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The method of small perturbations to Calculate Stiffness and damping in a Short Chumacera

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Abstract

In this paper, an alternative method to characterize a hydrodynamic bearing is presented. Based on the Reynolds' general lubrication equation, a perturbation is made on the center of the journal in order to partial pressures, so that be able to manage them to determine both the stiffness and damping dynamic coefficients. It is done the calculation for a classical case and it is generalized to situations that involve external excitations. The dynamic coefficients are gotten in an analytical way and they are plotted as a function of the balance eccentricity. The methodology presented in this document is of great value because it can be adapted for complex cases to get quite acceptable numerical solutions.

Reynolds, Rotordynamics, Journal Bearing.

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Introduction

The equations of motion of a system-rotor bearings contain coefficients corresponding to the lubricant film. These parameters change with the position having the shaft relative to the bearing center and the rotation speed. That's why the dynamic behavior is always heavily influenced by the values they can take these coefficients. It is in the literature as the operation speed increases, one of the stiffness coefficients can take negative values depending on its magnitude and the system could instability [1].

To study the behavior of the fluid in the hydrodynamic bearings Reynolds equation is used, which is a simplification of the Navier-Stokes equations for Newtonian fluids type. Reynolds equation relates the fluid pressure in the bearing with axial and circumferential coordinates, so it is possible to obtain the pressure field. It is not possible to analytically solve the Reynolds equation, but approaches are available depending on the length / diameter of the bearing (L / D) ratio. However, it is possible to determine the dynamic condition of the lubricant film taking the linear behavior of the pressure field and forces it along. Disturbing the equilibrium position of the stump to find modifying pressure increases the dynamic properties of the support and highlight the effect of stiffness and damping of the oil film.

Nomenclature

C: Clear radial bearing
c_{ij}: Damping coefficients
D: Diameter of the bearing
e: Eccentricity
H: Thickness o the lubricant film
h: Dimensionless film thickness
k_{ij}: Stiffness coefficients
L: Length of the bearing
N: Operating speed
p: Pressure of the lubricant film
S: Sommerfeld number
z: Axial Cartesian coordinate
ϕ: Angle of attitude.
θ: Circunferential coordinates of bearing
ε: Dimensionless exxentricity, ε=e/C
ω: Operating speed.
μ: Dynamic or Absotlute viscosity.

Development

The model describing the function of pressure in hydrodynamic bearings is the Reynolds equation; such an equation can be written generally as [2]:

$$\frac{1}{R_1^2}\frac{\partial}{\partial\tilde{\theta}}\left(\frac{H^3}{12\mu}\frac{\partial p}{\partial\tilde{\theta}}\right)+\frac{\partial}{\partial z}\left(\frac{H^3}{12\mu}\frac{\partial p}{\partial z}\right)=\frac{\omega}{2}\frac{\partial H}{\partial\tilde{\theta}}+\frac{\partial H}{\partial t} \tag{1}$$

This equation can be written in dimensionless form using the following parameters:

$$H = Ch = C[1+\varepsilon\cos(\tilde{\theta}-\phi)] \tag{2}$$

$$z=\frac{L}{2}\bar{z}\ ,\ p=\mu N\left(\frac{R_1}{C}\right)^2\bar{p}\ ,\qquad \omega=2\pi N \tag{3}$$

Substituting this in equation (1) is obtained in dimensionless form as:

$$\frac{\partial}{\partial\tilde{\theta}}\left(h^3\frac{\partial\bar{p}}{\partial\tilde{\theta}}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=12\pi\frac{\partial h}{\partial\tilde{\theta}}+\frac{24\pi}{\omega}\frac{\partial h}{\partial t} \tag{4}$$

With the boundary conditions:

$p=0$ for $z=\pm L/2$ y $\left(\frac{\partial p}{\partial z}\right)=0$ para $z=0$.

Note that: $\tilde{\theta}=\theta+\phi$, $h=1+\varepsilon\cos(\tilde{\theta}-\phi)$.

In Figure 1 a given rotor (stump in the bearing) illustrated position, the parameters included in Reynolds equation is:

