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# OFDM System and peak to average power ratio reduction methods

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## Abstract

The present article deals in a general way with the adverse effects of power peaks in multi-carrier communication systems, specifically, orthogonal frequency division modulation systems, each of the transformation stages is briefly described to generate a multi-carrier signal . In addition, the most common methods for reducing unwanted power peaks are discussed, and it is briefly explained how power peaks are measured using the ratio of maximum and average power. Additionally, the methods of signal predistortion and the simulation of the application of nonlinear predistortion methods in a multi-carrier communication system by orthogonal division of frequencies are explained quickly. Finally, some conclusions about the methods of asymmetric predistortion due to nonlinear transformations of the amplitude of the signal.

**OFDM, PAPR, companding, non-linearities, power amplifiers**

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## Introdution

The current society demands more capacity to exchange information and powerful networks in terms of high speed data transmission and bandwidth capacity. Optical networks are the predominant technology to achieve this objective [1]. Recently, there has been interest in investigating optical communication systems using multicarrier modulation formats, such as orthogonal frequency division multiplexing (OFDM), in their various configurations [2]. OFDM has the necessary characteristics to meet the needs of bandwidth efficiency and high speed data transmission; some of these researches are aimed to reduce the nonlinearities present in some electronic components, such as power amplifiers. These nonlinearities are caused by the presence of high peak power relative to the average power; this phenomenon is measured as Peak to Average Power Ratio (PAPR).

This paper presents in Section II the basic concepts about multicarrier modulation. Section III explains the typical orthogonal frequency division multiplexing system, and its special features when applied to optical communications systems. Section IV gives a briefly review to the optical amplifier features and its characteristics. Section V and VI treat the peak to average power ratio (PAPR), and the most recent methods to reduce this important issue in OFDM systems. Finally, Section VII draws the conclusions of this report.

## Multicarrier Modulation

Multicarrier Modulation (MCM) is a transmission scheme that uses several carriers to transmit data generated at the baseband by means of its translation in frequency to the passband of the channel [3]. Fig. 1 shows the MCM basic frequency diagram.

The most common MCM schemes are Frequency Division Multiplexing (FDM) and Orthogonal Frequency Division Multiplexing (OFDM) [4].

