# A Correlation Analysis of 2D-DCT coefficients of face images

SÁMANO-ABONCE, Obed Noé\*†, MONTECILLO-PUENTE, Francisco Javier, VILLASEÑOR-AGUILAR, Marcos Jesús and YÁÑEZ-VÁZQUEZ, Alejandra

Instituto Tecnológico Superior de Salvatierra. Calle Manuel Gómez Morín 300, Janicho, 38933 Salvatierra, Gto.

Received July 20, 2017; Accepted December 21, 2017

#### **Abstract**

In this paper, we present a correlation analysis performed on the coefficients of the discrete cosine transform (DCT) obtained from images of the ORL face database segmented into overlapping blocks. The primary objective of this work was to identify those coefficients that show big variance between images from different persons but, at the same time, keep the most similar behaviour across pictures of the same subject. This project was done in four stages. In stage one, we segmented face images in overlapping blocks. Then, in a second step, the DCT was applied to each of the blocks and the coefficients were stored. In stage three a variance analysis was made to identify the coefficients of higher variation. Finally, we calculated correlations between coefficients to distinguish those that maintain the most similar behaviour for each person. Results and conclusion of this work will be of central relevance in the face recognition research field, for example, to design novel supervised or unsupervised classification algorithms using a smaller number of coefficients than those reported in the literature.

# 2D-DCT Coefficients, Correlation, Variance

**Citation:** SÁMANO-ABONCE, Obed Noé, MONTECILLO-PUENTE, Francisco Javier, VILLASEÑOR-AGUILAR, Marcos Jesús and YÁÑEZ-VÁZQUEZ, Alejandra. A Correlation Analysis of 2D-DCT coefficients of face images. ECORFAN Journal-Democratic Republic of Congo 2017, 3-5: 10-17

<sup>\*</sup> Correspondencia al Autor (email: nosamano@itess.edu.mx)

<sup>†</sup> Researcher contributing first author.

#### Introduction

In the last years, face recognition has become a very active research area in the pattern recognition and computer vision field. This topic is of high interest for researchers because of the wide range of possible applications, for example in security, surveillance, commercial purposes, among others. Although big advances have been made in the last decades, there are still some challenges to solve, for example, changes in illumination, partial occlusion or rotation of the images. Moreover, the need to develop real time recognition systems require the design of fast algorithms.

An essential step in a face recognition system is feature extraction. Features are the characteristics that the face recognition system will use to discriminate across faces. Features can be extracted using many techniques, but in general, we can distinguish two approaches: geometric and appearance based. In the geometric approach points, lines, and curves are used to extract features. While, in the second one, pixel values and transformations can be used to select them.

After extracting the features, they must be selected. The problem of selecting features is a global optimisation problem that looks to reduce the number of features, removes irrelevant, noisy and redundant data. In literature, we can find many techniques that have been applied to accomplish this task. Among the most common we can find Principal Component Analysis (Song, Guo & Mei, 2010), Linear Discriminant Analysis (Guyon & Elisseeff, 2003) and Genetic Algorithms (Harandi et al., 2004). In the last decades, techniques as Particle Swarm Optimization Abdel-Kader, 2009)(Farag, (Ramadan & Elghazaly & Hefny, 2016) and Discriminative Power Analysis (Dabbaghchian Ghaemmaghami, 2010) has also been used.

Another technique widely used to extract features from face images is the Twodimensional Discrete Cosine Transform (2D-DCT). The success of this technique to extract features is principally due to its capacity to compress data in a small set of coefficients and perform data dimensionality reduction. However, not all the coefficients of this transformation have the same power of discrimination. Furthermore, opposite to what we may think, having a big number of coefficients does not guarantee recognition results. In fact, it could introduce redundant information reduce the efficiency of the system and increase the computation time (Swets & Weng, 1996). For these reasons, it is recommendable to accomplish a mathematical analysis to select the most appropriate coefficients before training a face recognition classifier.