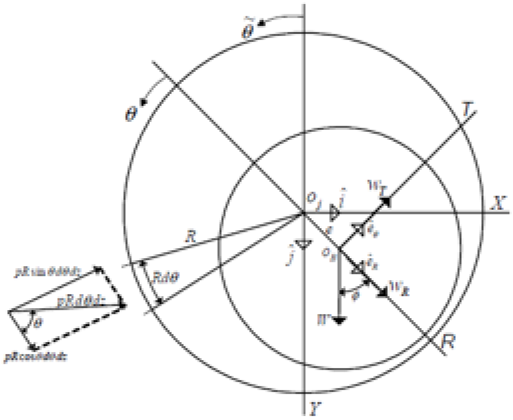

Figure 1 Instantaneous position of the rotor within the bearing

The proposed methodology is performed by making small perturbations around the equilibrium position. The fact only considering small perturbations is necessary because the equations of motion of rotor-bearings are highly nonlinear system. For example, consider the case of a mass 2M rotor supported by two identical bearings and properly aligned. The equations of motion are [1]:

$$\begin{Bmatrix} M & 0 \\ 0 & M \end{Bmatrix} \frac{\partial^2}{\partial t^2} \begin{Bmatrix} a\cos\phi \\ a\sin\phi \end{Bmatrix} = \begin{Bmatrix} W\cos\phi_t \\ W\sin\phi_t \end{Bmatrix} - \begin{Bmatrix} F_Y \\ F_X \end{Bmatrix} \tag{5}$$

Where F_X y F_Y are reaction forces in the bearings. These equations are highly nonlinear and even when known W and ϕ_t as functions of time. The method used to deal with this type of equations is linearized reaction forces of the bearings around its equilibrium position. Figure 2 depicts the effect produced by the load changes on the position of the bearing axis. The zero subscript refers to Figure steady state position and Δx , Δy indicate the displacements of the shaft about its equilibrium position or small displacements of disturbance. Calculating the change in these small perturbations with respect to time is obtained speeds disturbance $\Delta\dot{x}$ and $\Delta\dot{y}$ [3].

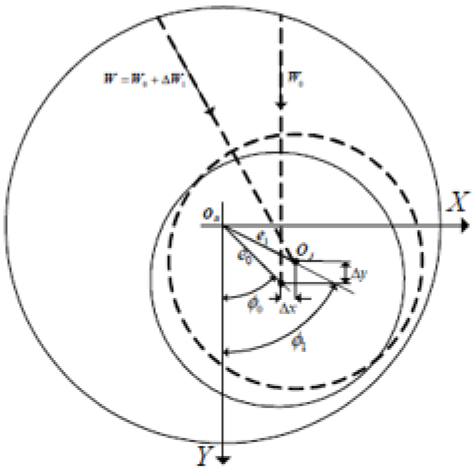

Figure 2 rotor position change made by disturbances.

The resulting reaction force F = W of Figure 2 has components F_X and F_Y. It made Taylor series expansions of the first order of these components is obtained:

$$F_X = (F_X)_0 + \left(\frac{\partial F_X}{\partial x}\right)_0 \Delta x + \left(\frac{\partial F_X}{\partial y}\right)_0 \Delta y + \left(\frac{\partial F_X}{\partial \dot{x}}\right)_0 \Delta\dot{x} + \left(\frac{\partial F_X}{\partial \dot{y}}\right)_0 \Delta\dot{y} \tag{6}$$

$$F_Y = (F_Y)_0 + \left(\frac{\partial F_Y}{\partial x}\right)_0 \Delta x + \left(\frac{\partial F_Y}{\partial y}\right)_0 \Delta y + \left(\frac{\partial F_Y}{\partial \dot{x}}\right)_0 \Delta\dot{x} + \left(\frac{\partial F_Y}{\partial \dot{y}}\right)_0 \Delta\dot{y} \tag{7}$$

The Y-axis direction is chosen such that $(F_X)_0 = 0$. Note that the partial derivatives of the forces with respect to the position and velocity, respectively represent the stiffness and damping in the lubricant film; then you can write:

$$k_{xx} = \left(\frac{\partial F_X}{\partial x}\right)_0 \qquad k_{xy} = \left(\frac{\partial F_X}{\partial y}\right)_0$$

$$k_{yx} = \left(\frac{\partial F_Y}{\partial x}\right)_0 \qquad k_{yy} = \left(\frac{\partial F_Y}{\partial y}\right)_0$$

$$c_{xx} = \left(\frac{\partial F_X}{\partial \dot{x}}\right)_0 \qquad c_{xy} = \left(\frac{\partial F_X}{\partial \dot{y}}\right)_0$$

$$c_{yx} = \left(\frac{\partial F_Y}{\partial \dot{x}}\right)_0 \qquad c_{yy} = \left(\frac{\partial F_Y}{\partial \dot{y}}\right)_0$$

This allows the equations (6) and (7) can be written as:

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$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = \begin{Bmatrix} 0 \\ (F_Y)_0 \end{Bmatrix} + \begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} + \begin{pmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} \begin{Bmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{Bmatrix} \quad (8)$$

Therefore, once the motion equations are linearized, they can easily solve the linearized after determining coefficients k_{ij} and c_{ij}.