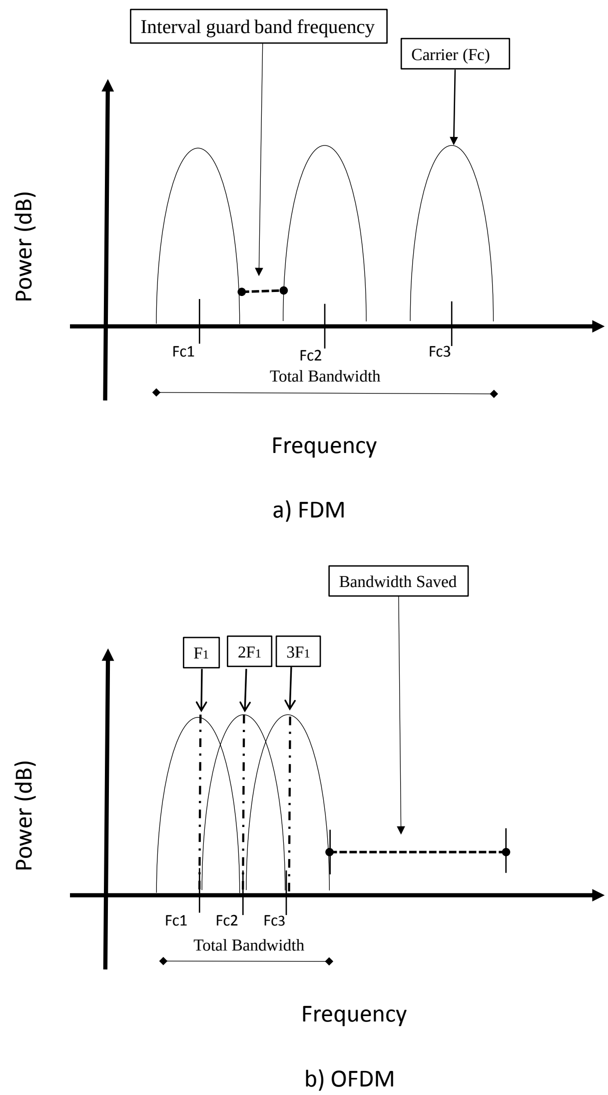


**Figure 1** Comparison of power spectrum between a) FDM and b) OFDM.

## Frequency Division Multiplexing (FDM)

FDM allows to transmit multiple signals in a singular channel to multiple users. FDM assigns to each user a different carrier frequency for transmission.

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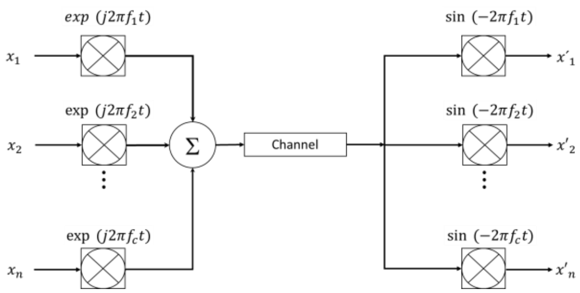
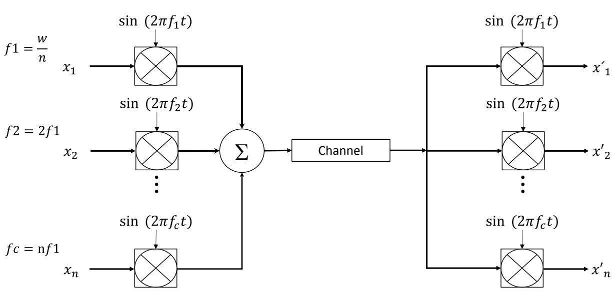
Fig. 2 illustrates the process to modulate a generic multicarrier FDM signal, where each signal $x_n$ modulates a separate carrier $f_c$ and each carrier is summed to be transmitted over the channel. Every single carrier is generated independently, however, the carrier frequency for each user is chosen so that the transmissions do not overlap in frequency. This results in a significant use of bandwidth as shown in Fig. 1a.




a) FDM




b)OFDM

**Figure 2** Multicarrier modulation FDM system basic diagram. a) FDM, b) OFDM.

Equation (1) shows the basic principle of an MCM system, where the multicarriers are added for transmission and must be separated in the receiver before demodulation [5].

$$s(t) = x_1\, sin(2\pi f_1 t) + x_2\, sin(2\pi f_2 t) + \cdots x_n\, sin(2\pi f_c t) \quad (1)$$

where $f_1$, $f_2$, … $f_c$ are the carriers frequencies, and $x_1$, $x_2$, … $x_n$ are the signals.

## Orthogonal Frequency Division Multiplexing (OFDM)

An alternative to improve the FDM scheme is the orthogonal frequency division multiplexing (OFDM). OFDM is a type of MCM and a special case of FDM. The principal advantage of OFDM over FDM is the bandwidth saved, this is possible because the OFDM signal presents orthogonality between subcarriers. This is made thanks to the use of subcarrier frequencies that are multiples of the first frequency $f_1$ (harmonics), as shown in Fig. 2b. The spectra between subcarriers overlap, but each subcarrier is in the spectral nulls of all other subcarriers [6]. Fig. 1b shows this principle in the frequency domain. A basic diagram of OFDM system is shown in Fig. 1b. Note that the main difference between FDM and OFDM is the way that the carrier frequencies are chosen, as seen in Fig. 2.

## The OFDM System

OFDM has been widely deployed in wireless communications and recently studied for optical communications. Some of its main characteristics are the low level of Inter Symbolic Interference (ISI), and the high spectral efficiency compared with other techniques of multicarrier modulation like FDM.