In this work, we present a correlation analysis applied to coefficients of the 2D-DCT obtained from face images segmented into overlapping blocks. The main objective of this work was to identify coefficients that keep the same behaviour across images of the same person and notable differences for distinct subjects in the database. In other words, we looked for coefficients with large variation across the classes and big correlation within the classes. This investigation was carried out using the ORL face dataset. Our main contribution is the identification of the coefficients that maintain the most similar behaviour across the images of the same person of this data set. This information will be useful to develop and apply supervised techniques perform to face recognition.

The investigation was carried out using the freely available ORL face dataset. The database has ten gray-level images of 40 different persons. Each image has 112 pixels of height (H) and 92 pixels in width (W).

Pictures were captured under controlled illumination conditions. However, including small variations in facial expression and orientation.

The content of this article is organized as follows. Section II is a brief review of related works. Section III describes the methodology. A theoretical framework can be found in section IV. Results are presented in Section V. Finally, in section VI we make our conclusions.

#### **Related Work**

In the field of face recognition, different approaches making use of 2D-DCT have been reported in the literature. In the early work of Nefian & Hayes (1998), face images were segmented into overlapping blocks and the 2D-DCT was applied to each block. Then, coefficients into a 3x13 window of the low frequencies were extracted and used as features to train Hidden Markov Models (HMM) and perform face recognition. Authors chose this window size because the major amount of information is concentred in these coefficients.

Hafed & Levine (2001) presented a face recognition system using the first 64 coefficients (a subset of 8x8) of the low-mid frequency 2D-DCT coefficients. The size of this subset was chosen such that it could sufficiently represent a face. Nevertheless, authors claimed that a small number of them could also give good results. Face recognition in this algorithm is accomplished comparing the feature vector of the input image to the feature vector of the dataset using the Euclidean distance nearest neighbour classifier.

In another method proposed to achieve face recognition, a Nearest Neighbour Discriminant Analysis (NNDA) applied to 2D-DCT coefficients was presented by Tyagi & Khanna (2012). First, authors applied the transformation to the whole image to extract features.

Then, they extracted a 16x16 window of low-frequency coefficients were used to train an algorithm using NNDA.

Dabbaghchian & Ghaemmaghami (2010) reported a method to select the coefficients that improve the recognition rate. They named this method Discriminative Power Analysis (DPA) and it is based on the idea of looking for those coefficients that show small variation inside classes and large variation inside classes of a de terminated dataset. They mention that the main difference of a DPA respect to techniques such as Principal Component Analysis (PCA) and Linear Discriminative Analysis (LDA) is that DPA keeps the original domain of the data.

Most recently, Chen et al. (2016), proposed a new face recognition method. This method uses histogram-based features in spatial and frequency domains. First, authors divide the face image into regions and then build a feature vector for every resulting area using a histogram of the low-frequency coefficients of the 2D-DCT, as well as Local Binary Pattern (LBP) histogram in the spatial domain. The DCT coefficients used in this work were obtained with a mask of size 4x4. Hence, they used only the first 16 frequency coefficients obtaining good results.

As can be verified in this brief review of the literature, there are many methods to select the 2D-DCT coefficients to be used as features for a classification task. While some authors use a feature selection technique, other just select the coefficients in a zigzag manner or extract a rectangular window throughout a mask. (Dabbaghchian and Ghaemmaghami, 2010).

## **Theoretical framework**

Two-dimensional Discrete Cosine Transform

If f(x,y) is an image of size MxN, the 2D-DCT of the image f(x,y), f(u,v), can be computed as defined in the equations 1 and 2.

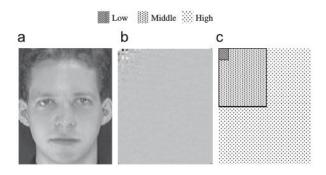
$$F(u,v) = \frac{1}{\sqrt{MN}} \alpha(u)\alpha(v) \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \times \cos\left(\frac{(2x+1)u\pi}{2M}\right)$$

$$\times \cos\left(\frac{(2y+1)v\pi}{2N}\right) \quad u = 0, 1, \dots, M, \quad v = 0, 1, \dots, N$$
(1)

Where:

$$\alpha(\omega) = \begin{cases} \frac{1}{\sqrt{2}} & \omega = 0\\ 1 & \text{otherwise} \end{cases}$$
 (2)

