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Comparative analysis of methods to determine deflection in steel beams: theoretical analysis, finite element and experimental

Análisis comparativo de métodos para determinar la deflexión en vigas de acero: análisis teórico, elemento finito y experimental

ALOR-SAVEDRA, Gabriela†, ALAFFITA-HERNÁNDEZ, Francisco Alejandro, ESCOBEDO-TRUJILLO, Beatris Adriana* and SILVA-ÁGUILAR, Oscar Fernando

Universidad Veracruzana, Faculty of Engineering, Coatzacoalcos campus, Mexico.
Universidad Veracruzana, Research Center on Energy and Sustainable Resources. Mexico.

ID 1st Author: *Gabriela, Alor-Saavedra*

ID 1st Co-author: *Francisco Alejandro, Alaffita-Hernández* / **ORC ID:** 0000-0002-9462-0160, **Researcher ID Thomson:** 57103583500

ID 2nd Co-author: *Beatris Adriana, Escobedo-Trujillo* / **ORC ID:** 0000-0002-8937-3019, **Scopus ID:** 54417142300, **CVU CONACYT ID:** 173174

ID 3rd Co-author: *Oscar Fernando, Silva-Aguilar* / **ORC ID:** 0000-0002-5109-3193, **CVU CONACYT ID:** 338659

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Abstract

This work makes a comparative study of two methods to determine deflection in steel beams: (a) Theoretical and (b) Finite element. For method (a) the solution of the differential equation associated with the modeling of the deflection of a beam is found, while for method (b) a simulation is made in Solidworks. Both methods are compared with experimental data in order to analyze which of the methods presents less uncertainty and show the usefulness of the theoretical part in the modeling of physical systems.

Steel beams, Deflection, Punctual load

Resumen

El trabajo hace un estudio comparativo de dos métodos para determinar la deflexión en vigas de acero: (a) Teórico y (b) Elemento finito. Para el método (a) se encuentra la solución de la ecuación diferencial asociada al modelado de la deflexión de una viga, mientras que, para el método (b) se hace una simulación en Solidworks. Ambos métodos son comparados con datos experimentales con la finalidad de analizar cuál de los métodos presenta menor incertidumbre y mostrar la utilidad de la parte teórica en el modelado de sistemas físicos.

Vigas de acero, Deflexión, Carga puntual

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* Correspondence to Author (E-Mail: bescobedo@uv.mx)
† Researcher contributing as first author.

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Introduction

Currently, for the design of some structural element, tools are used to simulate the deflection of a beam subjected to various loads, this detailed analysis is to more accurately visualize the response of the element under the loads to which it will be subjected during its life useful or some natural phenomenon. Detailed analysis of the deflection of a beam can be accomplished in two different ways: using software, both of which reduce the time for calculating the deflections. These programs, in general, work with the finite element method and in addition to the deflections they can provide other data. The second way is to do the analysis by means of mathematical formulas, mostly with differential equations, that is, look for what effect the element will have and find the formula that models this phenomenon.

In (Huo, 2017) they analyze steel beams, which, when subjected to a load in a laboratory test, the beam becomes deformed. 6 tests are carried out on 6 different profiles, once the results are displayed they propose another 4 profiles. The two types of profiles are cold rolled and hot rolled. The ABAQUS® program was also used to make the comparison with the test data. The experimental and simulated results are almost the same. In the work of (J. T. Katsikadelis, 2003) et al. A deflection analysis is made by means of Euler-Bernoulli equations contemplating a variable stiffness that undergoes large displacements under general boundary conditions that may be non-linear. As the properties of the beam's cross-section vary along its axis, the coefficients of the differential equations that govern the beam are variable as well. On the other hand, like (JT Katsikadelis, 2003), (Peijun Wang, 2016) et al., Suggest that the finite element method is very effective for the behaviors that beams may have when subjected to loads of mock tests. There are several works that do the deflection analysis of beams, we recommend the reader to review the references of the articles cited in this work.