## Elements of the OFDM system

Fig. 3 shows a basic block diagram of OFDM system. The first step is to convert the input data stream into a code-word, using coding and interleaving techniques to reduce the Bit Error Rate (BER).

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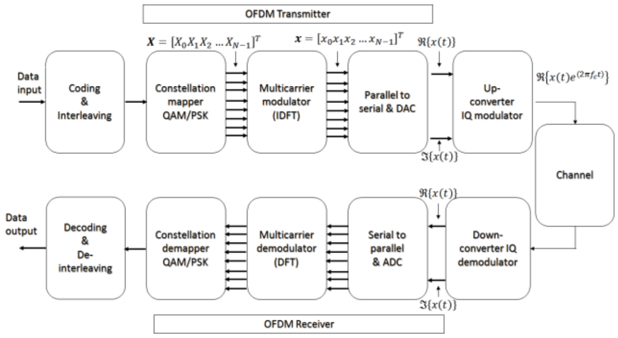


**Figure 3** Basic block diagram of an OFDM system.

The second step consists of mapping each of the data words using an M-ary modulation technique like Quadrature Amplitude Modulation QAM or Phase-Shift Keying PSK, this is made at the lowpass equivalent signal level so a complex signal is produced. The output of this functional block is a set of parallel data symbols, as shown in Fig. 4. The data symbols are oversampled introducing a set of null subcarriers. Each output symbol is independently modulated on a separate subcarrier frequency.

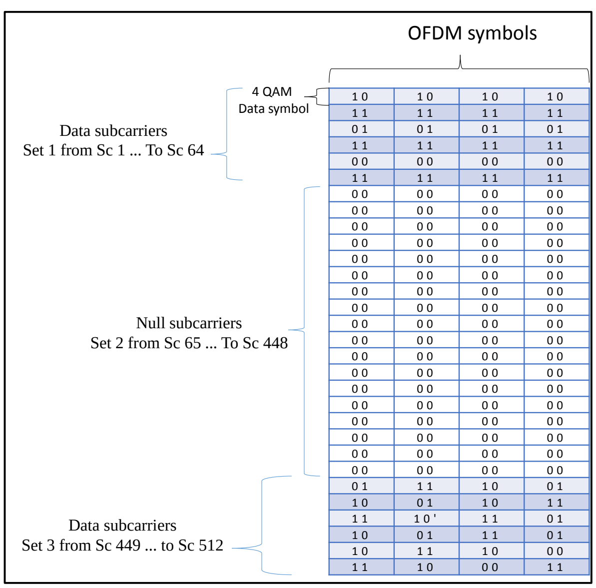


**Figure 4** OFDM symbol arrangement before multicarrier modulation

The multicarrier modulator and demodulator in OFDM, whose mathematical expression was presented in the last section, can be implemented by the use of a parallel bank of filters based on the Inverse Discrete Fourier transform (IDFT). This functional block eliminates the necessity of using local oscillators and filters per each signal, as shown in Fig. 1b. The definition of the Inverse Discrete Fourier Transform for the OFDM symbol sample is:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{\left(\frac{j2\pi kn}{N}\right)} \; for\ 0 \leq n \leq N-1 \qquad (2)$$

where $N$ is the total quantity of subcarriers, $k$ is the subcarrier number, and $X_k$ is the data carrier in "$k$" subcarrier.

The input to the IDFT block is the complex vector $\boldsymbol{X} = [X_0 X_1 X_2 \dots X_{N-1}]^T$, where $N$ is the size of the IDFT, and each of the array elements represents the data to be carried in the corresponding subcarrier. The resulting complex vector in the output of the IDFT is $\boldsymbol{x} = [x_0 x_1 x_2 \dots x_{N-1}]^T$ , as shown in Fig. 3.

OFDM modulation and demodulation based on IDFT/DFT can be performed efficiently by the use of Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform respectively. This implementation reduces computation complexity.