In the Cartesian coordinate system, the forces in the lubricant film are written as [4]:

$$F_X = \int_{-L/2}^{L/2} \int_{\tilde{\theta}_1}^{\tilde{\theta}_2} pR \sin \tilde{\theta}\, d\tilde{\theta}\, dz \quad (9)$$

$$F_Y = \int_{-L/2}^{L/2} \int_{\tilde{\theta}_1}^{\tilde{\theta}_2} pR \cos \tilde{\theta}\, d\tilde{\theta}\, dz \quad (10)$$

In dimensionless form it can be written:

$$\overline{F}_X = \frac{F_X}{\mu NLD(R/C)^2} = \frac{1}{4} \int_{-1}^{1} \int_{\tilde{\theta}_1}^{\tilde{\theta}_2} \overline{p} \sin \tilde{\theta}\, d\tilde{\theta}\, dz \quad (11)$$

$$\overline{F}_Y = \frac{F_Y}{\mu NLD(R/C)^2} = \frac{1}{4} \int_{-1}^{1} \int_{\tilde{\theta}_1}^{\tilde{\theta}_2} \overline{p} \cos \tilde{\theta}\, d\tilde{\theta}\, dz \quad (12)$$

To enter the effect of disturbance, note that in Figure 2 the position of the stump and the steady state position when there is associated a small perturbation. This can be quantified as:

$$\begin{aligned} e_0 \sin \phi_0 + \Delta x &= e \sin \phi \\ e_0 \cos \phi_0 + \Delta y &= e \cos \phi \end{aligned} \quad (13)$$

Substituting the above expressions into equation film thickness (2) and whereas $\varepsilon = e/C$ and $h = H/C$ dimensionless film thickness is obtained.

$$h = h_0 + \Delta X \sin \tilde{\theta} + \Delta Y \cos \tilde{\theta} \quad (14)$$

Where:

$h_0 = 1 + \varepsilon_0 \cos(\tilde{\theta} - \phi_0)$, $X = x/C$ y $Y = y/C$.

It derives equation (14) with respect to time yields:

$$\frac{dh}{dt} = \omega(\Delta X' \sin \tilde{\theta} + \Delta Y' \cos \tilde{\theta}) \quad (15)$$

Note that $\tau = \omega t$ and raw indicate the change from τ. Thus, to calculate the equilibrium conditions and the coefficients of the lubricant film, it is necessary to solve the Reynolds equation (4), subject to the boundary conditions given and disturbing with (14) and (15).

General methodology

Now the description of the methodology used in solving the Reynolds equation is performed by disturbances. Since the analysis is linear, the pressure in the oil film can be expressed as:

$$p = (p)_0 + \left(\frac{\partial p}{\partial x}\right)_0 \Delta x + \left(\frac{\partial p}{\partial y}\right)_0 \Delta y + \left(\frac{\partial p}{\partial \dot{x}}\right)_0 \Delta \dot{x} + \left(\frac{\partial p}{\partial \dot{y}}\right)_0 \Delta \dot{y} \quad (16)$$

Doing:

$$(p)_0 = p_0, \quad \left(\frac{\partial p}{\partial x}\right)_0 = p_x, \quad \left(\frac{\partial p}{\partial y}\right)_0 = p_y, \quad \left(\frac{\partial p}{\partial \dot{x}}\right)_0 = p_{\dot{x}}, \quad \left(\frac{\partial p}{\partial \dot{y}}\right)_0 = p_{\dot{y}}$$

The components of force in the bearings are by integrating the pressure on the bearing area, so we can write:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = \int_z \int_{\tilde{\theta}} (p_0 + p_x \Delta x + p_y \Delta y + p_{\dot{x}} \Delta \dot{x} + p_{\dot{y}} \Delta \dot{y}) \begin{Bmatrix} \sin \tilde{\theta} \\ \cos \tilde{\theta} \end{Bmatrix} R_1 d\tilde{\theta}\, dz \quad (17)$$

The terms of disturbance Δx, Δy, $\Delta \dot{x}$ and $\Delta \dot{y}$ are independent of integration variables, therefore, for (17):

$$\begin{Bmatrix} 0 \\ (F_Y)_0 \end{Bmatrix} = \begin{Bmatrix} \int_z \int_{\tilde{\theta}} p_0 \sin \tilde{\theta}\, R_1 dz d\tilde{\theta} \\ \int_z \int_{\tilde{\theta}} p_0 \cos \tilde{\theta}\, R_1 dz d\tilde{\theta} \end{Bmatrix} \quad (18)$$

$$\begin{Bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{Bmatrix} = \begin{Bmatrix} \int_z\int_{\tilde\theta} p_x \sin\tilde\theta R_1 dz d\tilde\theta & \int_z\int_{\tilde\theta} p_y \sin\tilde\theta R_1 dz d\tilde\theta \\ \int_z\int_{\tilde\theta} p_x \cos\tilde\theta R_1 dz d\tilde\theta & \int_z\int_{\tilde\theta} p_y \cos\tilde\theta R_1 dz d\tilde\theta \end{Bmatrix} \tag{19}$$