**Figure 1** is an example of a gray-level image and its 2D-DCT.



**Figure 1** a) A typical face image; b) its 2D-DCT transformed image c)typical division of the coefficients into low, middle and high frequencies. (Dabbaghchian et. al, 2012)

## Correlation

Correlation is a statistical technique that can show whether and how strongly pairs of variables are related. It is also used to determine how similar are two variables. There are several different techniques to calculate correlation, for instance, Pearson, Spearman, Kendall, product-momentum correlation, etc. In this work, we used the simple correlation function *corr* provided by octave software. If each row of  $\bf X$  and  $\bf Y$  is an observation and each column is a variable, then the (i,j)-th entry of  $\rho$  is given by the equation 3.

ISSN-On line: 2414-4924 ECORFAN® All rights reserved.

$$\rho(X,Y) = \frac{cov(X,Y)}{std(X)*std(Y)}$$
(3)

Where "cov" stands for covariance and "std" for standard deviation.

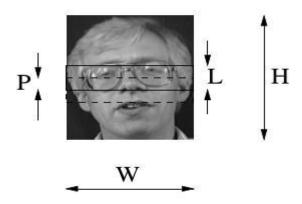
The result of the correlation is named "correlation coefficient" and it is commonly denoted by " $\rho$ ". The range of possible values of the correlation is from -1 to 1. A value near to -1 or 1 denotes high related variables, while a value close to 0 indicates poor or no linear relation among them.

## Methodology

This project was done in four stages. In stage one, we segmented face images in overlapping blocks. Then, in a second step, the 2D-DCT was applied to each of the blocks and the coefficients were stored. In stage three a variance analysis was made to identify the coefficients of higher variation. Finally, in the last stage, we calculated correlations between coefficients to distinguish those that maintain the most similar behaviour for each person.

#### **Coefficient extraction**

To extract the coefficients, we used the methodology reported by Nefian & Hayes (1998). In this method, blocks of size LxW pixels and overlap of P lines were extracted in a top to bottom direction, see **Figure 2**.



**Figure 2** Block extraction method as explained by Nefian & Hayes (1998).

SÁMANO-ABONCE, Obed Noé, MONTECILLO-PUENTE, Francisco Javier, VILLASEÑOR-AGUILAR, Marcos Jesús and YÁÑEZ-VÁZQUEZ, Alejandra. A Correlation Analysis of 2D-DCT coefficients of face images. ECORFAN Journal-Democratic Republic of Congo 2017

The total number of blocks Nb to extract in an image of LxW size and P overlapping lines can be calculated by the equation 4.

$$N_b = \frac{H - L}{L - P} + 1 \tag{4}$$

After extracting a block, the 2D-DCT transform was applied to it, and the coefficients were stored. The resulting 2D-DCT matrix of the blocks had the same size as the original block, that is, LxW pixels. With the objective to handle the information easier, the coefficients were rearranged in a single vector by concatenating all rows in a single row vector.

Nefian & Hayes (1998) claimed that best results in their method were obtained with L=10 and P=9, so we decided to use the same parameter values. With these parameters and the size of ORL face images, equation 4 can be used to determine Nb=103 blocks to extract. Moreover, authors suggested the vast amount of information of the 2D-DCT is concentrated in a window of 3x13 coefficients in the lowest frequencies range. Hence we limited our study to this window. In this way, the output of this stage was a four-dimensional array  $\bf C$  of size [Np x Ni x Nb x Nc].

Where:

 $N_p = \text{Number of persons in training set} = \\ 40.$ 

 $N_i = Number$  of images to use as training = 10.

 $N_b = Number of blocks = 103.$ 

 $N_c$  = Number of coefficients= 39.

## **Variance Analysis**

The objective of this stage was to determine the coefficients that present more variation in the whole database. This was done calculating the variance  $(\sigma^2)$  of each coefficient for the total data, that is, including all people.

Results can be consulted in Figure 3. As can be noted, higher variance appears in the low-frequency coefficients. However, special attention deserves coefficients 1 and 3.