The complex vector $\boldsymbol{x}$ generated by computing the IFFT is passed through a parallel to serial converter (P/S), and then, to a digital to analog converter (DAC), where the output is the OFDM signal waveform $x(t)$ . This analog signal corresponds to the sum of the baseband subcarriers. As said before, the modulation is made at the low pass equivalent signal level so $x(t)$ represents the baseband complex value signal having real $\Re\{x(t)\}$ and imaginary parts $\Im\{x(t)\}$.

$$x(t) = R\{x(t)\} + \Im\{x(t)\} \qquad (3)$$

Finally, the baseband complex value signal $x(t)$ is up-converted to passband signal $s(t)$ ready to be transmitted. Mathematically, this transformation involves a complex multiplier (mixer) that can be expressed by (4), where the passband signal $s(t)$ is a real-value signal at the center frequency carrier of $f_c$.

$$\mathrm{s(t)} = \Re\{x(t)\}\, e^{(2\pi f_c t)} \\ \Re\{x(t)\}\cos(2\pi f_c t) - \Im\{x(t)\}\sin(2\pi f_c t) \quad (4)$$

This kind of modulation uses an "In phase (I)" and "In quadrature (Q)" modulator (IQ modulator). IQ data inputs represents the complex value signal, where $I = \Re\{x(t)\}\, cos(2\pi f_c t)$ is the real part of the signal and $Q = \Im\{x(t)\}\, sin(2\pi f_c t)$ is the imaginary part of the signal.

The I/Q modulator mixes the I part of the signal with the $f_c$ carrier, and mixes the Q part of the signal with the same carrier at a 90-degree phase offset as shown in Fig. 5.The Q signal is subtracted from the output signal producing the final OFDM signal to be transmitted.

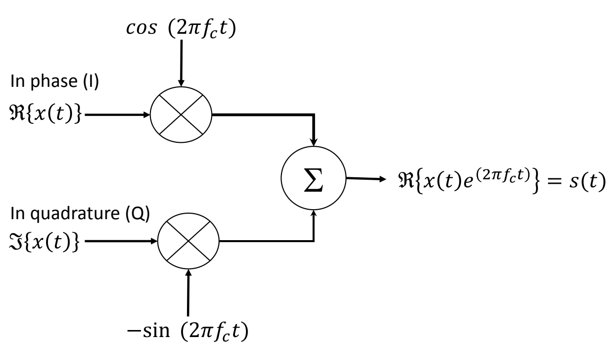


**Figure 5** Up-converter IQ Modulator block diagram, and the resulting OFDM signal $s(t)$.

The second part of the OFDM system is the receiver that is shown in Fig. 3. It is composed by the same functional blocks as the transmitter, but doing the inverse actions.

The aim of the receiver is to transform the analog passband signal that comes from the channel to baseband signal using a down-converter IQ demodulator. After this step, the baseband signal is demodulated and converted to serial digital data, which is the nature of the transmitter data input, as shown in Fig. 3.

| Application | Name of the Standard |
|---|---|
| Digital Audio Broadcasting | DAB Eureka 147 |
| Digital television | DVB-T/T2 (terrestrial), DVB-H (handheld), DMB-T/H, DVB-C2 (cable) |
| Wireless LAN | LAN IEEE 802.11a, IEEE 802.11g, IEEE 802.11n, IEEE 802.11ac, and IEEE 802.11ad |
| Worldwide Interoperability for Microwave Access | WiMAX (30-40 Gbps) |
| Asymmetric digital subscriber line | ADSL High speed data communications ADSL (G.dmt/ITUG.992.1) |
| Long Term Evolution | LTE and LTE Advanced 4G mobile phone standards |
| Modern narrow and BROADBAND power line communications. | IEEE 1901, ITU-T G.hn standard |

**Table 1** ofdm standards

Table I shows the different communication standards where OFDM is used. OFDM is present in different areas of communication technology like audio, video, data, smart grid, and it has a promising future in high speed data rate systems [7]. In fact, OFDM has recently been applied to optical communications, in consequence, a diversity of optical OFDM systems have been proposed, like Coherent Optical OFDM (CO-OFDM) for their special modulation scheme.