In this work we will analyze the deflection of a steel beam per finite element in Solidworks®, as well as, using the differential equation of the elastic. We compare the deflection results by the aforementioned methods with the experimental data obtained from the test carried out in the authors' work (Huo, 2017).

Deflection by laboratory test

The first data that are considered for the comparison of the deflection are those presented in (Huo, 2017). These data are obtained through the test described below:

Six impact tests were carried out on different type "I" profiles of cold rolled steel, welded at the ends to place the supports. The test was carried out with the machine shown in Figure 1 where a force was applied by means of the load control to the hammer with a weight of 980 kg and a maximum fall height of 16 m, the hammer falls to a certain height and impacts the midsection of each steel beam. A system was used to acquire the data which were captured by means of sensors distributed along the web of the beam.

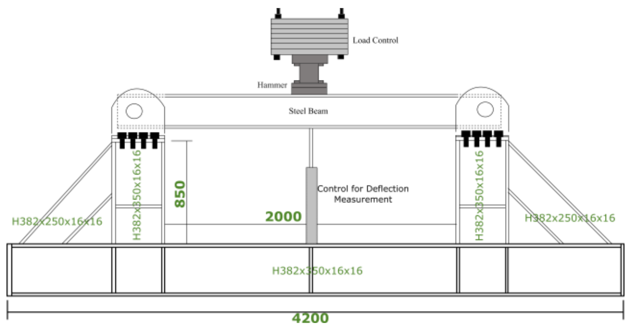

Figure 1 General view of the impact test

The area where the hammer will be impacted was reinforced with plates so that when the load was applied, the profile would not suffer a greater torsion. Since the test not only seeks to obtain the deflection that exists in the beam but also other phenomena, the applied loads are different. Once the tests were carried out, and taking into account the results of the six previous tests, they carried out four more tests, but on type "I" profiles of hot rolled steel.

This test was conducted at the Center for Integrated Protection of Engineering Structures Research (CIPRES) at Hunan University in Changsha, People's Republic of China.

The data found in Table 1 are some of the data presented in (Huo, 2017), they are not all since only those shown in the table are used.

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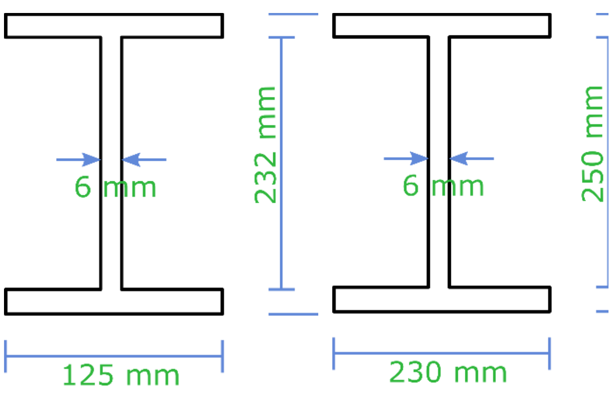
Profiles	Measurements (mm)	F_{umax} kN	F_{ue} kN	F_{uc} kN	F_p kN
Cold rolled					
HW11-58	H266x182x6x8	822	382	290	230
Hot rolled					
HR7-46	H250x125x6x9	790	260	176	136

Table 1 Steel Profile Details and Specifications

Table 1 shows the forces captured by the sensors. The forcé *Fumax* is the maximum impact force taken by the sensors, Fue is the average impact force of all the forces captured during the time of the test, Fuc is the static concentrated force and Fp is the plastic capacity of the beam. To calculate the deflection of the beam, both with Solidworks® and with a differential equation, the load Fue will be used, for the 2 beams HW11-58 and HR7-46.



a) Profile HR7-46 *b) Profile HW11-.58*

Figure 2 Profile measurements

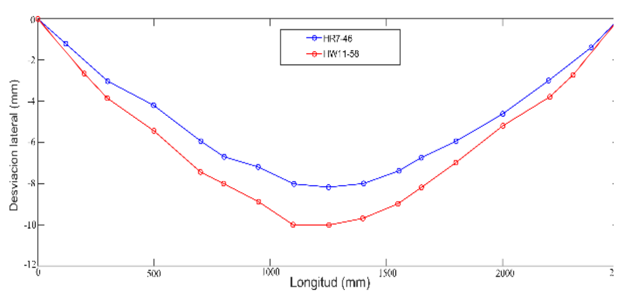


Figure 3 Impact test overview

The results of Figure 3 are the deflections of the two profiles under the loads by the average impact forces (*Fue*).