$$\begin{Bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{Bmatrix} = \begin{Bmatrix} \int_z\int_{\tilde\theta} p_{\dot x} \sin\tilde\theta R_1 dz d\tilde\theta & \int_z\int_{\tilde\theta} p_{\dot y} \sin\tilde\theta R_1 dz d\tilde\theta \\ \int_z\int_{\tilde\theta} p_{\dot x} \cos\tilde\theta R_1 dz d\tilde\theta & \int_z\int_{\tilde\theta} p_{\dot y} \cos\tilde\theta R_1 dz d\tilde\theta \end{Bmatrix} \tag{20}$$

To determine the coefficients k_{ij} and c_{ij}, you need to get disturbances pressure field first. Substituting the equations of disturbed film (14), (15) and (16) in the Reynolds equation (4) yields:

$$\frac{\partial}{\partial\tilde\theta}\left[(h_0+\Delta X\sin\tilde\theta+\Delta Y\cos\tilde\theta)^3\frac{\partial}{\partial\tilde\theta}(\bar p_0+\bar p_X\Delta X+\bar p_Y\Delta Y+\bar p_{X'}\Delta X'+\bar p_{Y'}\Delta Y')\right]+$$
$$+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left[(h_0+\Delta X\sin\tilde\theta+\Delta Y\cos\tilde\theta)^3\frac{\partial}{\partial\bar z}(\bar p_0+\bar p_X\Delta X+\bar p_Y\Delta Y+\bar p_{X'}\Delta X'+\bar p_{Y'}\Delta Y')\right]$$
$$=12\pi\frac{\partial}{\partial\tilde\theta}(h_0+\Delta X\sin\tilde\theta+\Delta Y\cos\tilde\theta)+24\pi(\Delta X'\sin\tilde\theta+\Delta Y'\cos\tilde\theta) \tag{21}$$

Developing the term $(h_0+\Delta X\sin\tilde\theta+\Delta Y\cos\tilde\theta)^3$ and removed the higher-order terms, the Reynolds equation is obtained with only first-order terms.

$$\frac{\partial}{\partial\tilde\theta}\left[(h_0^3+3h_0^2\sin\tilde\theta\Delta X+3h_0^2\cos\tilde\theta\Delta Y)\frac{\partial}{\partial\tilde\theta}(\bar p_0+\bar p_X\Delta X+\bar p_Y\Delta Y+\bar p_{X'}\Delta X'+\bar p_Y\Delta Y')\right]+$$
$$+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left[(h_0^3+3h_0^2\sin\tilde\theta\Delta X+3h_0^2\cos\tilde\theta\Delta Y)\frac{\partial}{\partial\bar z}(\bar p_0+\bar p_X\Delta X+\bar p_Y\Delta Y+\bar p_{X'}\Delta X'+\bar p_{Y'}\Delta Y')\right]$$
$$=12\pi\frac{\partial}{\partial\tilde\theta}(h_0+\Delta X\sin\tilde\theta+\Delta Y\cos\tilde\theta)+24\pi(\Delta X'\sin\tilde\theta+\Delta Y'\cos\tilde\theta) \tag{22}$$

Bringing terms of the order is obtained the following set of equations:

$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_0}{\partial\bar z}\right)=12\pi\frac{\partial h_0}{\partial\tilde\theta}$$

$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_X}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_X}{\partial\bar z}\right)=$$
$$=12\pi\cos\tilde\theta-\frac{\partial}{\partial\tilde\theta}\left(3h_0^2\sin\tilde\theta\frac{\partial p_0}{\partial\tilde\theta}\right)-\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^2\sin\tilde\theta\frac{\partial\bar p_0}{\partial\bar z}\right)$$
$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_Y}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_Y}{\partial\bar z}\right)=$$
$$=-12\pi\sin\tilde\theta-\frac{\partial}{\partial\tilde\theta}\left(3h_0^2\cos\tilde\theta\frac{\partial p_0}{\partial\tilde\theta}\right)-\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^2\cos\tilde\theta\frac{\partial\bar p_0}{\partial\bar z}\right)$$
$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_{X'}}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_{X'}}{\partial\bar z}\right)=24\pi\sin\tilde\theta$$
$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_{Y'}}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_{Y'}}{\partial\bar z}\right)=24\pi\cos\tilde\theta \tag{23}$$

