(t) = \pi\sin(10\pi t)$ y $w_u(t) = \pi\ cos(20\pi t)$.

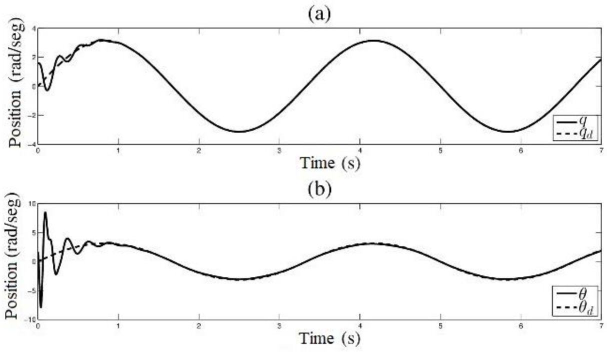

Figure 2 Performance of the system in closed loop (14) and (18), (a) Joint position and desired position, (b) Position of the actuator and position of the desired actuator

Source: Self Made

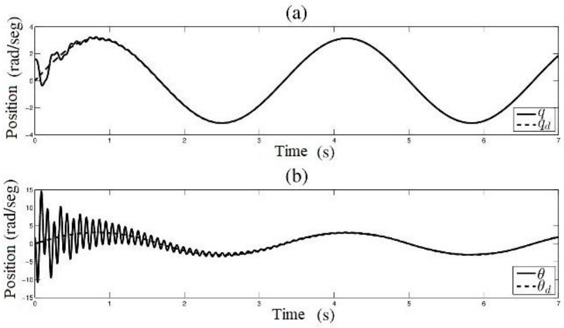

Figure 3 System performance in closed loop (14) and (18) without the non-linear control H_∞, (a) Articular position and desired position, (b) Position of the actuator and position of the desired actuator

Source: Self Made

Results

In Section I, the disturbed non-linear and time-varying plant was proposed, as well as the CGEV in which the theory is developed. Section II provides the controls that make up the CGEV in general for any system mentioned at the beginning. of the present paragraph, the controllers involved in the CGEV are the control by integral sliding mode (5) and control H_∞ non-linear (13), in section III the CGEV applied to a robot manipulator of l degrees of freedom with rotational articulations and with the elasticity effect in the presence of coupled and uncoupled perturbations, in the same way we present theorem 2 that gives solution to the non-linear H_∞ control (37) that is part of the CGEV, finally we analyze the proposed control in a pendulum of a degree of freedom with the rotational and flexible articulation and in the presence of coupled and uncoupled disturbances, the closed loop system (14) and (18) is analyzed with the help of M atLab / Simulink.

The performance of the aforementioned system is presented in figures 2 and 3, in figure 2 there is an underdamped result of the elasticity effect in the articulation and of the coupled and uncoupled disturbances to which the system was exposed, such effect and the disturbances were attenuated as time tends to infinity. Figure 3 shows a very marked underdamping in the position of the actuator, this is due to the fact that the non-linear controller H_∞ of the CGEV was eliminated, and the effects of the uncoupled perturbation take a little longer to be attenuated by the control discontinuous.

Conclusions

A variable structure global controller composed of integral sliding mode control and non-linear H_∞ control was proposed for non-linear, subacted and non-autonomous systems in the presence of coupled and uncoupled perturbations, the theory was validated with the regulation problem of movement for a robot manipulator of l degrees of freedom with rotational unions and with the effect of elasticity in each of them. In the proposed control structure, the integral sliding mode control keeps the path of the system in closed loop within the sliding mode and rejects the coupled disturbances and the control H_∞ in the sliding mode attenuates the uncoupled disturbances.

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