## Peak to Average Power Ratio

A major disadvantage of the OFDM systems is the high peak-to-average power ratio (PAPR) that is inherent in the transmitted signal. Large signal peaks occurs when the signals in the K subcarriers add constructively in phase. Such large signal peaks may saturate the power amplifier at the transmitter and, thus, cause intermodulation distortion in the transmitted signal. Fig. 6 shown the high peaks in an OFDM and the average magnitude.

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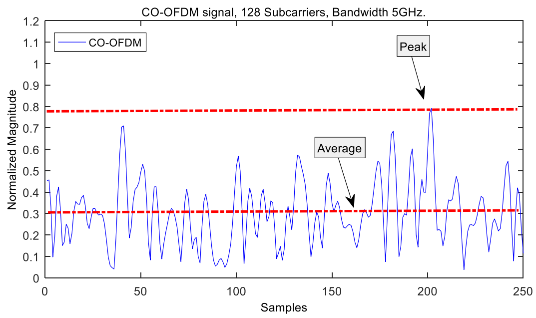


**Figure 6** OFDM signal, QAM baseband modulation, 128 subcarriers, 5 GHz, baseband bandwidth

The lowpass equivalent OFDM signal $x(t)$ has a PAPR defined as the ratio between the maximum instantaneous power and its average power, $Pav$.

$$PAPR[x(t)] = \frac{\max_{0 \le t \le NT} [x(t)^2]}{P_{av}} \quad (5)$$

where $N$ is the number of subcarrier and $T$ is the symbol period [12].

## Reduction Methods for PAPR

As mentioned earlier, a high PAPR drives into saturation the power amplifier (PA) in an OFDM transmitter producing nonlinearities in the output signal. One solution is to reduce the average power of the signal, but the performance of the amplification is also reduced. The most useful technique is to maintain the PA working with a good performance, high output power, but reducing the power peaks of the signal.

The literature presents many PAPR reduction techniques published since the high PAPR problem was detected. These techniques can be classified in three main categories, as shown in Fig. 12: Signal Distortion techniques, with special emphasis in the companding techniques, which have good results in PAPR reduction with low BER and a low power complexity. Multiple Signaling and Probabilistic techniques; and Coding techniques, which present a low performance in terms of BER and a high computational complexity.

A. Coding Techniques

As mentioned in Section II, coding is part of the first functional block in OFDM system, the main purpose being data protection and correction. The basic idea in a coding scheme to reduce PAPR is mapping the code word and drop those code words with a high probability to present a high PAPR.

Other coding technique works with phase adjustment, multiplying the code words by a phase adjustment vector, and selecting the code words with the lowest PAPR to be transmitted [8].

B. Multiple Signaling and Probabilistic Techniques

These techniques work in one of two ways. The first one is the selective mapping, which generates multiple permutations of the OFDM signal and transmits the code word with minimum PAPR. The other way is to modify the OFDM signal by introducing phase shifts, adding peak reduction carriers, or changing constellation points. The modification parameters are optimized to minimize PAPR.

a) Selective Mapping: Selective mapping (SLM) is a relatively simple approach to reduce PAPR. It generates a set of sufficiently different OFDM symbols, each of length *N*, all representing the same information as the original OFDM symbol *x*, then it transmits the one with the least PAPR. Information about the selected phase sequence should be transmitted to the receiver as side information to allow the recovery of the original symbol sequence at the receiver, which reduces the data transmission

b) Partial Transmit Sequence: In partial transmit sequence (PTS), an input data block of length $N$ is partitioned into a number of disjoint sub-blocks. The IDFT for each of these sub-blocks is computed separately and then weighted by a phase factor. The phase factors are selected in such a way as to minimize the PAPR of the combined signal of all the sub-blocks. The process of selecting the optimum phase factors is usually limited to a finite number of elements to reduce search complexity [9].