The second and third terms on the right side of the second of the above equations can be written as:

$$\frac{\partial}{\partial\tilde\theta}\left(3h_0^2\sin\tilde\theta\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^2\sin\tilde\theta\frac{\partial\bar p_0}{\partial\bar z}\right)$$
$$=\frac{\partial}{\partial\tilde\theta}\left(3h_0^3\frac{\sin\tilde\theta}{h_0}\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^3\frac{\sin\tilde\theta}{h_0}\frac{\partial\bar p_0}{\partial\bar z}\right)$$
$$=\frac{3\sin\tilde\theta}{h_0}\underbrace{\left[\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_0}{\partial\bar z}\right)\right]}_{12\pi\frac{\partial h_0}{\partial\tilde\theta}}+3h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\frac{\partial}{\partial\tilde\theta}\left(\frac{\sin\tilde\theta}{h_0}\right)$$

Therefore:

$$\frac{\partial}{\partial\tilde\theta}\left(3h_0^2\sin\tilde\theta\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^2\sin\tilde\theta\frac{\partial\bar p_0}{\partial\bar z}\right)=$$
$$=12\pi\frac{\partial h_0}{\partial\tilde\theta}\left(\frac{3\sin\tilde\theta}{h_0}\right)+3h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\frac{\partial}{\partial\tilde\theta}\left(\frac{\sin\tilde\theta}{h_0}\right)$$

Equal to the second and third terms on the right side of the third of the preceding equations:

$$\frac{\partial}{\partial\tilde\theta}\left(3h_0^2\cos\tilde\theta\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(3h_0^2\cos\tilde\theta\frac{\partial\bar p_0}{\partial\bar z}\right)=$$
$$=12\pi\frac{\partial h_0}{\partial\tilde\theta}\left(\frac{3\cos\tilde\theta}{h_0}\right)+3h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\frac{\partial}{\partial\tilde\theta}\left(\frac{\cos\tilde\theta}{h_0}\right)$$

Thus, the following set of equations is obtained for the pressure field and the steady state pressure field gradients.

$$\frac{\partial}{\partial\tilde\theta}\left(h_0^3\frac{\partial\bar p_0}{\partial\tilde\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar z}\left(h_0^3\frac{\partial\bar p_0}{\partial\bar z}\right)=12\pi\frac{\partial h_0}{\partial\tilde\theta}$$

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$$\frac{\partial}{\partial\tilde{\theta}}\left(h_0^3\frac{\partial\bar{p}_X}{\partial\tilde{\theta}}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_X}{\partial\bar{z}}\right)=$$
$$=12\pi\left(\cos\tilde{\theta}-\frac{3\sin\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)-3h_0^3\frac{\partial\bar{p}_0}{\partial\tilde{\theta}}\frac{\partial}{\partial\tilde{\theta}}\left(\frac{\sin\tilde{\theta}}{h_0}\right)$$
$$\frac{\partial}{\partial\tilde{\theta}}\left(h_0^3\frac{\partial\bar{p}_Y}{\partial\tilde{\theta}}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_Y}{\partial\bar{z}}\right)=$$
$$=-12\pi\left(\sin\tilde{\theta}+\frac{3\cos\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)-3h_0^3\frac{\partial\bar{p}_0}{\partial\tilde{\theta}}\frac{\partial}{\partial\tilde{\theta}}\left(\frac{\cos\tilde{\theta}}{h_0}\right)$$
$$\frac{\partial}{\partial\tilde{\theta}}\left(h_0^3\frac{\partial\bar{p}_{X'}}{\partial\tilde{\theta}}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_{X'}}{\partial\bar{z}}\right)=24\pi\sin\tilde{\theta}$$
$$\frac{\partial}{\partial\tilde{\theta}}\left(h_0^3\frac{\partial\bar{p}_{Y'}}{\partial\tilde{\theta}}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_{Y'}}{\partial\bar{z}}\right)=24\pi\cos\tilde{\theta} \quad (24)$$

The boundary conditions associated with the system of equations given by (24) will be:

$p_l = 0$ in $z = \pm L/2$; where $l = 0, x, y, \dot{x}, \dot{y}$

$\frac{\partial p_l}{\partial z} = 0$ in $z = 0$; where $l = 0, x, y, \dot{x}, \dot{y}$

Importantly for the aerodynamic coefficients through gradients p_x, p_y, $p_{\dot{x}}$ and $p_{\dot{y}}$, you need to calculate the first steady-state pressure p_0. The accuracy when calculating p_0 lead to the accuracy of the values of the dynamic coefficients.

Short Chumacera Case Analysis

To verify that the pressure field disturbances lead to correct numeric results dynamic coefficients for the case of infinitely short bearings are calculated.

Consider the two assumptions for short bearings made by Dubios and Ocvirk [5]. First, the pressure gradients in the x or θ are negligible when compared to the pressure gradients in the direction z (axial direction). Second, only the pressure in the convergent region clear ($0 < \theta < \pi$) It is considered for evaluating forces lubricant film.