Deflection by finite element method in Solidworks

The beam shown in (Huo, 2017) is drawn in the same way, with the same dimensions and the same material, see Figure 4 and proceed to do the hammer test. Unlike the data obtained by the hammer test in Huo, 2017, only two different profiles are used in Solidworks®.

A simulation of the two profiles is performed with the force It was given in Table 1. Once the required simulations are performed, the information is processed in Matlab®.

Figure 4 presents the beam with all the elements that are needed, the fasteners, loads and meshing.

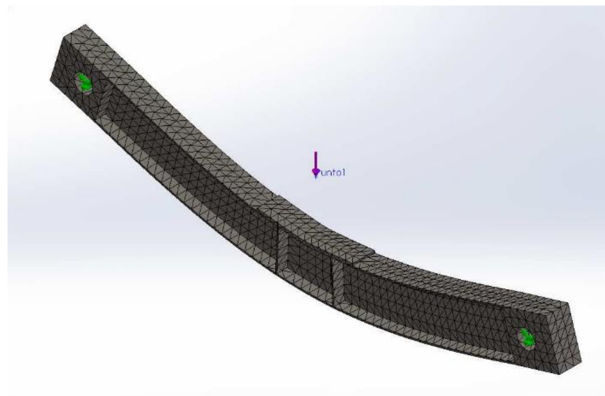


Figure 4 Beam model in Solidworks®

Deflection by analytical method (Elastic method or double integration)

For the elastic equation, it is a homogeneous material beam and has a uniform cross section throughout the length of the beam. At the center of the cross section, an imaginary line called the neutral axis or axis of symmetry is drawn. If force is applied in the plane perpendicular to the neutral axis, this axis presents a distortion, this distortion of the neutral axis is known as deflection or elastic curve, see Figure 5.

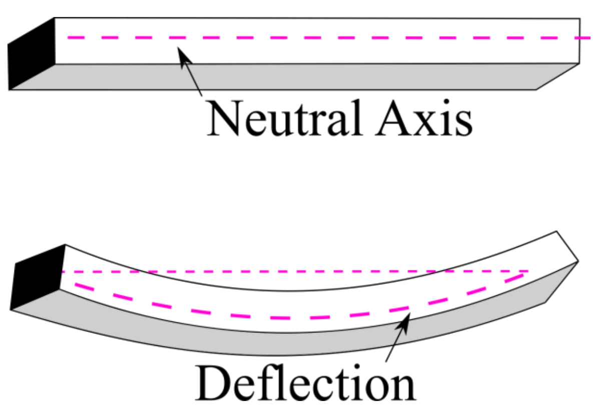


Figure 5 Representation of the elastic curve of a beam

In the elastic theory it is shown that the bending moment (Mx) at a point along the length of the beam (x) is related to the load to which the beam may be subjected by the following equation (Denis G. Zill, 2013):

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$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \qquad (1)$$

Equation (1) is an ordinary, linear, second-order differential equation, and governs the evolution of the elastic curve, which describes the deflections that a beam experiences when subjected to transverse loads.

The multiplication of EI represents the stiffness of the beam. The modulus of elasticity is taken from the data provided by Solidworks® when selecting the material of the beam. The moments of inertia (I) are as follows:

- 3.892×10^{-5} m^4 for profile HR7-46

- 5.628×10^{-5} m^4 for profile HW11-58

The M in equation (1) represents the bending moment.