C. Signal Distortion Techniques

These techniques introduces a distortion into the transmission signal before the amplification process. In addition, they can be classified in clipping and filtering, peak windowing, and peak cancellation. One of their characteristics is that the peaks are removed from the signal, but the drawback is an increase in the BER. A brief description about the most common signal distortion techniques follows.

a) Clipping and filtering: This method uses a clipper that limits the high peaks of the OFDM signal envelope to a predetermined clipping level (CL) if the signal exceeds that level; otherwise, the clipper passes the signal to the High Power Amplifier without change. This introduces in-band distortion and out-band distortion. Filtering the clipped OFDM signal can preserve the spectral efficiency by eliminating the out-of band distortion and, hence, improving the BER performance, but it can lead to peak power regrowth.

b) Peak windowing: The peak windowing limits such high peaks by multiplying them by a weighting function called a window function. Many window functions can be used in this process as long as they have good spectral properties, and they reduce the distortion as compared with clipping and filtering.

c) Peak cancellation: A peak cancellation waveform is appropriately generated, scaled, shifted and subtracted from the OFDM signal at those segments that exhibit high peaks. Peak cancellation can be carried out after the IFFT block of the OFDM transmitter. While performing the peak cancellation process, care should be taken to not create new peaks.

d) Companding Transforms: Companding transforms are typically applied to speech signals to optimize the required number of bits per sample. OFDM and speech signals behave similarly and these techniques have a relatively low computational complexity as compared to other PAPR reduction techniques. In this case companding complexity is not affected by the number of subcarriers. Companding transforms can be generally classified into four classes: linear symmetrical transform (LST), linear asymmetrical transform (LAST), nonlinear symmetrical transform (NLST), and nonlinear asymmetrical transform (NLAST). Fig. 13 depicts the profiles of these four classes [8]. The NLAST increases the mean power and reduces the peak power, and, consequently, reduces PAPR.

## Numerical simulation results

In this section we present numerical simulation result of application of the most used and effectiveness nonlinear companding transforms.

The first nonlinear companding transform is the $\mu$-law. The $\mu$-law belongs to the NLAST. It works compressing large signals and enlarging small ones, with the drawback of increasing the average power of the transmitted signal The $\mu$-law is shown as follows

$$s'_n = sgn(s_n) \cdot \frac{A}{\ln(1+u)} \ln\left(1 + \frac{u}{A}|s_n|\right) \quad (5)$$

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where $u$ is the companding parameter and $A$ is the maximum absolute value of the amplitude signal $s_n$.

The second one is the NLAST method wich is named as a wang's NLAST.

Another NLAST is presented by Wang et. al. in [10] and it's expressed as a piecewise function. This algorithm proposes to modify the statistics of the amplitude, defined by its Probability Density Function (PDF). The companding function is given by

$$s'_n = \begin{cases} sgn(s_n)\sqrt{\frac{2}{k_1}\left(1 - e^{\left(-\frac{|s_n|^2}{\sigma^2}\right)}\right)} & |s_n| \leq M \\ sgn(s_n)\frac{1}{k_2}\left((k_2 - k_1)c \cdot D + \sqrt{(k_2 - k_1)k_1 c^2 D^2 + 2k_2\left(1 - e^{\left(-\frac{|s_n|^2}{\sigma^2}\right)}\right)}\right) & |s_n| > M \end{cases} \quad (7)$$

where $k_1$, $k_2$ and c$(0 < c < 1)$ are the companding parameters, $M = \sigma\left(-\ln(1 - (k_1/2)c^2D^2)\right)^{1/2}$ is the inflexión point, , and $D$ $(D > 0)$is the cutoff point.