Therefore, in the case of short bearings, equations (24) reduce to the following expressions. $\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_0}{\partial\bar{z}}\right)=12\pi\frac{\partial h_0}{\partial\tilde{\theta}}$

$$\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_X}{\partial\bar{z}}\right)=12\pi\left(\cos\tilde{\theta}-\frac{3\sin\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)$$
$$\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_Y}{\partial\bar{z}}\right)=-12\pi\left(\sin\tilde{\theta}+\frac{3\cos\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)$$
$$\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_{X'}}{\partial\bar{z}}\right)=24\pi\sin\tilde{\theta}$$
$$\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(h_0^3\frac{\partial\bar{p}_{Y'}}{\partial\bar{z}}\right)=24\pi\cos\tilde{\theta} \quad (25)$$

Note that the right side of the above equations is independent of the steady-state pressure p_0, as if it was in the system (24) for general bearings.

Integrating twice each of these equations and boundary conditions using established:

$$\bar{p}_0=\frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2\left[12\pi\frac{\partial h_0}{\partial\tilde{\theta}}\right]$$
$$\bar{p}_X=\frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2\left[12\pi\left(\cos\tilde{\theta}-\frac{3\sin\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]$$
$$\bar{p}_Y=\frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2\left[-12\pi\left(\sin\tilde{\theta}+\frac{3\cos\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]$$
$$\bar{p}_{X'}=\frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2\left[24\pi\sin\tilde{\theta}\right] \quad (26)$$
$$\bar{p}_{Y'}=\frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2\left[24\pi\cos\tilde{\theta}\right]$$

Known p_0 and pressure gradients, the reaction forces and dynamic coefficients are calculated. In dimensionless form can be written:

$$\bar{F}_x=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_0\sin\tilde{\theta}\,d\theta\,d\bar{z} \qquad \bar{F}_y=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_0\cos\tilde{\theta}\,d\theta\,d\bar{z}$$
$$\bar{k}_{xx}=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_X\sin\tilde{\theta}\,d\theta\,d\bar{z} \qquad \bar{k}_{xy}=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_Y\sin\tilde{\theta}\,d\theta\,d\bar{z}$$
$$\bar{k}_{yx}=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_X\cos\tilde{\theta}\,d\theta\,d\bar{z} \qquad \bar{k}_{yy}=\frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi}\bar{p}_Y\cos\tilde{\theta}\,d\theta\,d\bar{z}$$

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$$\bar{c}_{xx} = \frac{\omega}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \bar{p}_{X'} \sin\tilde{\theta}\, d\theta d\bar{z} \quad \bar{c}_{xy} = \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \bar{p}_{Y'} \sin\tilde{\theta}\, d\theta d\bar{z}$$

$$\bar{c}_{yx} = \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \bar{p}_{X'} \cos\tilde{\theta}\, d\theta d\bar{z} \quad \bar{c}_{yy} = \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \bar{p}_{Y'} \cos\tilde{\theta}\, d\theta d\bar{z}$$

Results

Evaluating each of these integrals it is obtained for the forces in the lubricant:

$$\begin{aligned}\bar{F}_X &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 12\pi \frac{\partial h_0}{\partial\tilde{\theta}} \sin\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= -2\pi\left(\frac{L}{D}\right)^2 \int_{\phi}^{\phi+\pi} \frac{\partial h_0}{\partial\tilde{\theta}} \frac{\sin\tilde{\theta}}{h_0^3} d\tilde{\theta} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \int_0^{\pi} \frac{\varepsilon \sin\theta}{(1+\varepsilon\cos\theta)^3} \sin(\theta+\phi) d\theta \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi\varepsilon}{2(1-\varepsilon^2)^{3/2}}\cos\phi - \frac{2\varepsilon^2}{(1-\varepsilon^2)^2}\sin\phi\right] = 0\end{aligned} \quad (27)$$

$$\begin{aligned}\bar{F}_Y &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 12\pi \frac{\partial h_0}{\partial\tilde{\theta}} \cos\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{-2\varepsilon^2}{(1-\varepsilon^2)^2}\cos\phi - \frac{\pi\varepsilon}{2(1-\varepsilon^2)^{3/2}}\sin\phi\right] \\ &= \frac{-\varepsilon\pi}{(1-\varepsilon^2)^2}\left(\frac{L}{D}\right)^2 \sqrt{16\varepsilon^2+\pi^2(1-\varepsilon^2)}\end{aligned} \quad (28)$$