The load P applied to the beam is in the center of the beam span. The supports have vertical reactions P / 2, the normal forces cancel out since it is assumed that the beam is not subjected to a horizontal force. Taking the reactions in the supports, the bending moment is calculated. There are two bending moments (2) - (3), one before the middle of the beam (SECTION I), and the other is considered after the middle and as far as it ends (SECTION II). These moments are those that are substituted in equation (1), each one in its respective section.

$$M_{x1} = \frac{P}{2}x \quad 0 \leq x \leq \frac{L}{2} \qquad (2)$$

$$M_{x1} = \frac{P}{2}x - P\left(x - \frac{L}{2}\right) \quad 0 \leq x \leq \frac{L}{2} \qquad (3)$$

Then the deflections for the sections are as follows:

SECTION I

$$y = \frac{1}{EI}\left(-\frac{Px^3}{12} + \frac{PL^2}{16}x\right) \qquad (4)$$

SECTION II

$$y = \frac{1}{EI}\left(-\frac{Px^3}{12} - \frac{PLx^2}{4} + \frac{3PL^2}{16}x + \frac{PL^3}{48}\right) \qquad (5)$$

To find the maximum deflection in the beam, we substitute x = L / 2 in equations (4) and (5) and we have the following result for both sections:

$$y_{max} = \frac{PL^3}{48EI}$$

Results

Deflection in hot rolled steel profile (HR7-46) Table 1 shows the force Fue that was applied to the beam, and Figure 6 shows the Solidworks simulation with its respective load. The colors shown represent the deflections, as it is moving, blue represents the 0 displacement and red the maximum displacement.

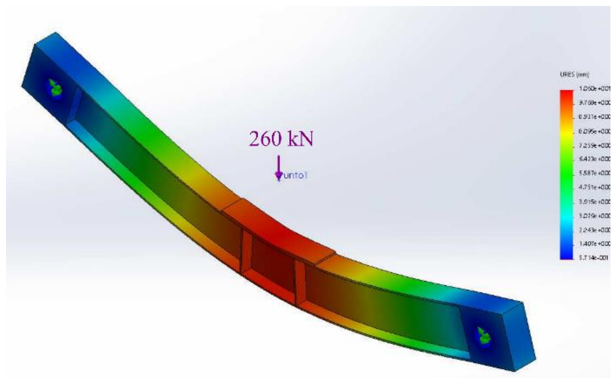

Figure 6 Deflection of a hot rolled steel beam subjected to a load of 260 kN

Table 2 shows the deflections that exist along the beam, with the experimental data given in (Huo, 2017) and Solidworks, as well as a column that contains the quadratic error at each node.

Length.	Exp.	Solidworks®	Quadratic Error
0	0	0	0
12	-0.120	-0.137	0.000289
30	-0.302	-0.358	0.003136
50	-0.419	-0.582	0.026569
70	-0.594	-0.781	0.034969
80	-0.670	-0.861	0.036481
95	-0.719	-0.951	0.053824
110	-0.802	-1.013	0.044521
125	-0.817	-1.034	0.047089
140	-0.808	-1.013	0.042025
155	-0.737	-0.951	0.045796
165	-0.674	-0.895	0.048841
180	-0.594	-0.780	0.034596
200	-0.461	-0.586	0.015625
220	-0.299	-0.362	0.003969
240	-0.138	-0.138	0
250	0	0	0

Table 2 Profile HR7-46. Deflection along the hot rolled steel beam with a load of 260 kN

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Deflection in cold rolled steel profile (HW11-58)

As for the beam with hot rolled profile, the force Was applied to the beam with a cold rolled steel profile and the simulation was carried out in Solidworks®, see Figure 7. Table 3 contains all the deflections, with their respective values. As can be seen, the quadratic error between the experimental data given in (Huo, 2017) and those obtained in the simulation is small, this indicates that the simulation in Solidworks gives a good approximation of the deflection in steel beams.