The variance for each person was also calculated. Again, as can be verified in Figure 4, the coefficients showing the greatest variation are 1 and 3.

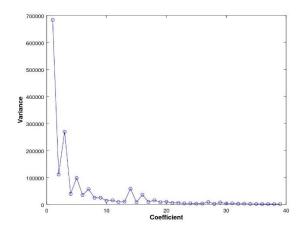


Figure 3 Coefficient variance for the total database.

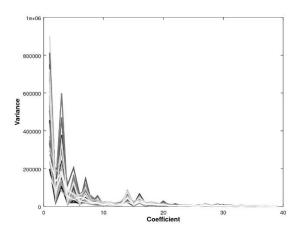


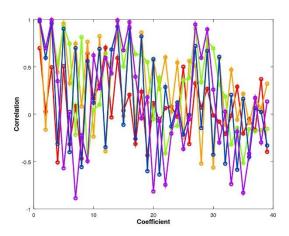
Figure 4 Coefficient variance per person.

#### Correlation

In this stage, a correlation was applied to the coefficients with the aim to detect those that maintain the most similar characteristics across the whole set of images of the same person.

ISSN-On line: 2414-4924 ECORFAN® All rights reserved. SÁMANO-ABONCE, Obed Noé, MONTECILLO-PUENTE, Francisco Javier, VILLASEÑOR-AGUILAR, Marcos Jesús and YÁÑEZ-VÁZQUEZ, Alejandra. A Correlation Analysis of 2D-DCT coefficients of face images. ECORFAN Journal-Democratic Republic of Congo 2017

The first image of each person was taken as the control curve. Then the correlation between each coefficient of this image and its correspondent coefficient of the subsequent 9 images were calculated. Hence the total number of correlation computed was 40 persons x 9 subsequent images x 39 coefficients = 14040 correlation coefficients computed. A correlation near 1 indicates a good that the coefficient kept almost the same behaviour across all the images of the subject. A low correlation value indicates big changes in the path and behaviour of that coefficients. Figure 5 shows the correlation of the coefficients for the first 5 persons of the database. We only present results for the first five subjects to keep graph clarity. However, the rest of curves are similar looking.



**Figure 5** Coefficient correlation for the first five persons in the database.

#### **Results**

As suggested in **Figure 5** the coefficients with the highest correlation are 1, 3, and 7. However, it is not true in all the cases. For this reason, in Figure 6 we present a histogram showing the percentage of times that each coefficient appeared ranked as the most correlated.

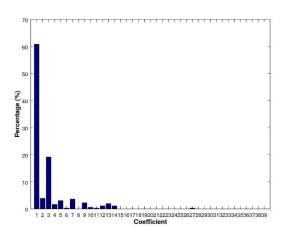
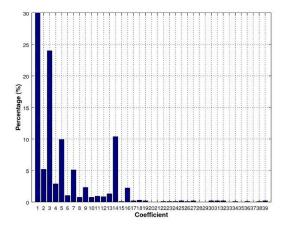


Figure 6 Percentage of times the given coefficient was the most correlated.

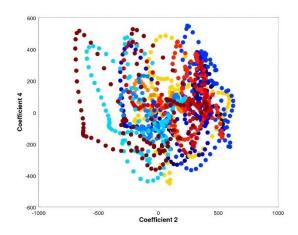
As can be verified in Figure 6 the coefficient number 1 appeared 61 % of times as the most correlated. It means that of the total of 360 tests (40 persons \* 9 images), the coefficient 1 was the most correlated with itself at least 219 times. The second most correlated coefficient is the third.

Figure 7 shows the percentage of times that the coefficient appeared in the top three of most correlated coefficients. According to the figure, coefficients 1, 3, 5, 7 and 14 are the most frequent in this top three. In fact, all they together sum a total of around 80 %.