Dynamic stiffness coefficients are obtained as follows:

$$\begin{aligned}\bar{k}_{xx} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[12\pi\left(\cos\tilde{\theta} - \frac{3\sin\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]\sin\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{2\varepsilon}{(1-\varepsilon^2)^2}\cos^2\phi - \frac{3\pi\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\sin\phi\cos\phi + \frac{4\varepsilon(1+\varepsilon^2)}{(1-\varepsilon^2)^3}\sin^2\phi\right] \\ &= \pi\left(\frac{L}{D}\right)^2 \frac{4\varepsilon}{(1-\varepsilon^2)^2}\frac{2\pi^2+(16-\pi^2)\varepsilon^2}{16\varepsilon^2+\pi^2(1-\varepsilon^2)}\end{aligned}$$

(29)

$$\begin{aligned}\bar{k}_{xy} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[-12\pi\left(\sin\tilde{\theta} + \frac{3\cos\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]\sin\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{-\pi(1+2\varepsilon^2)}{2(1-\varepsilon^2)^{5/2}}\cos^2\phi + \frac{2\varepsilon(1+3\varepsilon^2)}{(1-\varepsilon^2)^3}\sin\phi\cos\phi - \frac{\pi}{2(1-\varepsilon^2)^{3/2}}\sin^2\phi\right] \\ &= \pi\left(\frac{L}{D}\right)^2 \frac{1}{(1-\varepsilon^2)^{5/2}}\frac{\pi[-\pi^2+2\pi^2\varepsilon^2+(16-\pi^2)\varepsilon^4]}{16\varepsilon^2+\pi^2(1-\varepsilon^2)}\end{aligned}$$

(30)

$$\begin{aligned}\bar{k}_{yx} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[12\pi\left(\cos\tilde{\theta} - \frac{3\sin\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]\cos\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi}{2(1-\varepsilon^2)^{3/2}}\cos^2\phi + \frac{2\varepsilon(1+3\varepsilon^2)}{(1-\varepsilon^2)^3}\sin\phi\cos\phi + \frac{\pi(1+2\varepsilon^2)}{2(1-\varepsilon^2)^{5/2}}\sin^2\phi\right] \\ &= 2\pi\left(\frac{L}{D}\right)^2 \frac{\pi[\pi^2+(32+\pi^2)\varepsilon^2+2(16-\pi^2)\varepsilon^4]}{2(1-\varepsilon^2)^{5/2}[16\varepsilon^2+\pi^2(1-\varepsilon^2)]}\end{aligned}$$

(31)

$$\begin{aligned}\bar{k}_{yy} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[-12\pi\left(\sin\tilde{\theta} + \frac{3\cos\tilde{\theta}}{h_0}\frac{\partial h_0}{\partial\tilde{\theta}}\right)\right]\cos\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{4\varepsilon(1+\varepsilon^2)}{(1-\varepsilon^2)^3}\cos^2\phi + \frac{3\pi\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\sin\phi\cos\phi + \frac{2\varepsilon}{(1-\varepsilon^2)^2}\sin^2\phi\right] \\ &= \pi\left(\frac{L}{D}\right)^2 \frac{4\varepsilon[\pi^2+(32+\pi^2)\varepsilon^2+2(16-\pi^2)\varepsilon^4]}{(1-\varepsilon^2)^3[16\varepsilon^2+\pi^2(1-\varepsilon^2)]}\end{aligned}$$

(32)

Damping coefficients are:

$$\begin{aligned}\bar{c}_{xx} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[24\pi\sin\tilde{\theta}\right]\sin\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 4\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi}{2(1-\varepsilon^2)^{3/2}}\cos^2\phi - \frac{4\varepsilon}{(1-\varepsilon^2)^2}\sin\phi\cos\phi + \frac{\pi(1+2\varepsilon^2)}{2(1-\varepsilon^2)^{5/2}}\sin^2\phi\right] \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi[\pi^2+2(\pi^2-8)\varepsilon^2]}{(1-\varepsilon^2)^{3/2}[16\varepsilon^2+\pi^2(1-\varepsilon^2)]}\right]\end{aligned}$$

(33)

$$\begin{aligned}\bar{c}_{xy} = \bar{c}_{yx} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[24\pi\cos\tilde{\theta}\right]\sin\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 4\pi\left(\frac{L}{D}\right)^2 \left[\frac{-2\varepsilon}{(1-\varepsilon^2)^2}\cos^2\phi + \frac{3\pi\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\sin\phi\cos\phi + \frac{2\varepsilon}{(1-\varepsilon^2)^2}\sin^2\phi\right] \\ &= \pi\left(\frac{L}{D}\right)^2 \left[\frac{8\varepsilon[\pi^2+2(\pi^2-8)\varepsilon^2]}{(1-\varepsilon^2)^2[16\varepsilon^2+\pi^2(1-\varepsilon^2)]}\right]\end{aligned}$$