We used a OFDM transmitter simulator based on Matlab to measure the PAPR reduction per ODFM symbol generated, the OFDM frame is set up with 128 data subcarriers,4 QAM modulation, oversampling factor of 4, 5 GHz of base. Fig. 7 Shown a fraction of data samples in time where Wang compander has more compression level when the high peaks are presents compared with ulaw.

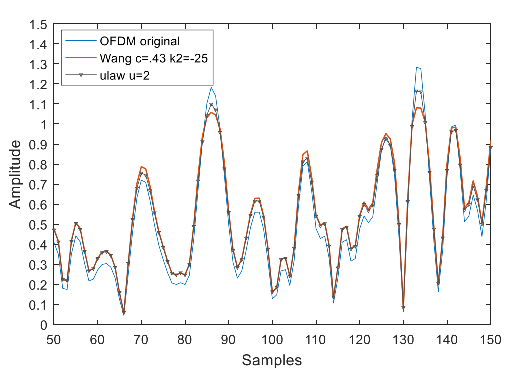


**Figure 7** OFDM signal companded by two NLAST, wang and ulaw.

As mentioned before companding methods changes the statistical distribution of the amplitude level of the OFDM signal, Fig. 8 shown the PDF of each companded method, u-law increase power average of the signal, instead of that Wang have a cut off point for high amplitude level.

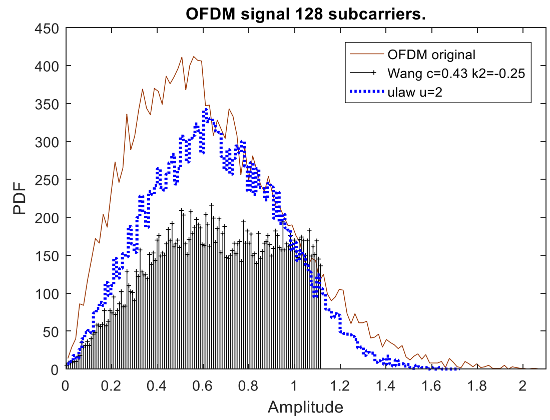


**Figure 8** OFDM companded signal PDF. Comparison between ulaw and Wang NLAST.

Finally the PAPR reduction is shown in Fig. 9. As a result each PAPR was calculated per OFDM symbol. In total we have 32 OFDM symbols. The lower PAPR level is obtained by Wang with a PAPR constant value of 4 dB, in contrast ulaw have proportional reduction approximated of 2 dB in each OFDM symbol.

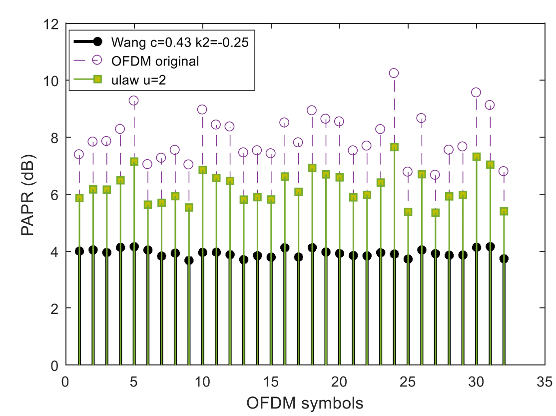


**Figure 9.**PAPR per OFDM symbol, Wang compander has more PAPR reduction level than ulaw.

## Conclusions

OFDM has a promising future in optical communications systems, hence it is attractive to search for novel techniques that increase performance, reduce PAPR, and thus prevent nonlinearities in optical power.Related to PAPR reduction, the Wang nonlinear companding methods exhibit a good performance compared with ulaw. However the NLAST companding methods shows a low computational complexity to be implemented, but little information has been reported in the literature. This open up new possibilities for research in novel techniques based on improving the properties of nonlinear companding techniques. In addition the future perspectives of this work is to implement more companding methods to compare, and increase the simulation to a optical OFDM system taking into account other interaction sichas optical amplifiers and digital to analog converters.

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