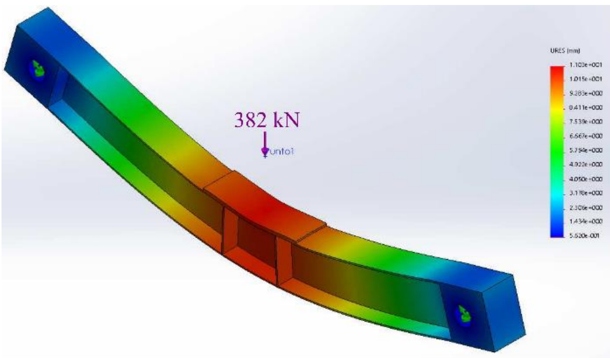

Figure 7 Deflection of a cold rolled steel beam under different loads

Length.	Exp.	Solidworks®	Quadratic Error
0	0	0	0
20	-0.265	-0.237	0.000784
30	-0.385	-0.361	0.000576
50	-0.543	-0.597	0.002916
70	-0.743	-0.795	0.002704
80	-0.800	-0.876	0.005776
95	-0.888	-0.973	0.007225
110	-1.000	-1.04	0.0016
125	-1.001	-1.103	0.010404
140	-1.000	-0.976	0.000576
155	-0.888	-0.976	0.007744
165	-0.800	-0.920	0.0144
180	-0.743	-0.794	0.002601
200	-0.543	-0.589	0.002116
220	-0.685	-0.363	0.103684
240	-0.265	-0.233	0.001024
250	0	0	0

Table 3 Profile HR11-58. Deflection along the cold rolled steel beam with a load of 382 kN

Deflection by analytical method

Tables 4 and 5 present the deflections that exist in the beam by experimental means and by means of an analytical method or differential equations, as well as a column of mean square errors at each node, for profiles HR7-46 and HR11-58., respectively. Similarly, in the Solidworks simulation it is found that the quadratic error between the experimental data given in (Huo, 2017) and those obtained with the solution of the differential equation (1) is small, this indicates that the solutions (4) - (5) also give a good approximation of deflection in steel beams.

Length.	Exp.	Analítico	Quadratic Error
0	0	0	0
12	-0.120	-0.147	0.000729
30	-0.302	-0.365	0.003969
50	-0.419	-0.588	0.028561
70	-0.594	-0.778	0.033856
80	-0.670	-0.859	0.035721
95	-0.719	-0.952	0.054289
110	-0.802	-1.009	0.042849
125	-0.817	-1.05	0.054289
140	-0.808	-1.014	0.042436
155	-0.737	-0.953	0.046656
165	-0.674	-0.893	0.047961
180	-0.594	-0.778	0.033856
200	-0.461	-0.588	0.016129
220	-0.299	-0.365	0.004356
240	-0.138	-0.148	1E-04
250	0	0	0

Table 4 Profile HR7-46. Deflection along the hot rolled steel beam with a load of 260 kN.

Length.	Exp.	Analítico	Quadratic Error
0	0	0	0
20	-0.265	-0.2628	4.84E-06
30	-0.385	-0.39	0.000025
50	-0.543	-0.627	0.007056
70	-0.743	-0.830	0.007569
80	-0.800	-0.945	0.021025
95	-0.888	-1.011	0.015129
110	-1.000	-1.082	0.006724
125	-1.001	-1.18	0.032041
140	-1.000	-1.085	0.007225
155	-0.888	-1.017	0.016641
165	-0.800	-0.953	0.023409
180	-0.743	-0.839	0.009216
200	-0.543	-0.616	0.005329
220	-0.685	-0.377	0.094864
240	-0.265	-0.249	0.000256
250	0	0	0

Table 5 Profile HR11-58. Deflection along the cold rolled steel beam with a load of 382 kN.

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Conclusions

As can be seen in Table 6, the mean square errors are small, even the difference between the two methods is not really significant, which is a very good indication that the analytical methods do not lose their validity. It is important to highlight the fact that using Solidworks increases precision, which is the most suitable for very large projects.

Profile	Solidworks	Analytical method
HR7-46	0.02574882	0.026221
HR11-58	0.00965471	0.01450081

Table 6 Mean square error for both profiles HR7-46 and HR11-58 for the two methods analyzed

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