(34)

$$\begin{aligned}\bar{c}_{yy} &= \frac{1}{4}\int_{-1}^{1}\int_{\phi}^{\phi+\pi} \frac{(\bar{z}^2-1)}{2h_0^3}\left(\frac{L}{D}\right)^2 \left[24\pi\cos\tilde{\theta}\right]\cos\tilde{\theta}\, d\tilde{\theta} d\bar{z} \\ &= 4\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi(1+2\varepsilon^2)}{2(1-\varepsilon^2)^{5/2}}\cos^2\phi + \frac{4\varepsilon}{(1-\varepsilon^2)^2}\sin\phi\cos\phi + \frac{\pi}{2(1-\varepsilon^2)^{3/2}}\sin^2\phi\right] \\ &= 2\pi\left(\frac{L}{D}\right)^2 \left[\frac{\pi[\pi^2+2(24-\pi^2)\varepsilon^2+\pi^2\varepsilon^4]}{(1-\varepsilon^2)^{5/2}[16\varepsilon^2+\pi^2(1-\varepsilon^2)]}\right]\end{aligned}$$

(35)

Tables 1 and 2 show these coefficients in the form $\tilde{k}_{ij} = (C/W)k_{ij}$ y $\tilde{c}_{ij} = (\omega C/W)c_{ij}$.

$\tilde{k}_{xx} = \dfrac{4[2\pi^2 + (16-\pi^2)\varepsilon^2]}{[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{k}_{xy} = \dfrac{\pi[-\pi^2 + 2\pi^2\varepsilon^2 + (16-\pi^2)\varepsilon^4]}{\varepsilon\sqrt{1-\varepsilon^2}[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{k}_{yx} = \dfrac{\pi[\pi^2 + (32+\pi^2)\varepsilon^2 + 2(16-\pi^2)\varepsilon^4]}{\varepsilon\sqrt{1-\varepsilon^2}[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{k}_{yy} = \dfrac{4[\pi^2 + (32+\pi^2)\varepsilon^2 + 2(16-\pi^2)\varepsilon^4]}{(1-\varepsilon^2)[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$

Table 1 Dynamic stiffness coefficients for a short bearing.

$\tilde{c}_{xx} = \dfrac{2\pi(1-\varepsilon^2)^{1/2}[\pi^2 + 2(\pi^2-8)\varepsilon^2]}{\varepsilon[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{c}_{xy} = \dfrac{8[\pi^2 + 2(\pi^2-8)\varepsilon^2]}{[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{c}_{yx} = \dfrac{8[\pi^2 + 2(\pi^2-8)\varepsilon^2]}{[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$
$\tilde{c}_{yy} = \dfrac{2\pi[\pi^2 + 2(24-\pi^2)\varepsilon^2 + \pi^2\varepsilon^4]}{\varepsilon\sqrt{1-\varepsilon^2}[\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}}$

Table 2 Dynamic damping coefficients for a short bearing.

You can view the behavior of these coefficients as a function of the eccentricity of balance. In Figures 3 and 4 the damping and stiffness variations appear; the dashed curves correspond to negative values of the coefficient.

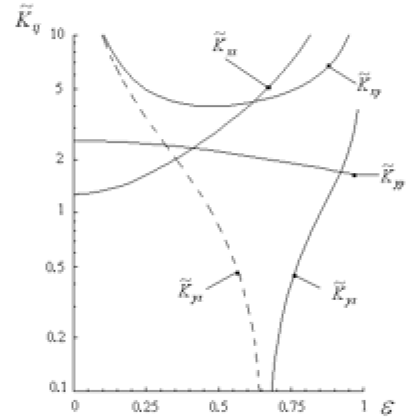

Figure 3 aerodynamic coefficients for a short Chumacera stiffness.

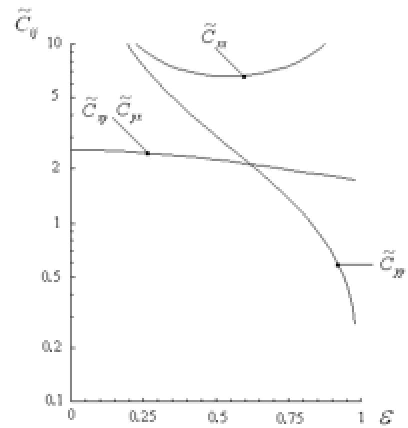

Figure 4 aerodynamic damping coefficients for a short Chumacera.

As noted, the coefficients obtained for the classical case are the same as they find alternatives [3] methodologies [4], [10]; meaning that the alternate method produces consistent results.

Conclusions

After showing the validity of this alternative solution, you might expect the perturbation technique can be used in more complex analysis that may involve external excitations in the bearings, the eternal problem of misalignment in the stands, and an option to open external pressurization ports lubricant.

Currently there are rough on some of these issues results but the perturbation technique can be adapted to become a numerical-analytical methodology to find rotordynamic coefficients as a function of misalignment and external pressurization bearings of any length.

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