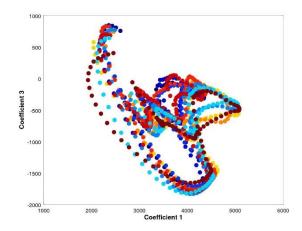


**Figure 7** Percentage of times that the specific coefficient appeared in the top three most correlated.

In Figure 8, we present a scatter plot of the coefficients 2 and 4 for the ten images of the subject 4 in the database, one colour for each image. Each one of the points corresponds to a different block. In total there are 103 points in the plot for each image. As we can see the curves are not similar, in other words, are poorly correlated. In contrast, Figure 9 presents a scatter plot of the same person but this time using coefficients 1 and 3. In agreement with our results, the curves show more similitudes and hence are better correlated.



**Figure 8** Scatter plot of the coefficients 2 and 4 for the subject 4 in the ORL database, one color for each image. Curves show low correlation.



**Figure 9** Scatter plot of the Coefficients 1 and 3 for the subject 4 in the ORL database, one colour for each image. Curves show high correlation.

### **Conclusions**

In this article, we performed a correlation analysis to identify those coefficients of the 2D-DCT that keeps most similarities in face images of the ORL Face database. The study shows that coefficients 1, 3, 5, 7 and 14 are the most correlated and hence they are good options to be used to train supervised or unsupervised face recognition algorithms. Also, the mentioned coefficients presented a great level of variance across subjects and therefore have potential to discriminate.

Nevertheless, it is important to clarify that the method here presented is database dependent and therefore, this procedure must be repeated for each one.

#### **Future Work**

As future work, we plan to use the results of this investigation to train Discrete Hidden Markov Models to recognize faces using a small number of coefficients.

#### References

Chen, Q., Kotani, K., Lee, F., & Ohmi, T. (2016). An Improved Face Recognition Algorithm Using Histogram-Based Features in **Spatial** and Frequency Domains. World Academy Science, Engineering of Technology, *International* Journal Computer, Electrical, Automation, Control and Information Engineering, 10(2), 385-389.

Dabbaghchian, S., Ghaemmaghami, M. P., & Aghagolzadeh, A. (2010). Feature extraction using discrete cosine transform and discrimination power analysis with a face recognition technology. *Pattern Recognition*, 43(4), 1431-1440.

ISSN-On line: 2414-4924 ECORFAN® All rights reserved.

Farag, M. M., Elghazaly, T., & Hefny, H. A. (2016, December). Face recognition system using HMM-PSO for feature selection. In *Computer Engineering Conference* (ICENCO), 2016 12th International (pp. 105-110). IEEE.

Guyon, I., & Elisseeff, A. (2003). An introduction to variable and feature selection. *Journal of machine learning research*, 3(Mar), 1157-1182.

Hafed, Z. M., & Levine, M. D. (2001). Face recognition using the discrete cosine transform. *International Journal of Computer Vision*, 43(3), 167-188.

Harandi, M. T., Ahmadabadi, M. N., Araabi, B. N., & Lucas, C. (2004, December). Feature selection using genetic algorithm and it's application to face recognition. In *Cybernetics and Intelligent Systems*, 2004 IEEE Conference on (Vol. 2, pp. 1368-1373). IEEE.

Nefian, A. V., & Hayes, M. H. (1998, May). Hidden Markov models for face recognition. In *Acoustics, Speech and Signal Processing, 1998. Proceedings of the 1998 IEEE International Conference on* (Vol. 5, pp. 2721-2724). IEEE.

Ramadn, R. M., & Abdel-Kader, R. F. (2009). Face recognition using particle swarm optimization-based selected features. *International Journal of Signal Processing, Image Processing and Pattern Recognition*, 2(2), 51-65.

Song, F.,Guo, Z., & Mei, D. (2010, November). Feature selection using principal component analysis. In *System science, engineering design and manufacturing informatization (ICSEM)*, 2010 international conference on (Vol. 1, pp. 27-30). IEEE.

Swets, D. L., & Weng, J. J. (1996). Using discriminant eigenfeatures for image retrieval. *IEEE Transactions on pattern analysis and machine intelligence*, 18(8), 831-836.

Tyagi, S. K., & Khanna, P. (2012). Face recognition using discrete cosine transform and nearest neighbor discriminant analysis. *International Journal of Engineering and Technology*, 4(3), 311.